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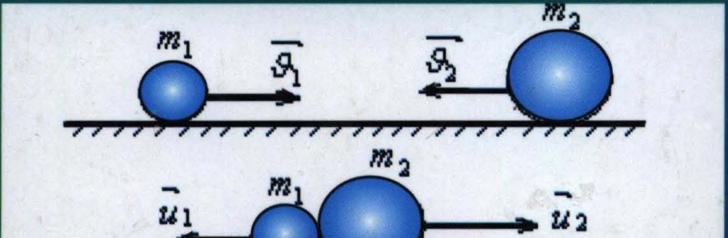

O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

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FIZIKADAN MASALALAR YECHISH

O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi tomonidan oliy o'quv yurtlarining bakalavriat ta'lim yo'nalishi talabalari uchun o'quv qo'llanma sifatida tavsiya etilgan

O'ZBEKISTON FAYLASUFLARI
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1. M.T.Normurodov

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Qo'llanmada 300 dan ortiq masalalarning original yechimlari ko'rsatilgan. Mustaqil yechish uchun 400 ta masala keltirilgan. Shuningdek, fizik kattaliklarning jadvallari, fizik kattaliklarning o'lchamlari, birliklar sistemasi, masala yechish uchun ko'rsatmalar ham mavjud.

Har bir mavzudan oldin qisqacha nazariy qism bayon qilingan, so'ngra esa oldin soddaroq, keyin esa murakkabroq masalalarning yechilishi bayon qilingan. Masala yechilishidan oldin ishchi formula topilgan, so'ngra esa uning to'g'riligi birliklar yordamida tekshirib ko'rsatilgan.

Qo'llanma oliy o'quv yurtlarining texnika yo'nalishlari uchun o'quv qo'llanma sifatida tavsiya etilmoqda.

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Fizikadan masalalar yechish uchun ko'rsatmalar

Fizikadan masalalar yechishda quyidagi ko'rsatmalarga rioya qilish maqsadga muvofiq:

— Masalani yechishga kirishishdan oldin, uning ma'nosiga, bayon qilinayotgan jarayonga yaxshilab tushunib olmoq kerak. Buning uchun masala shartini bir necha marta qayta-qayta o'qish, tahlil qilish, iloji boricha chizma yordamida tasvirlab olish kerak;

— Masala shartida berilgan kattaliklarning son qiymatlarini SI birliklar sistemasiga keltirish, zarur bo'lgan o'zgarmaslarning qiymatlarini esa jadvallardan olish kerak;

— Masalani umumiy holda yechib, ishchi formulani topish, ya'ni fizik qonunlar yordamida, topilishi kerak bo'lgan kattalikni berilgan kattaliklar orqali ifodalash kerak. Bu birinchidan, masalada bayon qilingan jarayonning fizik qonunlarni tushunishga yordam bersa, ikkinchidan ortiqcha oraliq hisob-kitoblarga zarurat qoldirmaydi;

— Hosil qilingan ishchi formula yordamida kattalikning o'lchamligini, ya'ni so'ralgan kattalikning birligi hosil bo'lishini tekshirib ko'rish kerak. Kattalik birligining noto'g'ri chiqishi masalaning noto'g'ri yechilganligini ko'rsatadi;

Masala yechishda foydalaniladigan fizik kattaliklarning son qiymatlari taxminiydir. Taxminiy sonlar bilan ishlashda esa ehtiyot bo'lish kerak. Aks holda juda ko'p vaqt behuda sarflanishi mumkin. Masalan, kattaliklardan birini o'ndan birgacha, boshqasini esa yuzdan birgacha aniqlikda olib, javobni yanada kattaroq aniqlikda hisoblash mutlaqo ma'noga ega emas. Chunki verguldan keyingi ikkinchi raqamdan boshlab natija shubhali va ularni aniqlash uchun qilingan mehnat befoydadir. Bunga yo'l qo'ymaslik uchun berilganlarning hammasini bir xil aniqlikka keltirish va undan keyingina hisob-kitobga kirishish kerak.

Fizik kattaliklarning o‘lchamliklari. Birliklar sistemasi

Akademik A.F.Ioffe iborasi bilan aytganda, *fizika — moddalar va maydonlarning umumiy xossalari va harakat qonunlarini o‘rganadigan fandir.*

Fizika fanining asosiy tekshirish uslubi — tajribadir. Tajribalar natijasini tushuntirish, asoslash maqsadida ilmiy nazariyalar yaratiladi. Bularning hammasi tabiatda mavjud bo‘lgan obyektiv qonunlarning ya’ni fizikaviy qonunlarning yaratilishiga olib keladi. Fizika qonunlari — fizik kattaliklar orasidagi munosabatlarni aniqlaydi.

Fizik kattalik deb — *miqdor jihatdan har bir fizik obyekt uchun xususiy, lekin sifat jihatidan ko‘plab obyektlar uchun umumiy bo‘lgan va bu obyektlarning biror xossasini ifodalovchi kattalikka aytiladi.*

Fizik kattalikni miqdor va sifat jihatdan to‘la ifodalaydigan kattalikka uning haqiqiy qiymati deyiladi va u doimo takomillashtirilib boriladigan tajribalar yordamida aniqlanadi.

Fizik kattaliklar sistemasi asosiy va hosilaviy kattaliklardan iboratdir. Asosiy fizik kattaliklar sifatida moddiy dunyoning asosiy xossalarini ifodalovchi: uzunlik, massa, vaqt kabi birliklar qabul qilingan. Qolgan to‘rtta asosiy kattaliklarning har biri fizikaning biror bo‘limidan olingan. Bunday kattaliklar tok kuchi, termodinamik harorat, modda miqdori va yorug‘lik kuchidir. Har bir asosiy kattalikka ramz sifatida lotin yoki grek alifbosining biror bosh harfi qabul qilingan. Bu ramzlar quyidagicha: uzunlik — *L*; massa — *M;* vaqt — *T;* elektr tokining kuchi — *I;* harorat — Θ; modda miqdori — ν ; yorug‘lik kuchi — *J*.

Fizik kattalikning o‘lchamligi deb, *uning asosiy fizik kattaliklari ramzlarining turli darajalari va proporsionallik koeffitsientlari orqali ifodasiga aytiladi.* Asosiy fizik kattalikning o‘ziga nisbatan o‘lchamligi birga teng va boshqa kattaliklarga bog‘liq emas. Ya’ni asosiy fizik kattalik o‘lchamligi uning ramzi bilan mos keladi. Masalan, uzunlikning o‘lchamligi — L, massaning o‘lchamligi — M va hokazo.

Kattalikning o‘lchamligi dim (inglizcha dimension — o‘lchamlik so‘zidan) bilan belgilanadi. Misol uchun: dim l = L.

Hosilaviy kattalikning o‘lchamligini topish uchun bu kattalikni aniqlovchi tenglamaning o‘ng tomonidagi kattaliklar belgilari o‘rniga ularning o‘lchamlari qo‘yiladi. Masalan, tekis harakat tezligi $V=S/t$ da, *S* ning o‘rniga uzunlik o‘lchamligi L, t ning o‘rniga vaqt o‘lchamligi T ni qo‘yib olamiz. $\dim V = L/T = LT^{-1}$.

Fizik kattaliklarning son qiymati, uning kattaligini ko'rsatuvchi son bo'lib, kattalikning tanlangan birligiga bog'liqdir. *Fizik kattalikning birligi deb, bir xil fizik kattalikni miqdoriy ifodalash uchun qo'llaniladigan, shartli ravishda son qiymati 1 ga teng deb belgilangan o'lchamli fizik kattaliklarga aytiladi.* Fizik kattalikning birligi, shu fizik kattalikning o'zidek kattalikdir.

Fizik kattaliklarning birliklar sistemasi (birliklar sistemasi) – *fizik kattaliklarning* yuqorida keltirilgan prinsipga asosan tuzilgan, sistema uchun hosil qilingan *asosiy va hosilaviy birliklarning majmuasidir.*

Mol (mol) – tarkibiy elementlari, 0,012 kg massali ^{12}C nuklidda mavjud bo'lgan tarkibiy elementlarga teng sistemaning modda miqdori.

Kelvin (K) – suv uchlamchi nuqtasi termodinamik haroratining 273,15 dan bir qismi.

Kandela (Kd) – $540\cdot10^{12}$ Hz chastotali monoxromatik nurlanish chiqaradigan manbaning, energetik kuchi $\frac{1}{683}\ \frac{W}{sr}$ bo'lgan yo'nalishdagi yorug'lik kuchi.

Radian (rad) – qarshisidagi yoyining uzunligi aylananing radiusiga teng bo'lgan ikkita radius orasidagi burchak.

Steradian (sr) – sfera sirtidan, tomoni, sfera radiusidek bo'lgan kvadratning yuzasiga teng yuzani ajratuvchi, uchi sfera markazida bo'lgan fazoviy burchak.

Hosilaviy birliklar fizikaviy qonunlardan foydalanib topiladi. Quyida asosiy va hosilaviy kattaliklarning o'lchamlari va birliklariga misollar keltiramiz:

Uzunlik:

$$\dim l = L;\ [l] = 1m$$

Tekis harakat tezligi: $v = \frac{S}{t}$;

$$\dim\ v = \frac{L}{T} = LT^{-1};\ [v] = \frac{[S]}{[t]} = \frac{1m}{1s} = 1\frac{m}{s}.$$

Nyutonning ikkinchi qonuni:

$$\dim F = MLT^{-2};\ [F] = [m][a] = 1kg\cdot1\frac{m}{s^2} = 1kg\cdot\frac{m}{s^2} = 1N.$$

I BOB. MEXANIKANING FIZIK ASOSLARI

1-§. Kinematika

Asosiy formulalar

Moddiy nuqta ilgarilanma harakatining kinematik tenglamasi

$$r=f(t),$$

yoki koordinatali shaklda

$$x=f_1(t);\ y=f_2(t);\ z=f_3(t)$$

bunda: t — vaqt.

O'rtacha tezlik

$$<\vec{V}>=\frac{\Delta\vec{r}}{\Delta t},\quad <v>=\frac{\Delta S}{\Delta t}$$

bunda: $\Delta\vec{r}$ — fazodagi o'rni $\vec{r}=\vec{i}x+\vec{j}y+\vec{k}z$ radius vektor bilan aniqlanuvchi nuqtaning Δt vaqtdagi ko'chishi, ΔS — o'tgan yo'li. ($\vec{i}$, $\vec{j}$, $\vec{k}$ — to'g'ri burchakli koordinatalar sistemasidagi birlik vektorlar (ortlar)).

Oniy tezlik

$$\vec{V}=\frac{d\vec{r}}{dt}=\vec{i}\,v_x+\vec{j}\,v_y+\vec{k}\,v_z=\vec{i}\,\frac{dx}{dt}+\vec{j}\,\frac{dy}{dt}+\vec{k}\,\frac{dz}{dt}.$$

Tezlik moduli

$$v=\sqrt{v_x^2+v_y^2+v_z^2}\,.$$

O'rtacha tezlanish

$$<\vec{a}>=\frac{\Delta\vec{V}}{\Delta t},\quad <a>=\frac{\Delta v}{\Delta t}.$$

Bunda, $\Delta\vec{V}$ — tezlik vektorining Δt vaqtdagi orttirmasi.

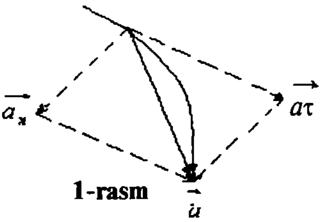
Oniy tezlanish

$$\vec{a}=\frac{d\vec{v}}{dt}=\vec{i}\,a_x+\vec{j}a_y+\vec{k}a_z=\vec{i}\,\frac{dv_x}{dt}+\vec{j}\,\frac{d\vec{v}_y}{dy}+\vec{k}\,\frac{dv_z}{dt}.$$

Tezlanish moduli

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} .$$

Egri chiziqli harakatda to‘la tezlanish (1-rasm)



1-rasm

$$\vec{a} = \vec{a_n} + \vec{a_\tau} ,$$

bu yerda

$$a_n = \frac{v^2}{R}; \quad a_\tau = \frac{dv}{dt}$$

– tezlanishning normal va tangensial tashkil etuvchilari,

$$a = \sqrt{a_n^2 + a_\tau^2} ,$$

bunda: R – trayektoriyaning shu nuqtasi uchun egrilik radiusi.

Moddiy nuqta aylanma harakatining kinematik tenglamasi:

$$\varphi = f(t) .$$

O‘rtacha va oniy burchak tezlik

$$< w >= \frac{\Delta\varphi}{\Delta t}, \quad w = \frac{d\varphi}{dt} ,$$

bunda $\Delta\varphi$ – moddiy nuqtaning Δt vaqt intervalida burilish burchagi.

O‘rtacha va oniy burchak tezlanish

$$< \varepsilon >= \frac{\Delta w}{\Delta t}, \quad \varepsilon = \frac{dw}{dt} ,$$

bunda: Δw – moddiy nuqta burchak tezligining Δt vaqt intervalida o‘zgarishi.

Tekis harakatning kinematik tenglamasi:

a) ilgarilanma (υ =const, a=0)

$$\vec{S} = \vec{S}_0 + vt ;$$

b) aylanma (ω =const, ε=0

$$\varphi = \varphi_0 + \omega t ,$$

bunda: S_0 – boshlang‘ich yo‘l, φ_0 – boshlang‘ich burchak.

Tekis o'zgaruvchan harakatning kinematik tenglamasi:
a) ilgarilanma (a=const)

$$S = \vartheta_0 t + \frac{at^2}{2}, \quad \vartheta = \vartheta_0 + at;$$

b) aylanma (ε =const)

$$\varphi = \omega_0 t + \frac{\varepsilon t^2}{2}, \omega = \omega_0 + \varepsilon t,$$

bunda: ϑ_0 va ω_0 – boshlang'ich chiziqli va burchakli tezliklar

Aylana trayektoriya bo'ylab harakatlanayotgan moddiy nuqta kinematikasining chiziqli va burchakli kattaliklari orasidagi munosabatlar: nuqtaning bosib o'tgan yo'li S va burilish burchagi φ orasidagi bog'lanish

$$S = \varphi R;$$

nuqtaning chiziqli tezligi

$$\vartheta = \omega R; \vec{V} = [\vec{\omega} \cdot \vec{R}].$$

Chiziqli tezlanish:
tangensial

$$a_\tau = \varepsilon R; \quad \vec{a}_\tau = [\vec{\varepsilon} \cdot \vec{R}];$$

normal

$$a_n = \omega^2 R; \quad \vec{a}_n = \omega^2 \vec{R} \quad .$$

Masala yechishga misollar

1-misol. To'g'ri chiziqli harakat tenglamasi $x = At + Bt^2$ ko'rinishga ega. Bu yerda $\mathrm{A} = 3\frac{\mathrm{m}}{\mathrm{s}}$, $\mathrm{B} = -0{,}25\frac{\mathrm{m}}{\mathrm{s}^2}$. Berilgan harakat uchun koordinataning va yo'lning vaqtga bog'lanish grafiklari tuzilsin.

Berilgan:
$x = At + Bt^2$;
$A = 3\frac{m}{s}$;
$B = -0{,}25\frac{m}{s^2}$,

$x(t) = ?$
$S(t) = ?$

Yechish: t ning turli qiymatlari uchun koordinataning qiymatlarini topib quyidagi jadvalga yozamiz:

Vaqt, s	0	1	2	4	6	8	10	11	12
Koordinata, m	0	2,75	5	8	9	8	5	2,75	0

Yo'l grafigini quyidagi mulohazalarga asoslanib tuzamiz.

1) tezlikning ishorasi o'zgarguncha koordinata va yo'l mos keladi. Tabiiyki, bu onda tezlik nolga teng bo'ladi, ya'ni:

$$\vartheta = \frac{dx}{dt} = 0,$$

$$\vartheta = \frac{d}{dt}(At + Bt^2) = A + 2Bt = 0$$

Bundan tezlik ishorasini o'zgartirish vaqtini topamiz:

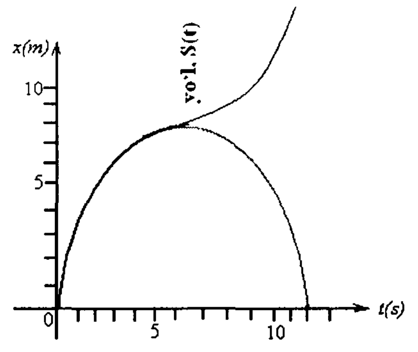
$$t = -\frac{A}{2B}.$$

Berilganlardan foydalansak, $t = -\frac{3}{-2 \cdot 0{,}25}$s=6s.

Demak, 6-sekundgacha yo'l koordinata bilan mos keladi.

2) $t = 6$ s. dan boshlab nuqta teskari yo'nalishda harakatlanadi. Natijada uning koordinatasi kamayib, o'tgan yo'li shu qonun bilan ortib boradi, yo'l grafigi esa koordinata grafigining ko'zgudagi tasviridek bo'ladi. (2-rasm)

2-misol. Moddiy nuqta ilgarilanma harakatining kinematik tenglamasi $S = At^4 + Bt^2 + C$ ko'rinishiga ega. Nuqtaning ikkinchi sekunddagi tezligi, tezlanishi, hamda dastlabki 2 s uchun o'rtacha tezlik topilsin. $A=4m/s^2$, $B=2m/s^2$.



2-rasm

Berilgan:

$S = At^4 + Bt^2 + C$

A=4m/s^4;

B =2m/s^2;

$t = 2s$;

$\Delta t = t - t_0 = 2s$.

$< \vartheta >= ?$

$\vartheta = ?$

$a = ?$

Yechish:

Moddiy nuqtaning o'rtacha tezligi

$$< \vartheta >= \frac{\Delta S}{\Delta t} \quad (1)$$

ifoda yordamida aniqlanadi.

Bu yerda:

$\Delta S = S(2s) - S(0),$

$\Delta t = t - t_0 = 2s - 0 = 2s.$

Unda (1) quyidagi ko'rinishni oladi:

$$< \vartheta >= \frac{\Delta S}{\Delta t} = \frac{S(2s) - S(o)}{\Delta t}. \quad (2)$$

Moddiy nuqtaning oniy tezligi:

$$\vartheta = \frac{ds}{dt} = 4At^3 + 2Bt. \quad (3)$$

Oniy tezlanishi:

$$a = \frac{d\vartheta}{dt} = 12At^2 + 2B. \quad (4)$$

(2) yoki (3) va (4) formulalar yordamida so'ralgan kattaliklarning birliklarini tekshiramiz:

$$[\vartheta]=\frac{[S]}{[t]}=\frac{1\text{m}}{1\text{s}}=1\frac{\text{m}}{\text{s}},$$

$$[a]=\frac{[\vartheta]}{[t]}=\frac{1\frac{\text{m}}{\text{s}}}{1\text{s}}=1\frac{\text{m}}{\text{s}^2}$$

va ularning to‘g‘riligiga ishonch hosil qilamiz. Berilganlarni yuqoridagi formulalarga qo‘yib quyidagini olamiz:

$$<\vartheta>=\frac{(4\cdot 2^4+2\cdot 2^2+C)\text{m}-(4\cdot 0+2\cdot 0+C)\text{m}}{2\text{s}}=\frac{72\text{m}}{2\text{s}}=36\frac{\text{m}}{\text{s}};$$

$$\vartheta=(4\cdot 4\cdot 2^3+2\cdot 2\cdot 2)\frac{\text{m}}{\text{s}}=136\frac{\text{m}}{\text{s}};$$

$$a=(12\cdot 4\cdot 2^2+2\cdot 2)\frac{\text{m}}{\text{s}^2}=196\frac{\text{m}}{\text{s}^2}.$$

Javob: $<\vartheta>=36\frac{\text{m}}{\text{s}}$; $\vartheta=136\frac{\text{m}}{\text{s}}$; $a=196\frac{\text{m}}{\text{s}^2}$.

3-misol. Odam poyezd bilan yonma-yon, poyezdning oldingi qalqonlari bilan bir chiziqda turibdi. Poyezd $0{,}1\frac{\text{m}}{\text{s}^2}$, tezlanish bilan harakat qilib boshlagan paytda, odam ham shu yo‘nalishda $1{,}5\frac{\text{m}}{\text{s}}$, o‘zgarmas tezlik bilan harakatlana boshladi. Qancha vaqtdan keyin poyezd odamga yetib oladi? Poyezdning shu ondagi tezligi ϑ_P va odamning bu vaqt ichida o‘tgan yo‘li S_{od} aniqlansin.

Berilgan:	**Yechish:**
$\vartheta_0=0$;	Masalaning shartiga ko‘ra t vaqtda
$a=0{,}1\frac{\text{m}}{\text{s}^2}$;	poyezd va odam o‘tgan
$\vartheta_{od}=0{,}1\frac{\text{m}}{\text{s}}$;	yo‘l teng bo‘ladi, ya’ni

$t = ?$

$\vartheta_p = ?$

$S_{od} = ?$

$$S_p = S_{od}. \qquad (1)$$

Bunda tezlanuvchan harakat qilayotgan poyezdning bosib o'tgan yo'li va tezligi:

$$S_p = \vartheta_o t + \frac{at^2}{2}, \quad \vartheta_p = V_0 + at.$$

Agar poyezdning boshlang'ich tezligi nolga tengligini ($V_0 = 0$) nazarda tutsak,

$$S_p = \frac{at^2}{2}, \quad \vartheta_p = at. \qquad (2)$$

Shu vaqtda o'zgarmas tezlik bilan harakatlanayotgan odamning bosib o'tgan yo'li

$$S_{od} = \vartheta_{od} \cdot t. \qquad (3)$$

(1) dan foydalanamiz:

$$\frac{at^2}{2} = \vartheta_{od} \cdot t.$$

Sodda o'zgartirishlardan keyin t ni topamiz:

$$t = \frac{2 \cdot \vartheta_{od}}{a}. \qquad (4)$$

(4) ni (2) ga qo'yib, poyezdning tezligini

$$\vartheta_p = a \cdot t = a \cdot \frac{2 \cdot \vartheta_{od}}{a} = 2 \cdot \vartheta_{od}, \qquad (5)$$

(4) ni (3) ga qo'yib odam bosib o'tgan yo'lni aniqlaymiz.

$$S_{od} = \vartheta_{od} \cdot \frac{2 \cdot \vartheta_{od}}{a} = 2 \cdot \frac{\vartheta_{od}^2}{a}. \qquad (6)$$

(4) va (6) lar asosida kattaliklarning birliklarini tekshiramiz ((5) uchun bunga zarurat yo'q)

$$[t]=\frac{[\vartheta]}{[a]}=\frac{1\frac{\text{m}}{\text{s}}}{1\frac{\text{m}}{\text{s}^2}}=1\text{s}\,; \qquad [S]=\frac{[\vartheta^2]}{[a]}=\frac{1\frac{\text{m}^2}{\text{s}^2}}{1\frac{\text{m}}{\text{s}^2}}=1\text{m},$$

va ularning to‘g‘riligiga ishonch hosil qilganimizdan keyin (4), (5) va (6) larga berilganlarni qo‘yamiz:

$$t=2\cdot\frac{1{,}5}{0{,}1}\text{s}=30\text{s}\,,$$

$$\vartheta_p=2\cdot 1{,}5\frac{\text{m}}{\text{s}}=3\frac{\text{m}}{\text{s}}\,,$$

$$S_{od}=2\cdot\frac{(1{,}5)^2}{0{,}1}=\frac{4{,}5}{0{,}1}\text{m}=45\text{m}.$$

Javob: t =30 s; $\vartheta_p=3\frac{\text{m}}{\text{s}}$; $S_{od}=45\text{m}$.

4-misol. Ikkita moddiy nuqta harakatlarining kinematik tenglamalari quyidagi ko‘rinishga ega: $x_1=A_1+B_1t+S_1t^2$; $x_2=A_2+B_2t+S_2t^2$, bunda: $B_1=B_2=3\,\text{m}/\text{s}$; $S_1=-5\,\text{m}/\text{s}^2$; $S_2=1{,}5\,\text{m}/\text{s}^2$. Vaqt t ning qanday qiymatida bu nuqtalarning tezliklari bir xil bo‘ladi? Vaqtning shu oni uchun nuqtalarning tezliklari ϑ_1 va ϑ_2, hamda tezlanishlari a_1 va a_2 lar aniqlansin.

Berilgan:	**Yechish:**
$x_1=A_1+B_1t+S_1t^2$;	Oniy tezlikni topish formulasidan
$x_2=A_2+B_2t+S_2t^2$;	foydalanib nuqtalarning t ondagi
$B_1=B_2=3\,\text{m}/\text{s}$;	tezliklarini topamiz

Berilgan	Yechish
$S_1 = -5\,{}^{m}\!/_{s^2}$;	$\vartheta_1 = \dfrac{dx_1}{dt} = B_1 + 2S_1 t$, (1)
$S_2 = 1{,}5\,{}^{m}\!/_{s^2}$;	$\vartheta_2 = \dfrac{dx_2}{dt} = B_2 + 2S_2 t$. (2)
$\vartheta_1(t) = \vartheta_2(t)$	Masalaning shartiga ko'ra
$t = ?$	$\vartheta_1(t) = \vartheta_2(t)$, demak,
$\vartheta_1 = \vartheta_2 = ?$	$B_1 + 2S_1 t = B_2 + 2S_2 t$,
$a_1 = ?$	bu ifodadan t ni aniqlasak,
$a_2 = ?$	

$$t = \frac{B_1 - B_2}{2(S_2 - S_1)} \qquad (3)$$

Agar $B_1 = B_2$ ligini nazarda tutsak, (3) dan $t=0$ paytda nuqtalarning tezliklari bir xil bo'lishini olamiz.

t ning bu qiymatini (1) va (2)ga qo'yamiz.

$$\vartheta_1 = B_1,\ \vartheta_2 = B_2 \quad \text{demak,}\ \vartheta_1 = \vartheta_2 = 3\,{}^{m}\!/_{s}.$$

Oniy tezlanishni topish formulasidan (1) va (2) yordamida nuqtalarning oniy tezlanishlarini topamiz.

$$a_1 = \frac{d\vartheta_1}{dt} = 2S_1; \quad (3) \qquad a_2 = \frac{d\vartheta_2}{dt} = 2S_2; \quad (4)$$

Berilganlarni (3) va (4) ga qo'ysak,

$$a_1 = 2\cdot(-5)\frac{m}{s^2} = -10\frac{m}{s^2}; \qquad a_2 = 2\cdot 1{,}5\frac{m}{s^2} = 3\frac{m}{s^2}$$

hosil bo'ladi.

Javob: $t = 0$; $\vartheta_1 = \vartheta_2 = 3\,{}^{m}\!/_{s}$; $a_1 = -10\dfrac{m}{s^2}$; $a_2 = 3\dfrac{m}{s^2}$.

5-misol. Nuqta 6 s davomida radiusi 0,8 m bo‘lgan aylana uzunligining yarmiga teng bo‘lgan yo‘lni o‘tdi. Shu vaqt uchun o‘rtacha yo‘l tezligi $<\vartheta>$ va o‘rtacha tezlik vektorining moduli $\left|<\vec{V}>\right|$ aniqlansin.

Berilgan:

$t = 6\text{s};$

$S = \pi R;$

$R = 0{,}8;$

$<\vartheta>= ?$

$\left|<\vec{V}>\right| = ?$

Yechish:

3-rasm

Nuqtaning o‘rtacha yo‘l tezligi

$$<\vartheta>= \frac{S}{t} \qquad (1)$$

formula yordamida aniqlanadi. Yoki mazkur masalada

$$<\vartheta>= \frac{\pi R}{t}. \qquad (2)$$

Rasmdan ko‘rinib turibdiki, berilgan t vaqt uchun o‘rtacha tezlik vektori $<\vec{V}>$. A nuqtadan B nuqtaga yo‘nalgan va $\overrightarrow{AB}$ vektorning uzunligi 2R ga teng. Demak,

$$\left|<\vec{V}>\right| = \frac{\left|\overrightarrow{AB}\right|}{t} = \frac{2R}{t}. \qquad (3)$$

Ko‘rinib turibdiki, topilgan ifodalardan tezlikning birligi hosil bo‘ladi:

$$[\vartheta]= \frac{[R]}{[t]} = \frac{1\text{m}}{1\text{s}} = 1\frac{\text{m}}{\text{s}}$$

berilganlarni (2) va (3) larga qo‘yib olamiz.

$$<\vartheta>= \frac{3{,}14 \cdot 0{,}8}{6}\frac{\text{m}}{\text{s}} = 0{,}42\frac{\text{m}}{\text{s}},$$

$$\left|<\vec{V}>\right| = \frac{2 \cdot 0{,}8}{6}\frac{m}{s} = 0{,}27\frac{\text{m}}{\text{s}}.$$

Javob: $<\vartheta>= 0{,}42\frac{\text{m}}{\text{s}}$; $\left|<\vec{V}>\right| = 0{,}27\frac{\text{m}}{\text{s}}$.

6-misol. Tosh $30\,{}^{m}\!/\!_{s}$ boshlang‘ich tezlik bilan minoradan gorizontal yo‘nalishda otildi. Harakat boshlanganining ikkinchi sekundi oxirida toshning tezligi ϑ tangensial a_τ va normal a_n tezlanishlari aniqlansin.

Berilgan:

$\vartheta_0 = 30\,{}^{0}\!/\!_{s}$;

$t = 2s$

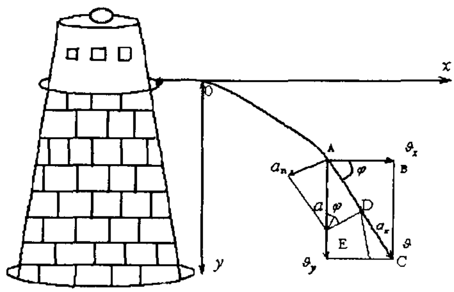
$\vartheta = ?$

$a_\tau = ?$

$a_n = ?$

Yechish.

Minoradan gorizontal otilgan toshning harakati murakkab harakat bo‘lib, u gorizontal yo‘nalishda tekis (chizmada x o‘qi yo‘nalishida) va tik yo‘nalishdagi tekis tezlanuvchan (y o‘qi yo‘nalishida) harakatlarning yig‘indisidan iborat.



4-rasm

Toshning tezligi esa shu yo‘nalishdagi tezliklar orqali aniqlanadi.

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2}\,. \qquad (1)$$

Bu yerda $\vartheta_x = \vartheta_0$; $\quad \vartheta_y = gt$.

Unda

$$\vartheta = \sqrt{\vartheta_0^2 + g^2 t^2}\,. \qquad (2)$$

Tezlanishning tangensial va normal tashkil etuvchilarini topish uchun ABC va ADE uchburchaklarining o‘xshashliklaridan foydalanib φ burchakning sinus va kosinuslarini topamiz:

$$\sin\varphi = \frac{\vartheta_y}{\vartheta} = \frac{a_\tau}{a}, \text{ yoki } \quad a_\tau = a\frac{\vartheta_y}{\vartheta} \qquad (3)$$

$$\cos\varphi = \frac{\vartheta_x}{\vartheta} = \frac{a_n}{a}, \text{ yoki } \quad a_n = a\frac{\vartheta_x}{\vartheta}\,. \qquad (4)$$

$a = g$ ekanligini va tezliklarning ifodalarini nazarda tutib, (3) va (4) ni qayta yozamiz.

$$a_\tau = g\frac{gt}{\vartheta} = \frac{g^2 t}{\vartheta};\qquad (5)$$

$$a_n = g\frac{\vartheta_0}{\vartheta}.\qquad (6)$$

Berilganlar va $g = 9{,}8\,\mathrm{m}/\mathrm{s}^2$ ni (2), (5), (6) ga qo'yib topamiz.

$$\vartheta = \sqrt{(30)^2 + (9{,}8)^2\cdot(2)^2}\,\frac{\mathrm{m}}{\mathrm{s}} = \sqrt{900 + 4\cdot 96}\,\frac{\mathrm{m}}{\mathrm{s}} = \sqrt{1284}\,\frac{\mathrm{m}}{\mathrm{s}} = 35{,}8\,\mathrm{m}/\mathrm{s};$$

$$a_\tau = \frac{(9{,}8)^2\cdot 2}{35{,}8}\,\frac{\mathrm{m}}{\mathrm{s}^2} = 5{,}37\,\mathrm{m}/\mathrm{s}^2;$$

$$a_n = 9{,}8\cdot\frac{30}{35{,}8}\,\frac{m}{s^2} = 8{,}21\,m/s^2.$$

Javob: $\vartheta = 35{,}8\,m/s$; $a_\tau = 5{,}37\,m/s^2$; $a_n = 8{,}21\,m/s^2$.

7-misol. G'ildirak tekis tezlanuvchan aylana bo'ylab 10 s vaqt oralig'ida 300 min $^{-1}$ aylanish chastotasiga erishdi. G'ildirakning burchak tezlanishi ε va shu vaqt ichidagi aylanishlar soni N aniqlansin.

Berilgan:

$\Delta t = 10s$;

$n = 300\,\mathrm{min}^{-1} = 5s^{-1}$;

$\omega_0 = 0$

$\varepsilon = ?$

$N = ?$

Yechish: G'ildirakning burchak tezlanishini

$$\varepsilon = \frac{\Delta\omega}{\Delta t} = \frac{\omega_1 - \omega_2}{\Delta t}\qquad (1)$$

formula yordamida aniqlaymiz. Agar burchak tezlik ω va aylanish chastotasi n orasidagi $\omega_t = 2\pi n$ bog'lanishni nazarda tutsak, (1)- quyidagi ko'rinishni oladi:

$$\varepsilon = \frac{2\pi n - \omega_0}{\Delta t}.\qquad (2)$$

Δt vaqt oralig'idagi to'la aylanishlar sonini esa o'rtacha aylanish chastotasini (tekis tezlanuvchan harakatda)

$$n_{o'rt} = \frac{n_0 + n}{2}$$

yordamida aniqlash mumkin. Mazkur masalada $n_0 = 0$.

$$N = n_{o'rt} \cdot \Delta t = \frac{n_0 + n}{2} \cdot \Delta t = \frac{n}{2} \cdot \Delta t \qquad (3)$$

Berilganlarni (2) va (3) ga qo‘yib quyidagini olamiz:

$$[\varepsilon] = \frac{[n]-[\omega]}{[t]} = \frac{\frac{1}{s}}{1s} = \frac{1}{1s^2} = 1s^{-2};$$

$$[N] = [n]\cdot[t] = \frac{1}{s}1s = 1;$$

$$\varepsilon = \frac{2\cdot 3{,}14\cdot 5 - 0}{10}\frac{1}{s^2} = \frac{31{,}4}{10}\frac{1}{s^2} = 3{,}14\frac{1}{s^2} = 3{,}14s^{-2};$$

$$N = \frac{5}{2}\cdot 10 = 25 \text{ ayl.}$$

Javob: $\varepsilon = 3{,}14s^{-2}$; N=25 ayl.

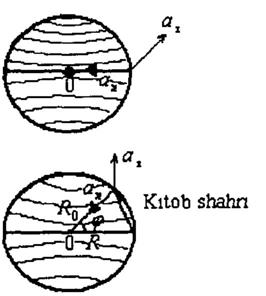
8-misol. Yer sirtida yotgan nuqtalarning: 1) ekvatorda $(\varphi_1 = 0)$; 2) Kitob shahri kengligida $(\varphi_2 = 39^0 8)$, – chiziqli tezligi ϑ va markazga intilma tezlanishi $a_{m\,i}$ aniqlansin.

Berilgan
$\varphi_1 = 0;$
$\varphi_2 = 39^0 8.$
$\vartheta = ?$
$a_{mi} = ?$

Yechish: Masalani yechishni soddalashtirish maqsadida dastlab ikkinchi holni ko‘raylik. Kitob shahri joylashgan kenglik uchun Yerning radiusini R_0 bilan belgilaymiz. Chizmadan ko‘rinib turibdiki u Yerning ekvatordagi radiusi R bilan quyidagicha bog‘langan:

$$R_0 = R\cos\varphi \;. \qquad (1)$$

Ekvatorda yotgan nuqta uchun $\varphi = 0$, $\cos\varphi = 1$ va demak, $R_0 = R$ bo‘ladi.



Yer nuqtasining chiziqli va burchak tezliklari orasidagi munosabatni yozamiz:

$$\vartheta = \omega R_0. \qquad (2)$$

Bu yerda: $\omega = \frac{2\pi}{T}$ Yerning aylanish chastotasi. T – Yerning bir marta o‘z o‘qi atrofida aylanishi uchun ketgan vaqt, ya’ni bir kecha-kunduzga teng vaqt.

Unda:

$$\vartheta = \frac{2\pi}{T} R_0 = \frac{2\pi}{T} R\cos\varphi. \qquad (3)$$

Markazga intilma tezlanish, tezlanishning normal tashkil etuvchisiga tengligini nazarda tutib, (3) ni hisobga olib yozamiz:

$$a_{m.i} = a_n = \frac{\vartheta^2}{R_0} = \frac{4\pi^2}{T^2} R_0 = \frac{4\pi^2}{T^2} R\cos\varphi. \qquad (4)$$

Bu yerda: $R = 6{,}371 \cdot 10^6 m$; $T = 86400s$ ekanligini nazarda tutsak, (3) va (4) formulalar yordamida quyidagilarni olamiz:

1) Ekvatorda $(\varphi_1 = 0)$:

$$\vartheta = \frac{2 \cdot 3{,}14}{86400} \cdot 6{,}371 \cdot 10^6 \cdot \cos 0 \frac{\text{m}}{\text{s}} = 463 \frac{\text{m}}{\text{s}};$$

$$a_{m\,i} = a_n = \frac{\vartheta^2}{R_0} = \frac{4\pi^2}{T^2} R_0 = \frac{4\pi^2}{T^2} R\cos\varphi.$$

2) Kitob shahri kengligida $(\varphi_2 = 39^0 8')$:

$$\vartheta = \frac{2 \cdot 3{,}14}{86400} \cdot 6{,}371 \cdot 10^6 \cdot cos 39^0 8' \frac{m}{s} \approx 359 \text{m/s};$$

$$a_{m.i.} = \frac{4 \cdot (3{,}14)^2}{(86400)^2} 6{,}371 \cdot 10^6 \cdot \cos 39^0 8' \frac{m}{s^2} = 2{,}6 \text{m/s}^2.$$

Javob: 1) $\vartheta = 463 \text{m/s}$; $a_{m.i} = 3{,}37 \text{ m/s}^2$; 2) $\vartheta \approx 359 \text{ m/s}^2$; $a_{m.i} = 2{,}6 \text{m/s}^2$.

MUSTAQIL YECHISH UCHUN MASALALAR

1. Jism yo‘lning birinchi yarmini 2 s davomida, ikkinchisini esa 8 s davomida o‘tdi. Agar yo‘lning uzunligi 20 m bo‘lsa, o‘rtacha yo‘l tezligi $<\vartheta>$ aniqlansin. [2 m/s].

2. Moddiy nuqta ilgarilanma harakatining kinematik tenglamasi $x = At + Bt^2$ ko‘rinishida berilgan. Bunda A=4 m/s, B=–0,05 m/s. Nuqtaning tezligi ϑ nolga teng bo‘lgan payt aniqlansin. Shu payt uchun koordinata va tezlanish topilsin. Shu harakat uchun koordinata, yo‘l, tezlik va tezlanishlarning vaqtga bog‘liqlik grafigi tuzilsin. [40 s; 80 m; –0,1 m/s²].

3. Bir joydan bir xil yo‘nalishda ikki nuqta tekis tezlanuvchan harakat qila boshladi. Ikkinchisi o‘z harakatini birinchisidan 2 s keyin boshladi. Birinchi nuqta 1 m/s boshlang‘ich tezlik va 2 m/s² tezlanish, ikkinchisi esa 10m/s boshlang‘ich tezlik va 1 m/s² tezlanish bilan harakatlana boshladi. Qancha vaqtdan keyin, va boshlang‘ich holatdan qancha masofada ikkinchi nuqta birinchisiga yetib boradi? [3,4 s; 15m; 10,6 s; 123m.]

4. Ikkita moddiy nuqtalar quyidagi tenglamalarga muvofiq harakatlanmoqda: $x_1 = A_1t + B_1t^2 + S_1t^3, x_2 = A_2t + B_2t^2 + S_2t^3$, bunda: $A_1 = 4\,{}^{m}\!/_{s}; B_1 = 8\,{}^{m}\!/_{s^2}; S_1 = -16\,{}^{m}\!/_{s^3}$; $A_2 = 2\,{}^{m}\!/_{s}; B_2 = -4\,{}^{m}\!/_{s^2}; S_2 = 1\,{}^{m}\!/_{s^3}$. Vaqt t ning qaysi qiymatida bu nuqtalarning tezlanishlari bir xil bo‘ladi? Shu qiymat uchun nuqtalarning tezliklari ϑ_1 va ϑ_2 aniqlansin. [0,235 s; 5,1 m/s; 0,286 m/s.]

5. Erkin tushayotgan jism o‘z harakatining oxirgi sekundida butun yo‘lning yarmini o‘tadi. 1) jismning qanday h balandlikda tushayotgani va; 2) yerga tushguncha ketgan vaqt topilsin. [1) 57 m; 2) 3,4 s].

6.Yuqoriga tik otilgan jism 8,6 m balandlikda 3 s oraliq bilan ikki marta bo‘ldi. Havọning qarshiligini hisobga olmay, otilgan toshning boshlang‘ich tezligini hisoblang. [19,6 m/s.]

7. Vagon –0,5 m/s² tezlanish bilan tekis sekinlanuvchan harakat qilmoqda. Vagonning boshlang‘ich tezligi 54 km/soat. Vagon qancha vaqtdan keyin va boshlang‘ich nuqtadan qancha uzoqlikda to‘xtaydi. [30 s; 225 m]

8. Jismning bosib o‘tgan yo‘li S ning vaqt t ga bog‘liqligi $S=A+Bt+Ct^2+Dt^3$ tenglama bilan berilgan. Bunda C=0,14 m/s², D=0,01 m/s². Harakat boshlangandan qancha vaqt o‘tgandan keyin jismning tezlanishi 1m/s² ga va shu vaqt oralig‘ida jismning o‘rtacha tezlanishi $<a>$ nimaga teng bo‘ladi. [12 s; 0,64 m/s².]

9. Agar jismning maksimal balandlikka ko‘tarilishi uning uchish

masofasining 1/4 qismiga teng bo'lsa, jism gorizontga nisbatan qanday φ burchak ostida otilganligi aniqlansin. Havoning qarshiligi inobatga olinmasin [45^0].

10. Tosh gorizontal yo'nalishda 10 m/s tezlik bilan otilgandan 3s o'tgach trayektoriyasining egrilik radiusi R topilsin. Havoning qarshiligi inobatga olinmasin. [305m].

11. Nuqta egri chiziq bo'ylab o'zgarmas 0,5m/s^2 tangensial tezlanish bilan harakatlanmoqda. Egri chiziqning egrilik radiusi 3m bo'lgan qismda nuqta 2m/s tezlik bilan harakatlansa, chiziqning shu qismida nuqtaning to'la tezlanish a topilsin. [142$^m/_{s^2}$].

12. Nuqta 10 sm radiusli aylana bo'ylab o'zgarmas tangensial tezlanish a_τ bilan harakatlanmoqda. Agar harakat boshlangandan keyingi beshinchi marta aylanish oxirida nuqtaning chiziqli tezligi 10 sm/s ga teng bo'lsa, harakat boshlangandan 20 s o'tgandan keyin nuqtaning normal tezlanishi a_n topilsin. [0,01 m/s^2.]

13. Moddiy nuqta aylanma harakatining kinematik tenglamasi $\varphi = A + Bt + St^2 + Dt^3$ ko'rinishga ega. Bunda B=1 rad/s, S=1 rad/s^2 va D=1 rad/s^3. Harakatning ikkinchi sekundining oxirida nuqtaning normal tezlanishi a_n=346 m/s^2 bo'lsa, nuqta trayektoriyasining egrilik radiusi topilsin. [1.2 m.]

14. Minoradan gorizontal ravishda tosh otdilar. Tosh 2s dan keyin minora asosidan 40 m masofada yerga tushdi. Toshning boshlang'ich ϑ_0 va oxirgi ϑ tezliklari aniqlansin. [20 m/s; 28 m/s.]

15. O'q gorizontga nisbatan 60^0 burchak ostida 200 m/s boshlang'ich tezlik bilan otiladi. Eng baland ko'tarilishi H, uchish uzoqligi S va trayektoriyaning eng yuqori nuqtasida egrilik radiusi R aniqlansin. Havoning qarshiligi hisobga olinmasin. [1,53 km, 3,53 km, 1,02 km.]

16. Avtomashina g'ildiragi tekis tezlanuvchan aylanmoqda. U 50 marta to'la aylanib, aylanish chastotasini 4 s^{-1} dan 6 s^{-1} gacha o'zgartirdi. G'ildirakning burchak tezlanishi ε aniqlansin. [1,26 rad/s^2.]

2-§. Moddiy nuqta va qatttiq jism ilgarilanma harakat dinamikasi

Moddiy nuqta harakati dinamikasining asosiy qonuni (Nyutonning ikkinchi qonuni):

Vektor shaklda

$$\frac{d\vec{p}}{dt} = \sum_{i=1}^{N} \vec{F_i}, \quad \text{yoki} \quad m\vec{a} = \sum_{i=1}^{N} \vec{F_i},$$

bu yerda: $\sum_{i=1}^{N} \vec{F_i}$ – moddiy nuqtaga ta'sir etayotgan kuchlarning geometrik yig'indisi: m – massa; $\vec{a}$ – tezlanish; $\vec{p} = m\vec{V}$ – impuls; $\vec{V}$ – tezlik; N – nuqtaga ta'sir etayotgan kuchlar soni.

Skalyar (koordinatalar) shaklda

$$ma_x = \sum F_{xi}, \quad ma_y = \sum F_{yi}, \quad ma_z = \sum F_{zi},$$

yoki

$$m\frac{d^2x}{dt^2} = \sum F_{xi}, \quad m\frac{d^2y}{dt^2} = \sum F_{yi}, \quad m\frac{d^2z}{dt^2} = \sum F_{zi},$$

bu yerda F_{ki}, F_{yi} va F_{ki} lar $\vec{F_i}$ ning koordinata o'qlaridagi proeksiyasi.

Agar harakat X o'qi yo'nalishida ro'y bersa va soddalik uchun uni ko'rsatmasak, $ma = F$ bo'ladi. Bu yerda F – kuchlarning teng ta'sir etuvchisi.

Og'irlik kuchi

$$\vec{P} = m\vec{g}.$$

$\vec{g}$ – erkin tushish tezlanishi.

Egri chiziqli trayektoriya bo'ylab harakatlanayotgan nuqtaga ta'sir etadigan kuchlar:

tangensial kuch $F_\tau = ma_t = m\frac{d\vartheta}{dt}$, normal kuch $F_n = ma_n = m\frac{\vartheta^2}{R} = m\omega^2 R$.

Sirpanishdagi ishqalanish kuchi

$$F_{ishq} = fN.$$

bunda: f — sirpanishda ishqalanish koeffitsienti, N — normal bosim kuchi.

Moddiy nuqtalar sistemasining (qattiq jismning) massa markazi (inersiya markazi)

$$\vec{r_c} = \frac{\sum_{i=1}^{N} m_i \vec{r_i}}{m},$$

m_i va $\vec{r_i}$ — mos ravishda i-moddiy nuqtaning massasi va radius-vektori.

Massa markazining koordinatalari

$$x_c = \frac{\sum m_i x_i}{m}, \quad y_c = \frac{\sum m_i y_i}{m}, \quad z_c = \frac{\sum m_i z_i}{m},$$

x_i, y_i, z_i lar i moddiy nuqtaning koordinatalari.

Impulsning (harakat miqdorining) saqlanish qonuni:

$$\sum_{i=1}^{N} \vec{P_i} = \text{const}, \quad \text{yoki} \quad \sum_{i=1}^{N} m_i \vec{V_i} = \text{const},$$

bu yerda: N — sistemaga kiruvchi moddiy nuqtalarning (yoki jismlarning) soni.

O'zgaruvchan massali jismning harakat tenglamasi (Mesherskiy tenglamasi):

$$m\vec{a} = \vec{F} + \vec{F_r},$$

bu yerda: $\vec{F_r} = -\vec{u}\frac{dm}{dt}$ — reaktiv kuch, $\vec{u}$ — ajralayotgan moddaning harakatlanayotgan jismga nisbatan tezligi.

Raketaning tezligi (Siolkovskiy formulasi)

$$\vartheta = u \cdot \ln \frac{m_o}{m},$$

bu yerda: m_0 – raketaning boshlang'ich, m – oxirgi massalari.

Inersial sanoq sistemasiga nisbatan $\vec{a}$ tezlanish bilan ilgarilanma harakat qilayotgan noinersial sanoq sistemasidagi m – massali jismga ta'sir etadigan inersiya kuchi

$$F_i = -ma.$$

Inersial sanoq sistemasiga nisbatan R – radiusli aylana bo'ylab o'zgarmas $\vec{\omega}$ burchak tezlik bilan harakatlanayotgan noinersial sanoq sistemasidagi harakatsiz jismga ta'sir etadigan inersiya kuchi:

$$\vec{F}_{mq} = -m\omega^2 \vec{R}.$$

Inersial sanoq sistemasiga nisbatan o'zgarmas $\vec{\omega}$ burchak tezlik bilan harakatlanayotgan noinersial sanoq sistemasida o'zgarmas $\vec{V'}$ tezlik bilan harakatlanayotgan jismga ta'sir etadigan inersiya kuchi (Koriolis kuchi):

$$\vec{F_k} = 2m[\vec{V'} \cdot \vec{\omega}]$$

Masala yechishga doir misollar

1-misol. 2 kg massali jism biror F kuch ta'sirida $x = A + Bt + St^2 + Dt^3$ tenglamaga muvofiq harakatlanadi.

Bu yerda: S=1 m/s², D=–0,2m/s³. Vaqtning $t_1 = 2$ s va $t_2 = 5$ s paytlari uchun kuchning qiymatlari topilsin. Vaqtning qaysi onida kuch nolga teng bo'ladi?

Berilgan:	**Yechish:** Jismga ta'sir etadigan kuchni Nyutonning ikkinchi qonuniga muvofiq aniqlaymiz.
$m = 2kg$;	$F = ma$. (1)
$x = A + Bt + St^2 + Dt$	Tezlanishni koordinatadan vaqt bo'yicha olingan ikkinchi tartibli hosiladek aniqlanadi:
$S=1$ m/s^2	
$D=-0{,}2$m/s^2	
$t^1=2$ s;	
$t^2=5$ s;	
$F=0$	
$F(t_1)=?$	
$F(t_2)=?$	
$t=0$	

$$a = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}\left(A + Bt + St^2 + Dt^3\right) = 2S + 6Dt. \quad (2)$$

(2) ni (1) ga qo'yib olamiz:

$$F = m(2S + 6Dt) \quad (3)$$

Kuchning nolga teng onini topish uchun (1) ni nolga tenglashtiramiz:

$$F = ma = 0.$$

Jismning massasi nolga teng bo'lmaganligidan

$$a=0.$$

(2) ga asosan

$$2S + 6Dt = 0,$$

$$t = -\frac{2S}{6D} = -\frac{S}{2D}. \quad (4)$$

berilganlarni (3) va (4) larga qo'yib quyidagini olamiz:

$F(t_1)=2 \cdot (2 \cdot 1-6 \cdot 0{,}2)$ N$=2(2-2{,}4)$ N$=-0{,}8$ N,

$F(t_2)=2 \cdot (2 \cdot 1-6 \cdot 0{,}2 \cdot 5)$ N$=2(2-6)$ N$=-0$ N,

$$\mathrm{t} = -\frac{1}{3 \cdot (-0{,}2)}\mathrm{s} = \frac{1}{0{,}6}\mathrm{C} = 1{,}67\mathrm{s}.$$

Javob: $F(t_1)=-0{,}8$ N; $F(t_2)=-8$ N; t=1,67 s.

2-misol. Agar 3 kg massali jism, tezlanishi $a = At + B$ qonun bilan aniqlanuvchi harakat qilayotgan bo'lsa, unda jismga 3 s dan keyin ta'sir etadigan kuch va jismning uchinchi sekund oxiridagi tezligi aniqlansin.

Jismning boshlang'ich tezligi nolga teng. $A = 5\frac{\mathrm{m}}{\mathrm{s}^3}$, $B = 3\frac{\mathrm{m}}{\mathrm{s}^2}$.

Berilgan:
$m = 3\text{kg};$
$a = At + B;$
$A = 5\frac{\text{m}}{\text{s}^3};$
$B = 3\frac{\text{m}}{\text{s}^2};$
$t = 3\text{s};$
$\vartheta_0 = 0.$

$F = ?$
$\vartheta = ?$

Yechish: Nyutonning ikkinchi qonuniga binoan

$$F = ma.$$

yoki mazkur masala holida

$$F = m(At + B). \qquad (1)$$

Tekis o'zgaruvchan harakat qilayotgan nuqtaning t vaqtdan keyingi tezligi

$$\vartheta = \vartheta_0 + at$$

formula bilan aniqlanadi. Bizning holimizda

$$\vartheta = \vartheta_0 + (At + B)t. \qquad (2)$$

(1) va (2)ga berilganlarning qiymatlarini qo'yib topamiz:

$$F=3(5\cdot3+3)\text{N}=3\cdot18\text{N}=54\text{N}.$$

$$\vartheta = [0 + (5\cdot3 + 3)\cdot3]\frac{\text{m}}{\text{s}} = 54\frac{\text{m}}{\text{s}}.$$

Javob: $F = 54\text{N},\ \vartheta = 54\frac{\text{m}}{\text{s}}.$

3-misol. Yer sirtidan qandaydir balandlikda muallaq turgan vertolyotdan massasi 100 kg bo'lgan yuk tashlandi. Havoning qarshilik kuchi tezlikka proporsional o'zgaradi deb hisoblab, qancha Δt vaqtdan keyin yukning tezlanishi a, erkin tushish tezlanishining yarmiga teng bo'lishi aniqlansin. Qarshilik koeffitsienti 10 kg/s.

Berilgan:
$m = 100\,\text{kg}$
$a = \frac{1}{2}g,$
$k = 10\frac{kg}{s}$

$\Delta t = ?$

Yechish. Tushayotgan jismga o'zaro qarama-qarshi yo'nalgan og'irlik kuchi $\vec{P} = m\vec{g}$ va havoning qarshilik kuchi $\vec{F_к} = k\vec{V}$ ta'sir qilishini nazarda tutib Nyutonning ikkinchi qonuniga muvofiq jismning harakat tenglamasini yozamiz:

$$m\frac{d\vec{V}}{dt} = m\vec{g} - k\vec{V}, \text{ yoki } m\frac{dV}{dt} = mg - kV. \qquad (1)$$

Tenglamani o'zgaruvchilarga ajratamiz:

$$\frac{dt}{m} = \frac{dV}{mg - kV}. \qquad (2)$$

(2) ni integrallash uchun o‘zgaruvchilarning chegara qiymatlarini aniqlaymiz. Yuk tashlangan vaqtni t_0 va yuk aytilgan tezlanishga erishgan vaqtini t desak,

$$t - t_0 = \Delta t \tag{3}$$

bo‘ladi. Tezlanishning chegaraviy qiymatini Δt dan so‘ng $a = g/2$ bo‘lishidan foydalanib topamiz:

$$ma = mg - kV\,; \qquad m\frac{g}{2} = mg - kV\,;$$

$$kV = \frac{m}{2}g\,; \qquad V = \frac{mg}{2k}. \tag{4}$$

Bu chegara qiymatlarini (2) ga qo‘yamiz:

$$\frac{1}{m}\int_{t_0}^{t} dt = \int_0^{\frac{m}{2k}} \frac{dv}{mg - kV}.$$

Integrallashdan keyin

$$\frac{1}{m}(\mathrm{t} - \mathrm{t}_0) = -\frac{1}{\mathrm{k}}\,|\,\ln(\mathrm{mg} - \mathrm{kV})\,|_0^{\frac{\mathrm{mg}}{2\mathrm{k}}}; \ \ \Delta\mathrm{t} = \frac{\mathrm{m}}{\mathrm{k}}\ln\left(\frac{mg}{mg - k\frac{mg}{2k}}\right) = \frac{m}{k}\ln 2 \ .$$

Minus ishora logarifm ostidagi kasrning maxraj va suratlarining o‘rnilarini almashtirdi.

Demak,

$$\Delta t = \frac{m}{k}\ln 2 \ . \tag{5}$$

Δt ning birligini tekshiramiz:

$$[t] = \frac{[m]}{[k]} = \frac{1\mathrm{kg}}{1\,\mathrm{kg}/\mathrm{s}} = 1\mathrm{s}\,.$$

Shunday qilib,

$$\Delta t = \frac{100}{10} \cdot \ln 2 \cdot s = 10 \cdot 0{,}69s = 6{,}9s$$

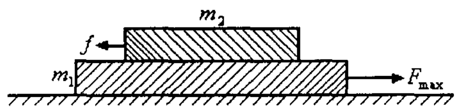
Javob: $\Delta t = 6{,}9$ s.

4-misol. 5 kg massali to'rt qirra jism gorizontal sirtda ishqalanishsiz erkin sirpanadi. Uning ustida 1 kg massali shunday jism turibdi. Jismlarning bir-birlariga tegib turgan sirtlari orasidagi ishqalanish koefitsiyenti $0{,}3$. Yuqoridagi jism sirpana boshlashi uchun pastdagi jismga qo'yilishi kerak bo'lgan kuchning maksimal qiymati F_{max} aniqlansin.

Berilgan:
$m_1 = 5kg$;
$m_2 = 1kg$;
$f = 0{,}3$;

$F_{max} = ?$

Yechish:



5-rasm

Ikkita to'rt qirra jismdan iborat sistemani harakatga keltirish uchun kamida ularning og'irligiga teng bo'lgan kuch qo'yilishi kerak:

$$F_{max} = fF = f(m_1 + m_2)g\,. \qquad (2)$$

Topilgan kattalikning birligini tekshirib,

$$[F] = [m] \cdot [g] = 1kg \cdot 1\frac{m}{s} = 1kg \cdot \frac{m}{s^2} = 1N\,,$$

va berilganlardan foydalanib, quyidagini olamiz:

$$F_{max} = 0{,}3(5+1)9{,}8N = 0{,}3 \cdot 6 \cdot 9{,}8N = 1{,}8 \cdot 9{,}8N = 17{,}7N.$$

Javob: $F_{max} = 17{,}7$ N.

5-misol. Prujinali taroziga chig'ir osilgan. Chig'irdan o'tkazib tashlangan chilvirga 1,5 kg ва 3 kg massali yuklar bog'langan. Yuklar harakatlana boshlaganda tarozining ko'rsatishi qanday bo'ladi? Chig'irning va chilvirning massasi inobatga olinmasin.

Berilgan:	**Yechish:**
m_1=1,5 kg;	Chig'irga va yuklarga ta'sir etayotgan kuchlar chizmada ko'rsatilgan.
m_2=3 kg.	Chig'ir harakatsiz bo'lganligidan tarozining ko'rsatishi
$F = ?$	

$$F = 2T \qquad (1)$$

bo'ladi.

Chilvirga ta'sir etadigan taranglik kuchi T ni aniqlash uchun yuklarning harakat tenglamalarini yozamiz. $P_1 = m_2 g$, $P_2 = m_2 g$ va $P_2 > P_1$ ligini hisobga olsak.

$$m_1 a = T - P_1 \quad \text{yoki} \quad m_1 a = T - m_1 g, \qquad (2)$$

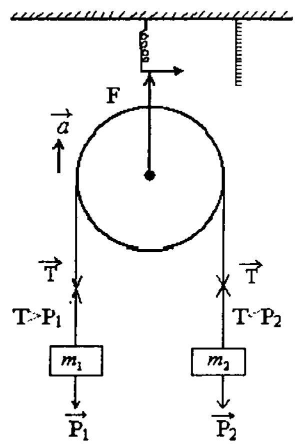
$$m_2 a = P_2 - T \quad \text{yoki} \quad m_2 a = m_2 g - T. \qquad (3)$$

Bu tenglamalardan a ni yo'qotish uchun (2) va (3) larni hadma-had bo'lib, T uchun quyidagi ifodani topamiz.

$$T = \frac{2m_1 m_2 g}{m_1 + m_2}. \qquad (4)$$

(4) ni (1) ga qo'yamiz:

$$F = 2T = \frac{4m_1 m_2 g}{m_1 + m_2}. \qquad (5)$$



6-rasm

F ning birligini hosil qilamiz:

$$[F] = \frac{[m][m][g]}{[m]} = 1\,\text{kg} \cdot 1\frac{\text{m}}{\text{s}^2} = 1\,\text{N}.$$

Berilganlar va g=9,8 m/s² ligidan quyidagini olamiz:

$$[F] = \frac{4 \cdot 15 \cdot 3 \cdot 9{,}8}{1{,}5 + 3} N = 39{,}2\text{N}.$$

Javob: F=39,2 N.

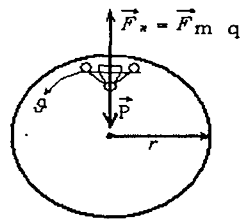
6-misol. Akrobat motosiklda radiusi r = 4 m bo'lgan "o'lim halqasini" yasamoqda. Yiqilib ketmaslik uchun, akrobat halqaning eng yuqori nuqtasidan qanday eng kichik ϑ_{min} tezlik bilan o'tmog'i kerak?

Berilgan:
r = 4 m.
ϑ_{min} = ?

Yechish:

Chizmadan ko'rinib turibdiki, "o'lim halqasining" yuqori nuqtasida akrobatga ikkita kuch ta'sir qiladi. Akrobat yiqilmasligi uchun bu kuchlar teng, ya'ni og'irlik kuchi $\vec{P}$ va markazdan qochma kuch $\vec{F}_{mq}$ larning yig'indisi nolga teng bo'lmog'i kerak:

$$F_{mq} = P.$$



7-rasm

Og'irlik kuchi $P = mg$, markazdan qochma kuch esa $F_{mq} = \frac{m\vartheta^2}{r}$ dek aniqlanadi. Agar tezlikning minimal qiymatini topish kerakligini hisobga olsak, (1)ni quyidagicha yozamiz:

$$\frac{m\vartheta_{min}^2}{r} = mg.$$

Bundan

$$\vartheta_{min} = \sqrt{g \cdot r}. \qquad (2)$$

Topilgan ifodaning to'g'riligiga ishonch hosil qilish uchun ϑ_{min} ning birligini tekshiramiz:

$$[\vartheta] = [g]^{1/2}[r]^{1/2} = \left[1\frac{m}{s^2} \cdot 1m\right]^{1/2} = 1\frac{m}{s};$$

$g = 9{,}8\frac{m}{s^2}$ ligini hisobga olib r ning qiymatini (2) ga qo'yib olamiz:

$$\vartheta_{min} = \sqrt{9,8 \cdot 4}\,\frac{m}{s} = \sqrt{39,2}\,\frac{m}{s} = 6,3\frac{m}{s}.$$

Javob: $\vartheta_{min} = 6,3\frac{m}{s}$.

7-misol. Uzunligi $l=50$ sm bo'lgan chilvirga bog'langan yukcha gorizontal tekislikda aylanalar chizmoqda. Agar aylanish chastotasi $n = 1s^{-1}$ ga teng bo'lsa, chilvir aylana markaziga o'tkazilgan tiklik bilan qanday φ burchak hosil qiladi?

Berilgan:
l =50 sm=0,5 m;
n =1 s^{-1}.

$\varphi = ?$

Yechish. Yukchaga ta'sir etayotgan kuchlar chizmada ko'rsatilgan. U og'irlik va chilvirning taranglik kuchlarining teng ta'sir etuvchisi ta'sirida harakatlanadi. Chizmadan ko'rinib turibdiki, yukcha tekis harakatlanishi uchun

$$F = F_n \qquad (1)$$

bo'lmog'i kerak. φ burchakning tangensidan topamiz:

$$\text{tg}\varphi = \frac{F}{P}, \text{ yoki } F = P \cdot tg\varphi = \text{mg} \cdot tg\varphi = \text{mg} \cdot \frac{\sin\varphi}{\cos\varphi}. \qquad (2)$$

Shuningdek,

$$F_n = \frac{m\vartheta^2}{R} = m\omega^2 R. \qquad (3)$$

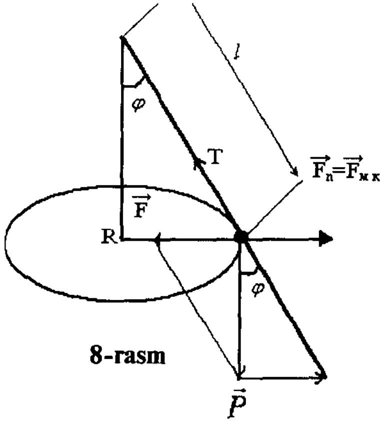
φ ning sinusini aniqlaymiz:

$$\sin\varphi = \frac{R}{l} \text{ yoki } R = l \cdot \sin\varphi. \qquad (4)$$

Burchak tezlik va aylanish chastotasi orasidagi bog'lanish:

$$\omega = 2\pi \cdot n \qquad (5)$$

(5) va (4) ni (3) ga qo'ysak,



8-rasm

$$F_n = m \cdot \omega^2 \cdot R = m \cdot (2\pi \cdot n)^2 \cdot l \cdot \sin\varphi = 4\pi^2 n^2 \cdot m \cdot l \cdot \sin\varphi . \quad (6)$$

(6) va (2) ni (1) ga qo‘yamiz:

$$m \cdot g \frac{\sin\varphi}{\cos\varphi} = 4\pi^2 n^2 \cdot m \cdot l \cdot \sin\varphi ,$$

bundan $\cos\varphi = \dfrac{g}{4\pi^2 \cdot n^2 \cdot l}$

yoki $l = \arccos(\dfrac{g}{4\pi^2 \cdot n^2 \cdot l})$. (7)

Berilganlar yordamida topamiz.

$$\varphi = \arccos(\frac{9{,}8}{4(3.14)^2 \cdot 1^2 \cdot 0{,}5}) = \arccos(0{,}50) = 60^0 .$$

Javob: $\varphi = 60^0$.

8-misol. 50 m/s tezlik bilan harakatlanayotgan 250 g massali to‘p devorga elastik urilib qaytadi. Natijada devor 2,2 kg m/s impuls oladi. Agar urilish 0,02 s davom etsa, to‘pning devorga urilish burchagi va urilish kuchi aniqlansin.

Berilgan:

$\vartheta = 50\frac{m}{s}$;

$m = 250$g$=0,25$ kg;

$P_d = 2,2$ kg$\frac{m}{s}$;

$\Delta t = 0,02$ s

$\alpha = ?$

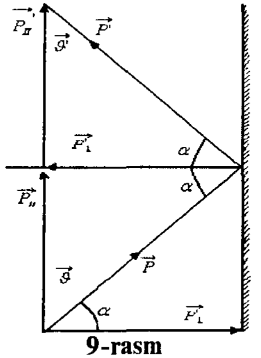
$F = ?$

Yechish: Impulsning saqlanish qonuniga muvofiq

$$\vec{P}_d = \vec{P} + \vec{P}'. \quad (1)$$

Rasmdan ko'rinib turibdiki, urilish elastik bo'lganligidan, urilish natijasida to'p impulsining devorga tik tashkil etuvchisi o'zgarmay qolaveradi. Demak, devor olgan impuls

$$P_d = P_\perp - P_\perp.$$



9-rasm

Agar $P_\perp' = -P_\perp$ ekanligini nazarda tutsak (chizmaga qarang):

$$P_d = P_\perp - (-P_\perp) = 2P_\perp. \quad (2)$$

Rasmdan osongina quyidagini aniqlash mumkin:

$$\cos\alpha = \frac{P_\perp}{P}; \quad P_\perp = P \cdot \cos\alpha = m\vartheta\cos\alpha. \quad (3)$$

(3) ni (2) ga qo'yamiz:

$$P_d = 2m\vartheta\cos\alpha.$$

Bundan

$$\cos\alpha = \frac{P_d}{2m\vartheta}, \quad \alpha = \arccos\left(\frac{P_d}{2m\vartheta}\right). \quad (4)$$

Impulsning saqlanish qonunini quyidagi ko'rinishda ham yozish mumkin:

$$\vec{F} \cdot \Delta t = \vec{P} + \vec{P}' = \vec{P}_d, \qquad \text{yoki} \qquad F \cdot \Delta t = P_d$$

Bundan

$$F = \frac{P_d}{\Delta t}. \quad (5)$$

F ning birligini tekshirib ko'ramiz:

$$[F]=\frac{[P]}{[t]}=\frac{1\text{kg}\frac{\text{m}}{\text{s}}}{1\text{s}}=1\text{kg}\cdot\frac{\text{m}}{\text{s}^2}=1\text{N}$$

Berilganlarni (4) va (5) formulalarga qo'yamiz:

$$\alpha=\arccos\left(\frac{2{,}2}{2\cdot 0{,}25\cdot 50}\right)=\arccos(0{,}1)\approx 85^0,$$

$$\text{F}=\frac{2{,}2}{0{,}02}\text{N}=110\text{N}.$$

Javob: $\alpha\approx 85^0$; $F=110$N.

9-misol. 200 m/s gorizontal tezlik bilan uchib ketayotgan zambarak o'qi ikkiga bo'linib ketdi. Birinchi bo'lakning massasi ikkinchisinikidan ikki marta katta. Katta massali bo'lak gorizont bilan 60^0, kichigi esa 30^0 burchak hosil qilib harakatlana boshlashdi. Agar kichik bo'lakning tezligi 400 m/s bo'lsa, katta bo'lakning tezligi topilsin. Og'irlik kuchi inobatga olinmasin.

Berilgan:

$\vartheta=200\frac{m}{s}$;

$\alpha_1=60^0$;

$\alpha_2=30^0$;

$\vartheta_2=400\,m/s$;

$m_1=\frac{2}{3}m$;

$m_2=\frac{1}{3}m$.

$\vartheta_1=?$

Yechish:

Jarayon uchun impulsning saqlanish qonunini yozamiz:

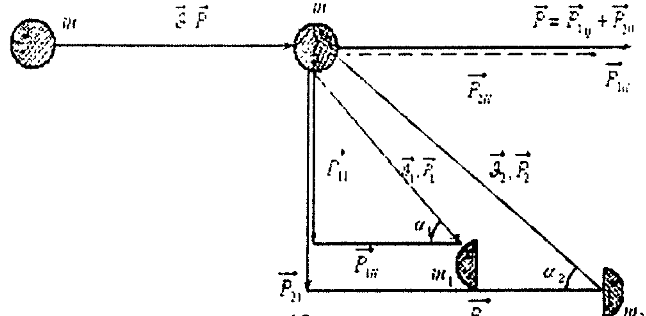
$$\vec{P}=\vec{P_1}+\vec{P_2}.$$

Zambarak o'qi gorizontal yo'nalishda harakatlanayotganligini nazarda tutsak, unda impulsning saqlanish qonuni

$$\vec{P}=\vec{P_{1II}}+\vec{P_{2II}},\ \text{yoki}\ P=P_{1II}+P_{2II}, \quad (1)$$

ko'rinishni oladi (chizmaga qarang). O'z navbatida

$$P_{1II}=P_1\cdot\cos\alpha_1,\ P_{2II}=P_2\cdot\cos\alpha_2 \quad (2)$$



10-rasm

Impuls $P = m \cdot \vartheta$ ko‘rinishda aniqlanishini nazarda tutib (2) yordamida (1) ni qayta yozsak:

$$m\vartheta = m_1\vartheta_1 \cos\alpha_1 + m_2\vartheta_2 \cos\alpha_2 ,$$

yoki

$$m\vartheta = \frac{2}{3} m\vartheta_1 \cos\alpha_1 + \frac{1}{3} m\vartheta_2 \cos\alpha_2 .$$

Bundan $3\vartheta = 2 \cdot \vartheta_1 cos\alpha_1 + \vartheta_2 \cos\alpha_2$,

$$\vartheta_1 = \frac{3\vartheta - \vartheta_2 \cdot cos\alpha_2}{2 \cdot \cos\alpha_1} . \qquad (3)$$

Berilganlarni (3) ga qo‘yib, quyidagini topamiz:

$$\vartheta_1 = \frac{3 \cdot 200 - 400 \cdot \frac{\sqrt{3}}{2}}{2 \cdot \frac{1}{2}} \frac{\text{m}}{\text{s}} = 254 \text{m/s} .$$

Javob: $\vartheta_1 = 254$ m/s.

10-misol. 240 kg massali qayiqda 60 kg massali odam bor. Qayiq 2 m/s tezlik bilan suzmoqda. Odam gorizontal yo‘nalishda 4 m/s tezlik bilan (qayiqqa nisbatan) qayiqdan sakraydi. Agar odam: 1) qayiq harakati yo‘nalishida; 2) qayiq yo‘nalishiga teskari yo‘nalishda sakrasa, qayiqning keyingi tezligi topilsin.

Berilgan:

$m_1 = 240\text{kg}$;

$m_2 = 60\text{kg}$;

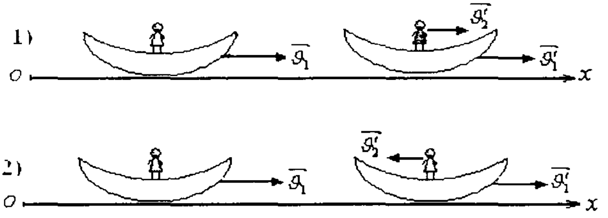
$\vartheta_1 = 2\,\text{m/s}$;

1) $\vartheta_2' = 4\,\text{m/s}$;

2) $\vartheta_2' = -4\,\text{m/s}$.

$\vartheta_1' = ?$

Yechish:



11-rasm

Son o'qini chizmadagidek tanlaymiz va jarayon uchun impulsning saqlanish qonunini yozamiz. Sakrashgacha qayiq-odam sistemasining to'la impulsi sakragandan keyingi impulslar yig'indisiga teng bo'lishi kerak.

$$\vec{P}_q + \vec{P}_{od} = \vec{P}_q + \vec{P}'_{od}. \qquad (1)$$

Impulsning aniqlanishiga muvofiq:

$$\vec{P}_q = m_1\vec{\vartheta}_1,\ \overrightarrow{P_{od}} = m_2\vec{\vartheta}_1,\ \vec{P}_q' = m_1\vec{\vartheta}_1',\ \vec{P}_{od}' = m_2(\vec{\vartheta}_2' + \vec{\vartheta}_1). \qquad (2)$$

$\overrightarrow{P'_{od}}$ ni aniqlashda ϑ_2' odamning qayiqqa nisbatan tezligi, suvga nisbatan tezligi esa $\left(\overrightarrow{\vartheta_2'} + \overrightarrow{\vartheta_1}\right)$ ekanligi nazarga olingan.

(2) ni (1) ga qo'yib, undan ϑ_1' ni aniqlaymiz:

$$m_1\overrightarrow{\vartheta_1} + m_2\overrightarrow{\vartheta_1} = m_1\overrightarrow{\vartheta_1'} + m_2(\overrightarrow{\vartheta_2'} + \overrightarrow{\vartheta_1}),$$

$$\overrightarrow{\vartheta_1'} = \frac{(m_1 + m_2)\overrightarrow{\vartheta_1} - m_2(\overrightarrow{\vartheta_2'} + \overrightarrow{\vartheta_1})}{m_1}. \qquad (3)$$

Tanlangan son o'qi holida ϑ_2' dastlab 4 m/s ga, keyingi holda esa −4 m/s ga teng bo'ladi. Berilganlarni (3) ga qo'yib, quyidagini olamiz:

1) $$\vartheta_1' = \frac{(240 + 60)\cdot 2 - 60(4 + 2)}{240}\frac{\text{m}}{\text{s}} = \frac{600 - 360}{240}\frac{\text{m}}{\text{s}} = 1\frac{\text{m}}{\text{s}};$$

2) $$\vartheta_1' = \frac{(240 + 60)\cdot 2 - 60(-4 + 2)}{240}\frac{\text{m}}{\text{s}} = \frac{600 + 120}{240}\frac{\text{m}}{\text{s}} = 3\frac{\text{m}}{\text{s}}.$$

Javob: 1) $\vartheta_1' = 1\frac{\text{m}}{\text{s}}$; 2) $\vartheta_1' = 3\frac{\text{m}}{\text{s}}$.

Mustaqil yechish uchun masalalar

1. Jism 10N o‘zgarmas kuch ta’sirida chiziqli harakatlanadi va u o‘tgan *S* yo‘lning t vaqtga bog‘liqligi $S=A-Bt+Ct^2$ tenglama bilan berilgan. Bunda $C=1$m/s. Jismning massasi m aniqlansin. [5 kg.]

2. Stolda 4 kg massali aravacha turibdi. Aravachaga, ikkinchi uchi chig‘ir orqali tashlangan chilvir bog‘lab qo‘yilgan. Agar chilvirning ikkinchi uchiga 1 kg massali tosh osilsa, aravacha qanday a tezlanish bilan harakat qiladi. [1,96 m/s.]

3. Vagon tormozlanib, 3,3 s vaqt oralig‘ida uning tezligi 47,5 km/soatdan 30km/soatgacha bir tekis o‘zgaradi. Vagon tormozlanganda tokchadagi chamadon siljiy boshlashi uchun, chamadon va tokcha orasidagi ishqalanish koeffitsientining chegaraviy qiymati qanday bo‘lishi kerak? [0.15.]

4. 600 m/s tezlik bilan uchayotgan $4{,}65\cdot10^{-26}$kg massali molekula idish devoriga normal bilan 60^0 burchak hosil qilib uriladi va shunday burchak ostida tezligini o‘zgartirmay elastik qaytadi. Urilishda idish devoriga berilgan kuch impulsi topilsin. [$2{,}8\cdot10^{-23}$N $\cdot$ s.]

5. 1 *t* massali bosqon 2 m balandlikdan sandonga tushadi. Zarbaning ta’sir etish vaqti $t=0{,}01$ s. Zarba kuchining o‘rtacha qiymati $\langle F\rangle$ aniqlansin. [626 kN.]

6. Yer sirtidan tik yuqoriga qarab uchirilgan 1t massali fazoviy kema 2 g tezlanish bilan ko‘tarilmoqda. Soplodan otilib chiqayotgan gaz oqimining tezligi 1200 m/s. Sarflanayotgan yoqilg‘i miqdori Q_m topilsin. [24,5 kg/s.]

7. Gorizont bilan 30^0 burchak hosil qiladigan qiya tekislikning eng yuqori nuqtasiga vaznsiz chig‘ir mahkamlangan. Massalari teng, $m_1=m_2=1$kg bo‘lgan yuklar bir-birlari bilan ip yordamida birlashtirilib, chig‘irga osilgan. 1) yuklarning harakat tezlanishi; 2) ipning taranglik kuchi, – topilsin. Chig‘irdagi ishqalanish hamda yukning qiya tekislikka ishqalanishi hisobga olinmasin. [1) 2,45m/s^2; 2) 7,35N.]

8. Chig‘ir, gorizont bilan 30^0 va 45^0 burchaklar hosil qiladigan qiya tekisliklarning cho‘qqisiga mahkamlangan. Massalari $m_1=m_2=2$kg bo‘lgan yuklar chig‘ir orqali oshirib tashlangan ip yordamida biriktirilgan. Ip va chig‘irlarni vaznsiz deb hisoblab, chig‘irning ishqalanishini inobatga olmay, yuklarning qiya tekislikdagi ishqalanish koeffitsientlarini $f_1=f_2=f=0{,}1$ deb qabul qilib: 1) yuklarning tezlanishlari. 2) ipning taranglik kuchi, – topilsin. [0,24 m/s^2; 2) 12 N.]

9. Suv oqimi, oqim yo‘nalishiga 60^0 burchak ostida qo‘yilgan harakatsiz tekislikka uriladi. Oqimning tezligi 20 m/s ga, ko‘ndalang kesimining yuzasi 5sm^2 ga teng. Oqimning tekislikka bosim kuchi aniqlansin. [346 N.]

10. O‘qning boshlang‘ich tezligi 800 m/s. Havoda harakatlanishda 0,8 s vaqt davomida uning tezligi 200 m/s gacha kamaydi. O‘qning massasi 10 g. Havoning qarshilik kuchi tezlik kvadratiga proporsional deb hisoblab, qarshilik koeffitsienti k aniqlansin. Og‘irlik kuchining ta’siri hisobga olinmasin. [$4{,}7 \cdot 10^{-5}$ kg/m.]

11. 10 kg massali o‘q trayektoriyasining eng yuqori nuqtasida 200 m/s tezlikka ega bo‘lgan. Shu nuqtada u ikki bo‘lakka bo‘linib ketdi.

3 kg massali kichik bo‘lak dastlabki yo‘nalishda 400 m/s tezlik oldi. Bo‘linishdan keyingi katta, ikkinchi bo‘lakning tezligi ϑ_2 topilsin. [114 m/s.]

12. Samolyot 200 m radiusli Nesterov halqasini yasamoqda. Agar samolyotning tezligi 100 m/s bo‘lsa, uchuvchining eng pastki nuqtada o‘rindiqqa bosim kuchi F, uning og‘irlik kuchi P dan necha marta katta bo‘ladi? [6.1 marta.]

13. Massasi 5 t bo‘lgan avtomobil 10 m/s tezlik bilan qavariq ko‘prikdan harakatlanmoqda. Agar ko‘prikning egrilik radiusi 50 m bo‘lsa, eng yuqori qismda avtomobilning ko‘prikka bosim kuchi F aniqlansin. [39 kN.]

14. 5t massali motorli qayiqda, 7 m/s tezlik bilan 25 m/s suvni motorli qayiq orqasiga otuvchi suvotgich bor. Motorli qayiq harakatiga qarshilikni inobatga olmay: 1) harakat boshlanganidan 3 min. dan keyin motorli qayiqning tezligi; 2) motorli qayiqning mumkin bo‘lgan eng katta tezligi aniqlansin. [1) 6,6 m/s; 2) 7 m/s.]

15. Radiusi 2m bo‘lgan gorizontal holatdagi diskning o‘rtasida nishon, chetida esa havo to‘pponchasi o‘rnatilgan. Disk harakatsiz bo‘lganda sharcha nishonining markaziga tegadi. Disk o‘zining markazidan o‘tgan tik o‘q atrofida o‘zgarmas 0,5 rad/s burchak tezlik bilan aylanganda esa, sharcha nishon markazidan 10 sm masofada joylashgan nuqtaga borib tegadi. Sharchaning tezligini toping. [20m/s.]

3-§. Ish va energiya

Asosiy formulalar

O‘zgarmas kuchining bajargan ishi:

$$A = \vec{F} \cdot \vec{r} = F \cdot r \cdot \cos\alpha \text{, ёки } A = F \cdot S \cdot \cos\alpha ,$$

bu yerda: $\vec{r}$ – ko‘chish vektori, $|\vec{r}| = r = S$, α – kuch vektori $\vec{F}$ va ko‘chish vektori $\vec{r}$ yo‘nalishlari orasidagi burchak.

O‘zgaruvchan kuchning ishi:

$$A = \int_L F(r) \cdot \cos\alpha \cdot dr .$$

Integrallash ko‘chish trayektoriyasi L bo‘ylab olib boriladi. Δt vaqt intervali uchun o‘rtacha quvvat:

$$< N >= \frac{\Delta A}{\Delta t} .$$

Oniy quvvat:

$$N = \frac{dA}{dt} \text{, yoki } N = \frac{\vec{F} \cdot \vec{dr}}{dt} = \vec{F} \cdot \vec{V} = F \cdot \vartheta \cdot \cos\alpha ,$$

bu yerda: dA – dt vaqtda bajarilgan ish.

Ilgarilanma harakat qilayotgan moddiy nuqtaning (yoki jismning) kinetik energiyasi:

$$T = \frac{m\vartheta^2}{2} \text{, yoki } T = \frac{P^2}{2m} .$$

Maydonning biror nuqtasida turgan jismga ta’sir etuvchi kuch va bu jismning potensial energiyasi orasida quyidagi bog‘lanish mavjud:

$$\vec{F} = -\text{grad}P = -\left(\vec{i}\frac{dP}{dx} + \vec{j}\frac{dP}{dy} + \vec{k}\frac{dP}{dz} \right).$$

Sharsimon simmetrik maydon holida (masalan, gravitatsion maydon)

$$F = -\frac{dP}{dr}.$$

Bir jinsli og'irlik kuchi maydonida turgan jismning potensial energiyasi

$$P = mgh.$$

Bunda: h –jismning nolinchi sifatida qabul qilingan sathdan balandligi. Formula $h << R$ (R – Yerning radiusi) hol uchun o'rinli.

Konservativ kuchlar ta'sir etadigan yopiq sistema uchun to'la mexanik energiyaning saqlanish qonuni

$$T + P = E = \text{const}.$$

Energiyaning va impulsning saqlanish qonunlarini qo'llash to'g'ri, markaziy urilishda sharlarning tezliklari uchun quyidagi ifodalarni topishga imkon beradi:
absolut noelastik urilish

$$u = (m_1\vartheta_1 + m_2\vartheta_2)/(m_1 + m_2);$$

absolut elastik urilish

$$u_1 = \frac{\vartheta_1(m_1 - m_2) + 2m_2\vartheta_2}{m_1 + m_2},$$

$$u_2 = \frac{\vartheta_2(m_2 - m_1) + 2m_1\vartheta_1}{m_1 + m_2},$$

bu yerda: m_1 va m_2 – sharlarning massalari, ϑ_1 va ϑ_2 – ularning urilishgacha bo'lgan tezliklari.

Masala yechishga doir misollar

1-misol. Tekis o'suvchi kuchning 12 m yo'lni o'tishda bajargan ishi A hisoblansin. Agar kuch yo'lning boshida 10 N bo'lsa, yo'lning oxirida 46 N bo'ladi.

Berilgan:
S=12 m;
F_1=10 N;
F_2=46 N.
A=?

Yechish:
Tekis o'zgaruvchan kuchning ishi

$$A = \langle F \rangle \cdot S \cdot \quad (1)$$

formula yordamida aniqlanadi. Bu yerda $\langle F \rangle$ kuchning o'rtacha qiymati

$$< F > = \frac{F_1 + F_2}{2}. \qquad (2)$$

(2)ni (1)ga qo'yib quyidagini topamiz:

$$A = \frac{F_1 + F_2}{2} \cdot S. \qquad (3)$$

Berilganlarni (3)ga qo'ysak,

$$A = \frac{10+46}{2} \cdot 12\text{J} = 28 \cdot 12\text{J} = 336\text{J}.$$

Javob: A =336 J.

2-misol. Massasi 1 kg bo'lgan jism o'zgarmas F kuch ta'sirida $S=Bt^2+Ct+D$ kinematik qonunga muvofiq to'g'ri chiziqli harakat qiladi. Bu yerda $\text{B}=1\,\frac{\text{m}}{\text{s}^2} \cdot \text{C}=2\,\frac{\text{m}}{\text{s}}$. Ta'sir boshlanganidan 5 s dan keyin kuchning ishi aniqlansin.

Berilgan:

$m = 1\text{kg};$

$S = Bt^2 + Ct + D;$

$B = 1\,{}^{\text{m}}/_{\text{s}^2};$

$C = 2\,{}^{\text{m}}/_{\text{s}};$

$t_0 = 5\,\text{s}.$

$A=?$

Yechish:

Jism harakatining kinematik qonuni berilgan holda elementar ishni aniqlash maqsadga muvofiq

$$\text{dA} = \text{F} \cdot \text{dS}. \qquad (1)$$

To'la ishni topish uchun bu ifodani *0* dan t_0 gacha integrallash (barcha elementar ishlarni uzluksiz qo'shib chiqish) kerak:

$$A = \int_0^{t_0} dA = \int_0^{t_0} F \cdot dS. \qquad (2)$$

Jismga ta'sir etadigan kuchni topish uchun Nyutonning ikkinchi qonunidan foydalanamiz:

$$F = m \cdot a = m \cdot \frac{d^2S}{dt^2}.$$

Bizning holimizda u quyidagicha bo'ladi:

$$F = m \cdot \frac{d^2}{dt^2}(Bt^2 + Ct + D) = m \cdot \frac{d}{dt}(2Bt + C) = 2Bm \qquad (3)$$

dS ni esa differensiallash qoidasiga muvofiq quyidagini aniqlaymiz:

$$dS = S_t' \cdot dt = (Bt^2 + Ct + D)_t' \cdot dt = (2Bt + C) \qquad (4)$$

(3) va (4) larni (2) ga qo‘yib olamiz:

$$A = 2\int_0^{t_0} Bm(2Bt + C)dt = \int_0^{t_0}(4B^2mt + 2BCm)dt = \left[4B^2m\frac{t^2}{2} + \right.$$

$$\left. + 2BCmt\right]\Big|_0^{t_0} = 2Bmt(Bt + C)\Big|_0^{t_0} = 2Bmt_0(Bt_0 + C).$$

Demak,

$$A = 2Bmt_0(Bt_0 + C)\,. \qquad (5)$$

Hosil qilingan ifodaning to‘g‘riligini tekshirish maqsadida kattaliklarning birliklarini qo‘yamiz.

$$[A] = [B][m][t]([B][t] + [c]) = 1\frac{m}{s^2}\cdot 1kg\cdot 1s\cdot 1\frac{m}{s} =$$

$$= 1kg\cdot\frac{m}{s^2}\cdot 1m = 1\cdot N\cdot m = 1J.$$

Endi berilganlarni (5) ga qo‘yib, quyidagini olamiz:

$$A = 2\cdot 1\cdot 1\cdot 5(1\cdot 5 + 2)J = 10\cdot(5 + 2)J = 70J\,.$$

Javob: A=70 J.

3-misol. Massasi 2kg bo‘lgan moddiy nuqta *ox* o‘qi bo‘ylab yo‘nalgan kuch ta’sirida $x = A + Bt + Ct^2 + Dt^3$ tenglamaga binoan harakat qiladi. Bu yerda B=−2m/s, C=1m/s^2, D=−0,2m/s^3. Vaqtning $t_1 = 2s$ va $t_2 = 5s$ onlarida kuch erishtira oladigan quvvat N topilsin.

Berilgan:

$m = 2\text{kg}$;

$x = A + Bt + Ct^2 + Dt^3$

$B = -2\,{}^{m}/_{s}$;

$S = 1\,{}^{m}/_{s^2}$;

$D = -0{,}2\,{}^{m}/_{s^3}$;

$t_1 = 2s$;

$t_2 = 5s$.

$N_1 = ?$

$N_2 = ?$

Yechish: Oniy quvvat

$$N = F \cdot \vartheta \cdot \cos\alpha$$

formula bilan aniqanadi. Berilgan masalada kuch va ko'chish yo'nalishi mos keladi, ya'ni $\alpha = 0$, $\cos\alpha = 1$. Unda

$$N = F \cdot \vartheta. \qquad (1)$$

Kuchning qiymatini esa Nyutonning ikkinchi qonuni $F = ma$ ga muvofiq aniqlaymiz

$$N = m \cdot a \cdot \vartheta. \qquad (2)$$

Oniy tezlik va tezlanishlarni esa koordinatadan vaqt bo'yicha olingan birinchi va ikkinchi tartibli hosilalardek aniqlaymiz:

$$\vartheta = \frac{dx}{dt} = \frac{d}{dt}\left(A + Bt + Ct^2 + Dt^3\right) = B + 2Ct + 3Dt^2. \qquad (3)$$

$$a = \frac{d^2x}{dt^2} = \frac{d\vartheta}{dt} = \frac{d}{dt}\left(B + 2Ct + 3Dt^2\right) = 2C + 6Dt. \qquad (4)$$

(3) va (4) larni (2) ga qo'yib, quyidagini topamiz:

$$N = m(2C + 6Dt)\left(B + 2Ct + 3Dt^2\right). \qquad (5)$$

(2) formula asosida N ning birligini tekshirib ko'ramiz:

$$[N] = [m][a][\vartheta] = 1\text{kg} \cdot 1\frac{\text{m}}{\text{s}^2} \cdot 1\frac{\text{m}}{\text{s}} = 1\text{N} \cdot 1\frac{\text{m}}{\text{s}} = 1\frac{\text{J}}{\text{s}} = 1\text{W}$$

Endi berilganlarni (5) ga qo'yib olamiz:

$$N_1 = 2(2 \cdot 1 - 6 \cdot 0{,}2 \cdot 2)(-2 + 2 \cdot 1 \cdot 2 - 3 \cdot 0{,}2 \cdot 2^2)$$
$$\text{W} = 2 \cdot (-0{,}4)(-0{,}4)\text{W} = 0{,}32\text{W};$$

$$N_2 = 2(2 \cdot 1 - 6 \cdot 0{,}2 \cdot 5)(-2 + 2 \cdot 1 \cdot 5 - 3 \cdot 0{,}2 \cdot 5^2)$$
$$\text{W} = 2 \cdot (-0{,}4)(-7)\text{W} = 56\text{W}.$$

Javob: $N_1 = 0{,}32W$, $N_2 = 56W$.

4-misol. Nasos diametri 2 sm bo'lgan suv oqimini 20 m/s tezlik bilan chiqara oladi. Suvni chiqarish uchun kerak bo'lgan o'rtacha quvvat topilsin.

Berilgan:

d=2 sm=2 · 10^{-2} m;

$\vartheta = 20$ m/s

$< N >= ?$

Yechish: O‘rtacha quvvatni

$$< N >= F \cdot < \vartheta > \qquad (1)$$

formula yordamida aniqlaymiz.

Bu yerda $< \vartheta >$ suvning o‘rtacha tezligi. Suvning dastlabki tezligi nolga teng: $\vartheta_0 = 0$.

Suv tekis tezlanuvchan harakat qiladi, deb olish mumkin

$$< \vartheta >= \frac{\vartheta_0 + \vartheta}{2} = \frac{\vartheta}{2} . \qquad (2)$$

Unda

$$< N >= \frac{F \cdot \vartheta}{2} . \qquad (3)$$

Kuchni esa Nyutonning ikkinchi qonuniga muvofiq topamiz

$$F = m \cdot a \cdot \qquad (4)$$

Tekis tezlanuvchan harakatda tezlanish

$$a = \frac{\vartheta - \vartheta_0}{t} = \frac{\vartheta}{t} , \qquad (5)$$

bu yerda: $\vartheta_0 = 0$ ligi hisobga olingan, t – suvning harakatlanish vaqti. Suvning massasi esa

$$m = \rho V = \rho \cdot sl = \rho \frac{\pi d^2}{4} \vartheta t \qquad (6)$$

kabi aniqlanadi. Bu formulani hosil qilishda oqimning hajmi $V = S \cdot l$, oqimning ko‘ndalang kesimi $S = \frac{\pi d^2}{4}$, oqim uzunligi $l = \vartheta \cdot t$ ekanligidan foydalanildi. (5) va (6) ni (4) ga qo‘yib, quyidagini olamiz:

$$F = \rho \frac{\pi d^2}{4} \cdot \vartheta \cdot t \cdot \frac{\vartheta}{t} = \frac{\pi d^2}{4} \rho \vartheta^2 . \qquad (7)$$

(7) ni (3) ga qo‘yib o‘rtacha quvvat uchun

$$<N>=\frac{\pi d^2}{8}\rho\vartheta^3. \qquad (8)$$

ni olamiz. Olingan ifoda yordamida quvvatning birligini tekshirib,

$$[N]=[d]^2\cdot[\rho][\vartheta]^3=1\text{m}^2\cdot1\frac{\text{kg}}{\text{m}^3}\cdot1\frac{\text{m}^3}{\text{s}^3}=1\text{kg}\frac{\text{m}}{\text{s}^2}\cdot\frac{\text{m}}{\text{s}}=1\frac{\text{N}\cdot\text{m}}{\text{s}}=1\text{W}$$

uning to‘g‘riligiga ishonch hosil qilgandan keyin, $\rho=10^3\frac{kg}{m^3}$ va berilganlarni (8) ga qo‘yib, quyidagini olamiz:

$$<N>=\frac{3{,}14\cdot(2\cdot10^{-2})^2}{8}\cdot10^3\cdot(20)^3\,\text{W}=\frac{3{,}14\cdot32}{8}\cdot10^2\,\text{W}=12{,}6\cdot10^2\,\text{W}=1{,}2\text{kW}.$$

Javob: <N>=1,26 kW.

5-misol. Jism 3m/s tezlik bilan harakat qilardi. So‘ngra 5 s davomida unga 4 N kuch ta’sir qiladi. Bu vaqt davomida jismning kinetik energiyasi 100J ga ortdi. Jismning massasi va kuch ta’sirining oxirida uning tezligi topilsin.

Berilgan:	Yechish:
$\vartheta_0=3^{\text{m}}/_{\text{s}}$;	Masalani yechish uchun impulsning saqlanish qonunidan foydalanamiz.
Δt=5 s;	
F=4 N	
ΔT=100.	Impulsning o‘zgarishi:
m=?	$F\cdot\Delta t=P-P_0=m\vartheta-m\vartheta_0=m(\vartheta-\vartheta_0)$. (1)
$\vartheta=?$	Jism kinetik energiyasining o‘zgarishi:
	$\Delta T=T-T_0=\frac{m\vartheta^2}{2}-\frac{m\vartheta_0^2}{2}$.

Bundan

$$2\cdot\Delta T=m(\vartheta^2-\vartheta_0^2)=m(\vartheta-\vartheta_0)(\vartheta+\vartheta_0). \qquad (2)$$

(2) ning o‘ng tomonidagi $m(\vartheta - \vartheta_0)$ ifoda o‘rniga (1) dan foydalanib $F \cdot \Delta t$ ni qo‘yamiz:

$$2 \cdot \Delta T = F \cdot \Delta t \cdot (\vartheta + \vartheta_0).$$

Bu ifodadan ϑ ni topib, quyidagini olamiz:

$$\vartheta = \frac{2 \cdot \Delta T}{F \cdot \Delta t} - \vartheta_0. \tag{3}$$

(1) ifodadan m ni topamiz:

$$m = \frac{F \cdot \Delta t}{\vartheta - \vartheta_0}. \tag{4}$$

(3) va (4) yordamida kattaliklarning birliklarini tekshirib ko‘ramiz ((3) ning birinchi hadini olish kifoya):

$$[\vartheta] = \frac{[T]}{[F][t]} = \frac{1\text{J}}{1\text{N} \cdot 1\text{s}} = \frac{1\text{N} \cdot \text{m}}{1\text{N} \cdot \text{s}} = 1\frac{\text{m}}{\text{s}};$$

$$[m] = \frac{[F][t]}{[\vartheta]} = \frac{1\text{N} \cdot 1\text{s}}{1\frac{\text{m}}{\text{s}}} = 1\text{kg} \cdot \frac{\text{m}}{\text{s}^2}\frac{\text{s}^2}{\text{m}} = 1\text{kg}.$$

Berilganlarni bu formulalarga qo‘yamiz:

$$\vartheta = \left[\frac{2 \cdot 100}{4 \cdot 5} - 3\right]\frac{\text{m}}{\text{s}} = (10 - 3)\frac{\text{m}}{\text{s}} = 7\frac{\text{m}}{\text{s}};$$

$$m = \frac{4 \cdot 5}{7 - 3}\text{kg} = \frac{4 \cdot 5}{4}\text{kg} = 5\text{kg}.$$

Javob: $\vartheta = 7\frac{\text{m}}{\text{s}}$; m=*5* kg.

6-misol. 100 kg massali yukni 2 s vaqt davomida 4 m balandlikka tekis tezlanuvchan ko‘tarishda bajarilgan A ish hisoblansin.

Berilgan:	Yechish:
$m=100$ kg $t=2$ s; $h=4$ m. $A=?$	Bajarilgan ish yukni tekis tezlanish bilan harakatlantirish va unga potensial energiya berish (og'irlik kuchini yengish) uchun sarflanadi

$$A = A_0 + \Pi . \qquad (1)$$

bu yerda A_0 yukni tekis tezalnish bilan harakatlantirish uchun bajarilgan ish

$$A_0 = F \cdot h . \qquad (2)$$

Kuchni Nyutonning ikkinchi qonuniga muvofiq $F = ma$

tezlanishni esa, $h = \frac{at^2}{2}$ formuladan aniqlaymiz. $a = \frac{2h}{t^2}$. Unda kuch quyidagi ko'rinishni oladi:

$$F = \frac{2h}{t^2} \cdot m . \qquad (3)$$

(3) ni (2) ga qo'yamiz:

$$A_0 = \frac{2mh^2}{t^2} . \qquad (4)$$

Yukning potensial energiyasi esa:

$$P = mgh \qquad (5)$$

ifoda yordamida aniqlanadi.

A_0 va P lar uchun topilgan ifodalarni (1) ga qo'yamiz

$$A = \frac{2mh^2}{t^2} + mgh = mh\left(\frac{2h}{t^2} + g\right) . \qquad (6)$$

(6) yordamida A ning birligini tekshirib ko'ramiz:

$$[A] = [f][m][g][s] = 1 \cdot 1\text{kg} \cdot 1\frac{\text{m}}{\text{s}^2} \cdot 1\text{m} = 1\text{N} \cdot \text{m} = 1\text{J} ,$$

va berilganlarni unga qo'yamiz:

$$A=100\cdot 4\left(\frac{2\cdot 4}{2^2}+9{,}8\right)\text{J}=400(2+9{,}8)\text{J}=400\cdot 11{,}8\text{J}=4720\text{J}.$$

Javob: A=4720J.

7-misol. 1 t massali bosqon 2 m balandlikdan sandonga tushadi. Urilish 0,01 s davom etadi. Urilishning o‘rtacha kuchi $< F >$ aniqlansin.

Berilgan:
m=1 t=10^3kg;
h=2m;
$\Delta t = 0{,}01\text{s}$.

$< F >= ?$

Yechish: Jarayon uchun impulsning saqlanish qonuni quyidagi ko‘rinishga ega bo‘ladi:

$$< F > \Delta t = m\vartheta,$$

bundan esa $< F >=\dfrac{m\vartheta}{\Delta t}$. (1)

Shuningdek, energiyaning saqlanish qonunini yozamiz. Bosqonning h balandlikda turgandagi potensial energiyasi $P=mgh$, u sandonga urilayotganidagi kinetik energiyasi $T=\dfrac{m\vartheta^2}{2}$ ga teng.

$$P=T,\quad \text{yoki}\quad mgh=\frac{m\vartheta^2}{2}$$

bundan ϑ ni aniqlaymiz:

$$\vartheta=\sqrt{2gh}. \qquad (2)$$

(2) ni (1) ga qo‘yamiz:

$$< F >=\frac{m}{\Delta t}\sqrt{2gh}. \qquad (3)$$

(3) yordamida kuchning birligini tekshirib ko‘ramiz:

$$< F >=\frac{[m]}{[t]}[g]^{1/2}=[\text{h}]^{1/2}=\frac{1\text{kg}}{1\text{s}}\left[1\frac{\text{m}}{\text{s}^2}\right]^{1/2}[1\text{m}]^{1/2}=1\text{kg}\frac{\text{m}}{\text{s}^2}=1\,\text{N},$$

va uning to‘g‘riligiga ishonch hosil qilgach, berilganlarni qo‘yamiz:

$$<F>=\frac{10^3}{0{,}01}\sqrt{2\cdot 9{,}8\cdot 2}\text{N}=10^5\sqrt{4\cdot 9{,}8}\text{N}=6{,}3\cdot 10^5\text{N}.$$

Javob: $<F>=6{,}3\cdot 10^5\text{N}=630\text{kN}.$

8-misol. 10 g massali po'lat sharcha 1 m balandlikdan po'lat taxtaga tushdi va urilishdan keyin sakrab 0,8 m balandlikka ko'tarildi. Sharcha impulsining o'zgarishi aniqlansin.

Berilgan:

$m=10\text{g}=10^{-2}\text{kg}$;

$h_1=1\text{m}$;

$h_2=0{,}8\text{m}$.

$\Delta p=?$

Yechish: Jarayon uchun impulsning saqlanish qonunini yozamiz. Bunga muvofiq, sharchaning urilishgacha bo'lgan impulsi $\vec{P}_1$, uning urilishdan keyingi impulsi $\vec{P}_2$ va po'lat taxta olgan turtki impulsi, ya'ni sharcha impulsining o'zgarishi $\Delta\vec{P}$ larning yig'indisiga teng. $\vec{P}_1$ va $\vec{P}_2$ larning qarama-qarshi yo'nalganligini qayd etish kerak:

$$\vec{P}_1=\vec{P}_2+\Delta\vec{P},$$

bundan

$$\Delta\vec{P}=\vec{P}_1-\vec{P}_2=m\vec{\vartheta}_1-m\vec{\vartheta}_2=m(\vec{\vartheta}_1-\vec{\vartheta}_2). \qquad (1)$$

ϑ_1 va ϑ_2 larning qarama-qarshi yo'nalganligidan quyidagini yozamiz:

$$\Delta P=m[\vartheta_1-(-\vartheta_2)]=m(\vartheta_1+\vartheta_2). \qquad (2)$$

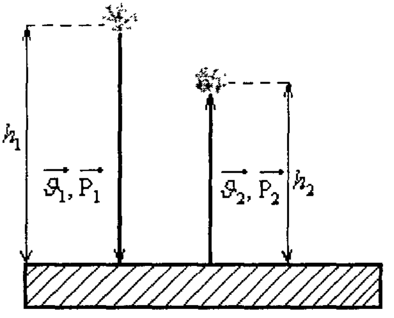
Endi energiyaning saqlanish qonunini qo'llaymiz:

1) h_1 balandlikdagi sharning potensial energiyasi uning urilish oldidagi kinetik energiyasiga teng:

$$P_1=T_1, \quad \text{yoki} \quad mgh_1=\frac{m\vartheta_1^2}{2},$$

bundan

$$\vartheta_1=\sqrt{2gh_1}; \qquad (3)$$



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2) sharchaning sakrayotgandagi kinetik energiyasi uning h_2 balandlikdagi potensial energiyasiga teng:

$$T_2 = P_2, \quad \text{yoki} \quad \frac{m\vartheta_2^2}{2} = mgh_2,$$

bundan
$$\vartheta_2 = \sqrt{2gh_2}\,. \qquad (4)$$

(3) va (4) ni (2) ga qo'yib quyidagini topamiz:

$$\Delta p = m\left(\sqrt{2gh_1} + \sqrt{2gh_2}\right). \qquad (5)$$

(5) yordamida impulsning birligini tekshirib ko'ramiz:

$$[P] = [m][gh]^{1/2} = [m][g]^{1/2}\cdot[h]^{1/2} = 1\text{kg}\left[1\frac{\text{m}^2}{\text{s}^2}\right]^{1/2} = 1\text{kg}\cdot\frac{\text{m}}{\text{s}}$$

va berilganlarni o'rniga qo'yamiz:

$$\Delta P = 10^{-2}\ \sqrt{2\cdot 9{,}8\cdot 1} + \sqrt{2\cdot 9{,}8\cdot 0{,}8}\ \text{kg}\cdot\frac{\text{m}}{\text{s}} = 10^{-2}\cdot\left(\sqrt{19{,}60} + \sqrt{15{,}68}\right)\text{kg}\cdot\frac{\text{m}}{\text{s}} \approx$$

$$\approx 10^{-2}\cdot(4{,}43 + 3{,}96)\text{kg}\cdot\frac{\text{m}}{\text{s}} = 8{,}39\cdot 10^{-2}\,\text{kg}\frac{\text{m}}{\text{s}}.$$

Javob: $\Delta P = 0{,}84\text{kg}\cdot\frac{\text{m}}{\text{s}}$.

9-misol. 6 kg massali shar 4 kg massali boshqa harakatsiz sharga urildi. Birinchi sharning impulsi $5\text{kg}\cdot\frac{\text{m}}{\text{s}}$ ga teng. Urilish to‘g‘ri, noelastik. Urilishdan bevosita keyingi onda:

1) birinchi sharning P_1' va ikkinchi sharning P_2' impulslari; 2) birinchi shar impulsining o‘zgarishi ΔP_1; 3) birinchi sharning T_1' va ikkinchi sharning T_2' kinetik energiyalari; 4) birinchi shar kinetik energiyasining o‘zgarishi ΔT_1; 5) birinchi shar kinetik energiyasining ikkinchi sharga berilgan qismi ω_1 va kinetik energiyaning birinchi sharda qolgan qismi ω_2; 6) sharlar ichki energiyalarining o‘zgarishi ΔU; 7) birinchi shar kinetik energiyasining sharlar ichki energiyasiga aylangan qismi ω, – aniqlansin.

Berilgan:

$m_1 = 6\text{kg};$

$m_2 = 4\text{kg};$

$P_1 = 5\text{kg}\cdot\frac{\text{m}}{\text{s}};$

$P_2 = 0.$

1) $P_1' = ?$

2) $\Delta P_1 = ?$

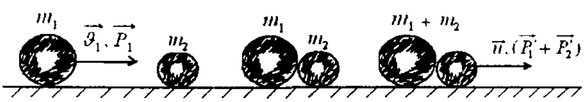
3) $T_1' = ?\ \ T_2' = ?$

4) $\Delta T_1 = ?$

5) $\omega_1 = \frac{T_2'}{T_1} = ?;\ \ \omega_2 = \frac{T_1'}{T_1} = ?$

6) $\Delta U = ?$

7) $\omega = \frac{\Delta U}{T_1} = ?$



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Yechish: Impulsning saqlanish qonuniga muvofiq birinchi sharning urilishgacha bo‘lgan impulsi sharlarning urilishdan keyingi impulslari yig‘indisiga teng bo‘lishi kerak, ya’ni

$$\overrightarrow{P_1} = \overrightarrow{P_1'} + \overrightarrow{P_2'}, \qquad \text{yoki} \qquad P_1 = (m_1 + m_2)u.$$

Bundan

$$u = \frac{P_1}{m_1 + m_2}. \tag{1}$$

1. Urilishdan keyingi onda sharlarning impulslari

$P_1' = m_1 u$ va $P_2' = m_2 u$. (1) ni keltirib qo'ysak,

$$P_1' = \frac{m_1 P_1}{m_1 + m_2}, \qquad P_2' = \frac{m_2 P_1}{m_1 + m_2}. \tag{2}$$

Berilganlarni qo'yib topamiz

$$P_1' = \frac{6\cdot 5}{6+4}\text{kg}\cdot\frac{\text{m}}{\text{s}} = \frac{30}{10}\text{kg}\cdot\frac{\text{m}}{\text{s}} = 3\text{kg}\cdot\frac{\text{m}}{\text{s}};$$

$$P_2' = \frac{4\cdot 5}{6+4}\text{kg}\cdot\frac{\text{m}}{\text{s}} = \frac{20}{10}\text{kg}\cdot\frac{\text{m}}{\text{s}} = 2\text{kg}\cdot\frac{\text{m}}{\text{s}}.$$

2. Birinchi shar impulsining o'zgarishi uning urilishdan keyingi va oldingi impulslarining farqiga teng

$$\Delta P_1 = P_1' - P_1 = -P_2',$$

$$\Delta P_1 = (3-5)\text{kg}\cdot\frac{\text{m}}{\text{s}} = -2\text{kg}\cdot\frac{\text{m}}{\text{s}}. \tag{3}$$

3. Sharlarning urilishdan keyingi kinetik energiyalari

$$T_1' = \frac{m_1}{2}u^2 = \frac{m_1}{2}\frac{P_1^2}{(m_1+m_2)^2}, \tag{4}$$

$$T_2' = \frac{m_2}{2}u^2 = \frac{m_2}{2}\frac{P_1^2}{(m_1+m_2)^2}. \tag{5}$$

Berilganlarni qo'yib olamiz:

$$T_1' = \frac{6}{2}\frac{5^2}{(6+4)^2}\text{J} = \frac{6\cdot 25}{2\cdot 100}\text{J} = 0{,}75\text{J};$$

$$T_2' = \frac{4}{2}\frac{5^2}{(6+4)^2}\,\text{J} = \frac{4\cdot 25}{2\cdot 100}\,\text{J} = 0{,}5\,\text{J}\ .$$

4. Birinchi shar kinetik energiyasining o‘zgarishi ΔT_1, sharning dastlabki T_1 va keyingi T_1' kinetik energiyalarining farqiga teng:

$$T_1 = \frac{m_1 \vartheta_1^2}{2} = \frac{m_1^2 \vartheta_1^2}{2m_1} = \frac{P_1^2}{2m_1}\ . \qquad (6)$$

(4) ifodadan:

$$\Delta T_1 = T_1 - T_1' = \frac{P_1^2}{2m_1} - \frac{m_1}{2}\frac{P_1^2}{(m_1+m_2)^2} = \frac{P_1^2\cdot m_2(2m_1+m_2)}{2m_1(m_1+m_2)^2}\ ;$$

$$\Delta T_1 = \left(\frac{5^2}{2\cdot 6} - 0{,}75\right)J = \left(\frac{25}{12} - 0{,}75\right)J = (2{,}08 - 0{,}75)J = 1{,}33J\ .$$

5. (5) va (6) lardan foydalanib yozamiz:

$$\omega_1 = \frac{T_2'}{T_1} = \frac{\dfrac{m_2}{2}\dfrac{P_1^2}{(m_1+m_2)^2}}{\dfrac{P_1^2}{2m_1}} = \frac{m_1\cdot m_2}{(m_1+m_2)^2}\ . \qquad (8)$$

(4) va (6) lar yordamida quyidagini olamiz:

$$\omega_2 = \frac{T_1'}{T_1} = \frac{\dfrac{m_2}{2}\dfrac{P_1^2}{(m_1+m_2)^2}}{\dfrac{P_1^2}{2m_1}} = \frac{m_1^2}{(m_1+m_2)^2}\ . \qquad (9)$$

Berilganlarni (8) va (9) larga qo‘yib olamiz:

$$\omega_1 = \frac{6\cdot 4}{(6+4)^2} = \frac{24}{100} = 0{,}24\ ;$$

$$\omega_2 = \frac{6^2}{(6+4)^2} = \frac{36}{100} = 0{,}36\ .$$

6. Sharlar ichki energiyalarining o'zgarishi ΔU, sharlarning urilishigacha $T_1 (T_2 = 0)$ va urilishdan keyingi $T_1' + T_2'$ kinetik energiyalarining farqiga teng. Chunki energiyaning saqlanish qonuniga muvofiq

$$T_1 = T_1' + T_2' + \Delta U ,$$

bundan

$$\Delta U = T_1 - (T_1' + T_2') . \qquad (10)$$

(6), (4) va (5) larni (10) ga qo'yib quyidagini olamiz:

$$\Delta U = \frac{P_1^2}{2m_1} - \left(\frac{m_1}{2} \frac{P_1^2}{(m_1 + m_2)^2} + \frac{m_2}{2} \frac{P_1^2}{(m_1 + m_2)^2} \right) = \frac{P_1^2}{2} \frac{m_2}{m_1 (m_1 + m_2)} . \qquad (11)$$

Berilganlar yordamida quyidagini topamiz:

$$\Delta U = \frac{5^2}{2} \frac{4}{6(6+4)} \text{J} = \frac{25 \cdot 4}{12 \cdot 10} \text{J} = \frac{2{,}5}{3} \text{J} = 0{,}83\text{J}$$

7. (6) va (11) dan foydalanib

$$\omega = \frac{\Delta U}{T_1} = \frac{\dfrac{P_1^2}{2} \dfrac{m_2}{m_1 (m_1 + m_2)}}{\dfrac{P_1^2}{2m_1}} = \frac{m_2}{m_1 + m_2} . \qquad (12)$$

ni olamiz. Berilganlarning o'rniga qo'yib, hisoblaymiz:

$$\omega = \frac{4}{6+4} = \frac{4}{10} = 0{,}4 .$$

Javob:

1) $P_1' = 3\text{kg} \cdot \frac{\text{m}}{\text{s}}$; $P_2' = 2\text{kg} \cdot \frac{\text{m}}{\text{s}}$; 2) $\Delta P_1 = -2\text{kg} \cdot \frac{\text{m}}{\text{s}}$;

3) $T_1' = 0{,}75\text{J}$; $T_2' = 0{,}5\text{J}$; 4) $\Delta T_1 = 1{,}33\text{J}$;

5) $\omega_1 = 0{,}24$; $\omega_2 = 0{,}36$; 6) $\Delta U = 0{,}83\text{J}$; 7) $\omega = 0{,}4$.

MUSTAQIL YECHISH UCHUN MASALALAR

1. 3 kg massali jismni 2 m balandlikka tikka ko'tarilganda 120J ish bajarildi. Jism qanday tezlanish bilan ko'tarilgan? [10,2 m/s^2.]

2. Massasi 5kg bo'lgan qadoqtosh biror balandlikdan 3s da yerga tushadi. Yo'lning o'rta nuqtasida qadoqtoshning kinetik va potensial energiyalari topilsin. Havoning qarshiligi hisobga olinmasin. [855 J.]

3. 20 kg massali yuk tik yuqori yo'nalgan 400N o'zgarmas kuch ta'sirida 15m balandlikka ko'tarilgan. Ko'tarilgan yukning potensial energiyasi va kuch bajargan ish topilsin. [2,94 kJ; 6kJ.]

4. Minoradan, gorizontal yo'nalishda 20m/s tezlik bilan otilgan 1kg massali jism, 3s dan keyin yerga tushdi. Yerga tushish paytida jism ega bo'lgan kinetik energiya aniqlansin. Havoning qarshiligi hisobga olinmasin. [633 J.]

5. Massasi 3t bo'lgan vertolyot havoda muallaq turibdi. Rotor diametrining ikki qiymati: 1) d_1=18m; 2) d_2=8m– uchun vertolyot motorining shu holatda erishishi mumkin bo'lgan quvvati *N* aniqlansin. Hisoblashda rotor o'z diametriga teng diametrli silindrik havo oqimini pastga otadi deb hisoblansin. [1)139W; 2)313 W.]

6. 10 kg massali yuk, gorizontga nisbatan burchak tashkil qiluvchi, uzunligi 2m bo'lgan qiya tekislikdan 2 s vaqtda ko'tarildi. Ishqalanish koeffisiyenti 0,1. 1) yukni qiya tekislikdan ko'tarishda bajarilgan ish; 2) ko'taruvchi qurilmaning o'rtacha; va 3) maksimal quvvatlari aniqlansin. [1) 170 J; 2) 85 W; 3)173 W.]

7. Konkida uchuvchi, muz ustida turgancha 5kg massali qadoqtoshni oldinga otdi va tepki natijasida 1m/s tezlik bilan orqaga qarab siljidi. Konki uchuvchining massasi 60 kg. Konki uchuvchining qadoqtoshni otishda bajargan ishi aniqlansin. [390 J.]

8. Yuqoridan tushgan 20g massali po'lat sharcha 81 sm balandlikka sakradi. 1) urilishda taxtacha olgan kuch impulsi; 2) urilishda ajralgan issiqlik miqdori aniqlansin. [1) 0,17 Ns; 2) $37{,}2 \cdot 10^{-3}$ J.]

9. Gorizontal yo'nalishda 500 m/s tezlik bilan uchayotgan 10g massali o'q, uzunligi 1m va massasi 5kg bo'lgan ballastik mayatnikka tegdi va tiqilib qoldi. Mayatnikning og'ish burchagi aniqlansin. 18^0 30".

10. 5 m/s tezlik bilan harakatlanayotgan m_1 massali shar, m_2 massali harakatsiz sharga uriladi. Urilish to'g'ri, noelastik. Urilishdan keyin sharlarning tezligi *V*, hamda harakatlanayotgan shar kinetik energiyasining qancha ω qismi sharlar ichki energiyalarini orttirishga sarflanishi aniqlansin. Ikki hol: 1) m_1=2 kg, m_2=8 kg; 2) m_1=8 kg, m_2=2 kg qaralsin. [1) 1 m/s; 0,8; 2) 4m/s; 0,2.]

11. Harakatlanayotgan m_1 massali jism m_2 massali tinch turgan jismga markaziy, absolut elastik urilishi natijasida, birinchi jismning tezligi 1,5 marta kamayadi. 1) $\frac{m_1}{m_2}$ nisbat; 2) agar birinchi jismning dastlabki kinetik energiyasi

1000 J bo‘lsa, ikkinchi jismning harakatlanib boshlagandagi kinetik energiyasi T'_2 aniqlansin. [1) 5; 2) 555 J.]

12. Massasi 1 kg bo‘lgan bolg‘acha bilan massasi 7 g bo‘lgan mix devorga qoqilmoqda. Bolg‘acha zarbasining fikh aniqlansin. [0,93.]

13. 3 m/s tezlik bilan harakatlanayotgan 4 kg massali jism, shunday massali harakatsiz boshqa jismga uriladi. Urilishni markaziy va noelastik deb hisoblab, urilish natijasida ajralib chiqadigan issiqlik miqdori Q hisoblansin. [9 J.]

14. 10^{-27} kg massali, 9 nJ kinetik energiyali zarra, $4 \cdot 10^{-27}$ kg massali harakatsiz zarra bilan elastik to‘qnashadi va unga 5 nJ kinetik energiya beradi. Zarraning dastlabki yo‘nlishdan chetlanish burchagi aniqlansin. [144^0.]

4-§ QATTIQ JISM MEXANIKASI

Asosiy formulalar.

Aylanish o'qiga nisbatan inersiya momentlari:

a) moddiy nuqtaniki

$$J = mr^2,$$

bunda: m – nuqtaning massasi; r – undan to aylanish o'qigacha bo'lgan masofa;

b) diskret qattiq jismniki

$$J = \sum_{i=1}^{n} \Delta m_i \cdot r_i^2,$$

bunda, Δm_i jismning i – qismning massasi; r_i – aylanish o'qidan shu qismgacha bo'lgan masofa; n – jism qismlarining soni;

Yaxlit qattiq jism uchun:

$$J = \int r^2 dm.$$

Agar jismning zichligi ρ uning hajmi bo'yicha bir xil, ya'ni bir jinsli jism bo'lsa

$$dm = \rho dV \quad \text{va} \quad J = \rho \int r^2 dV$$

bunda: V – jismning hajmi.

Jismning ixtiyoriy o'qqa nisbatan inersiya momenti, Shteyner teoremasi

$$J = J_0 + ma^2.$$

Bunda: J_0 – shu jismning berilgan o'qqa parallel va og'irlik markazidan o'tuvchi o'qqa nisbatan inersiya momenti; a – o'qlar orasidagi masofa; m – jism massasi.

Aylanayotgan jismning o'qqa nisbatan impuls momenti:

$$L = J \cdot \omega.$$

Impuls momentining saqlanish qonuni:

$$\sum_{i=1}^{n} L_i = \text{const},$$

bunda: L_i – sistema tarkibiga kiruvchi i – jismning impuls momenti.

Jism	Inersiya momenti aniqlanadigan o‘q	Inersiya momentining formulasi
m massali va l uzunlikli bir jinsli ingichka tayoqcha.	Tayoqchaga tik ravishda uning og‘irlik markazidan o‘tadi. Tayoqchaga tik ravishda uning uchidan o‘tadi.	$\frac{1}{12}ml^2$. $\frac{1}{3}ml^2$.
R – radiusli va m massali ingichka halqa yoki massasi m gardishi bo‘ylab taqsimlangan g‘ildirak	Asos tekisligiga tik ravishda markazidan o‘tadi.	mR^2
R – radiusli va m massali bir jinsli aylanma disk (silindri)	Asos tekisligiga tik ravishda disk markazidan o‘tadi	$\frac{1}{2}mR^2$
R radiusli va m massali *bir* jinsli shar	Shar markazidan o‘tadi.	$\frac{2}{5}mR^2$

O‘zaro ta’sirlashuvchi ikkita jismlar uchun impuls momentining saqlanish qonuni:

$$J_1\omega_1 + J_2\omega_2 = J_1'\omega_1' + J_2'\omega_2',$$

bunda: $J_1 J_2 \omega_1 \omega_2$ – jismlarning o‘zaro ta’sirigacha inersiya momentlari va burchak tezliklari; $J_1', J_2', \omega_1', \omega_2'$ – shu kattaliklarning o‘zaro ta’sirdan keyingi qiymatlari.

Inersiya momenti o‘zgaradigan bitta jism uchun impuls momentining saqlanish qonuni:

$$J_1\omega_1 = J_2\omega_2 .$$

bunda: J_1 va J_2 – dastlabki va keyingi inersiya momentlari,

ω_1 va ω_2 – dastlabki va keyingi burchak tezliklar.

Jismga ta’sir etayotgan $\vec{F}$ kuchning aylanish o‘qiga nisbatan momenti:

$$M = F_1 \cdot l ,$$

bunda: F_1 – $\vec{F}$ kuchning aylanish o‘qiga tik tekislikdagi proyeksiyasi;

l – $\vec{F}$ kuchning yelkasi (aylanish o‘qidan kuch ta’sir chizig‘igacha bo‘lgan eng qisqa masofa).

Qo‘zg‘almas o‘qqa nisbatan qattiq jism aylanma harakat dinamikasining asosiy qonuni:

$$\vec{M}dt = d(J \cdot \vec{\omega}),$$

bunda: $\vec{M} - dt$ vaqt davomida jismga ta’sir etuvchi kuch momenti; J – jismning inersiya momenti; $\vec{\omega}$ – burchak tezlik; $J\vec{\omega}$ – impuls momenti.

Kuch momenti va inersiya momenti o‘zgarmas bo‘lganda tenglama quyidagi ko‘rinishni oladi:

$$\vec{M} \cdot \Delta t = J \cdot \Delta\vec{\omega} \cdot$$

Inersiya momenti o‘zgarmas bo‘lganda aylanma harakat dinamikasining asosiy qonuni:

$$\vec{M} = J \cdot \vec{\varepsilon},$$

bunda: $\vec{\varepsilon}$ – burchak tezlanish.

Jismni aylantirayotgan M kuch momentining ishi:

$$A = M \cdot \varphi,$$

bunda: φ – jismning burilish burchagi.

Jism aylanayotganda erishiladigan oniy quvvat:

$$N = M \cdot \omega \cdot$$

Aylanma harakat qilayotgan jismning kinetik energiyasi:

$$T = \frac{1}{2}J\omega^2 .$$

Sirpanishsiz dumalayotgan jismning kinetik energiyasi:

$$T = \frac{1}{2}m\omega^2 + \frac{1}{2}J\omega^2,$$

bunda: $\frac{1}{2}m\omega^2$ – jismning ilgarilanma harakat kinetik energiyasi;

$\frac{1}{2}J\omega^2$ – jismning aylanma harakat kinetik energiyasi.

Jism aylanishida bajariladigan ish va uning kinetik energiyasining o‘zgarishi quyidagi munosabatlar bilan bog‘langan:

$$A = \frac{1}{2}J\omega_2^2 - \frac{1}{2}J\omega_1^2 .$$

Quyida aylanma harakat dinamikasiga taalluqli kattaliklar va tenglamalarning, ilgarilanma harakatning mos kattaliklari va tenglamalariga o‘xshashligi keltiriladi.

Ilgarilanma harakat	Aylanma harakat
Dinamikaning asosiy qonuni	
$F\cdot\Delta t = m\vartheta_2 - m\vartheta_1$; $F = ma$	$M\cdot\Delta t = J\omega_2 - J\omega_1$; $M = J\varepsilon$
Saqlanish qonuni	
Impulsning	Impuls momentining
$\sum_{i=1}^{n} m_i\vartheta_i = \text{const}$	$\sum_{i=1}^{n} J_i\omega_i = \text{const}$
Ish va quvvat	
$A = F\cdot S$ $N = F\cdot\vartheta$	$A = M\cdot\varphi$ $N = M\cdot\omega$
Kinetik energiya	
$T = \frac{1}{2}m\vartheta^2$	$T = \frac{1}{2}J\omega^2$

Masala yechishga doir misollar

1-misol. Uzunligi 30 sm va massasi 100 g bo‘lgan ingichka bir jinsli tayoqchaning, tayoqchaga tik va uning: 1) uchlaridan; 2) o‘rtasidan; 3) tayoqcha uchidan uzunligining 1/3 qismiga teng masofadagi nuqtadan, o‘tuvchi o‘qqa nisbatan inersiya momenti aniqlansin.

Berilgan:
l =30 sm=0,3 m;
m=100 g=0,1 kg;

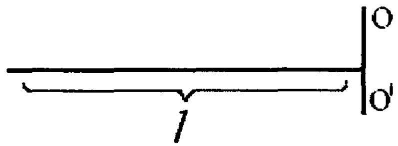
$$r = \frac{1}{2}l.$$

$J_1 = ?$

$J_2 = ?$

$J_3 = ?$

Yechish: Bir jinsli ingichka tayoqchaning, tayoqchaga tik va uchidan o‘tgan o‘qqa nisbatan inersiya momenti



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$$J_1 = \frac{1}{3}ml^2. \qquad (1)$$

Bir jinsli ingichka tayoqchaning, tayoqchaga tik va uning o‘rtasidan o‘tgan o‘qqa nisbatan inersiya momenti:

$$J_2 = \frac{1}{2}ml^2. \qquad (2)$$

Uchinchi hol uchun Shteyner formulasidan foydalanamiz:

$$J_3 = J_2 + ma^2. \qquad (3)$$

Bu yerda a berilgan o‘qdan, tayoqchaning o‘rtasidan o‘tgan o‘qqacha bo‘lgan masofa.

Rasmdan ko‘rinib turibdiki:

$$a = \frac{l}{2} - r = \frac{l}{2} - \frac{1}{3}l = \frac{1}{6}l. \qquad (4)$$

(2) va (4) yordamida (3) ni quyidagicha o‘zgartiramiz:

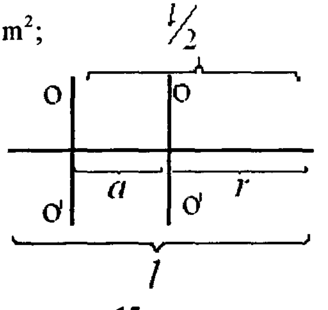
$$J_3 = \frac{1}{2}ml^2 + \frac{1}{36}ml^2 = \frac{1}{9}ml^2. \qquad (5)$$

Berilganlarni (1), (2) va (5) larga qo‘yamiz:

$$J_1 = \frac{1}{3}\cdot 0.1\cdot(0{,}3)^2\text{kg}\cdot\text{m}^2 = 3\cdot10^{-3}\text{kg}\cdot\text{m}^2;$$

$$J_2 = \frac{1}{2}\cdot 0{,}1\cdot(0{,}3)^2\text{kg}\cdot\text{m}^2 = 0{,}75\cdot10^{-3}\text{kg}\cdot\text{m}^2;$$

$$J_3 = \frac{1}{9}\cdot 0{,}1\cdot(0{,}3)^2\text{kg}\cdot\text{m}^2 = 10^{-3}\text{kg}\cdot\text{m}^2.$$



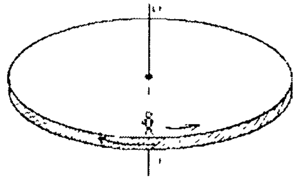
15-rasm

Javob:

$$J_1 = 3\cdot10^{-3}\text{kg}\cdot\text{m}^2; J_2 = 0{,}75\cdot10^{-3}\text{kg}\cdot\text{m}^2; J_3 = 10^{-3}\text{kg}\cdot\text{m}^2.$$

2-misol. Disk shakliga ega platforma tik o'qi atrofida aylanishi mumkin. Platformaning chekkasıda 60 kg massali odam turibdi. Agar odam platforma chekkasidan yurib va uni aylanib, platformaning dastlabki nuqtasiga qaytsa, platforma qanday burchakka buriladi? Platformaning massasi 240 kg ga teng. Odamning inersiya momenti J_1 moddiy nuqtanikidek deb hisoblansin.

Berilgan:	Yechish:
$m_1 = 60\,\text{kg}$; $m_2 = 240\,\text{kg}$; $J_1 = m_1 R^2$. $\varphi = ?$	Masalani yechish uchun impuls momentining saqlanish qonunidan foydalanamiz.



16-rasm

Odam-platforma sistemasi uchun to'la impuls momenti nolga teng bo'lishi kerak, chunki jarayon boshlanishigacha ular harakatsiz bo'lishgan, ya'ni:

$$L_p - L_0 = 0, L_p = L_0$$

yoki

$$J_1\omega_1 = J_2\omega_2 . \qquad (1)$$

Rasmdan ko'rinib turibdiki, harakat davomida odam $2\pi - \varphi$, platforma esa φ burchakka burilganligidan ularning burchak tezliklari mos ravishda

$$\omega_1 = \frac{2\pi - \varphi}{t} \quad \text{va} \quad \omega_2 = \frac{\varphi}{t}$$

bo'ladi. Platformaning inersiya momenti:

$$J_2 = \frac{1}{2} m_2 R^2$$

ekanini hisobga olib (1) ni qayta yozamiz:

$$m_1 \cdot R^2 \frac{(2\pi - \varphi)}{t} = \frac{1}{2} m_2 R^2 \frac{\varphi}{t}$$

yoki

$$4\pi m_1 - 2m_1\varphi = m_2\varphi ,$$

bundan esa

$$\varphi = \frac{4\pi m_1}{m_2 + 2m_1}. \qquad (2)$$

(2) dan burchak radianlarda chiqishi ko'rinib turibdi. Berilganlarni qo'yib olamiz:

$$\varphi = \frac{4 \cdot \pi \cdot 60}{240 + 2 \cdot 60} \text{rad} = \frac{240}{360} \pi \text{rad} = \frac{2}{3} \pi \text{ rad}.$$

Javob: $\varphi = \frac{2}{3} \pi$ rad.

3-misol. Uzunligi 50 sm va massasi 400 g bo'lgan ingichka bir jinsli tayoqcha, o'rtasidan tayoqchaga tik o'tgan o'q atrofida 3 rad/s² burchak tezlanish bilan aylanadi. Aylantiruvchi moment M aniqlansin.

Berilgan:

$l = 50\text{sm} = 0{,}5\text{m}$;

$m = 400\text{g} = 0{,}4\text{kg}$;

$\varepsilon = 3\frac{\text{rad}}{\text{s}^2} = 3\frac{1}{\text{s}^2}$.

$M = ?$

Yechish: O'qning inersiya momenti o'zgarmasligini nazarda tutib aylanma harakat dinamikasining asosiy qonunini yozamiz:

$$M = J \cdot \varepsilon, \qquad (1)$$

bunda, M – aylantiruvchi moment, ε – burchak tezlanish. J – ingichka bir jinsli tayoqchaning, o'rtasidan tayoqchaga tik o'tgan o'qqa nisbatan inersiya momenti bo'lib, quyidagicha aniqlanadi:

$$J = \frac{1}{12} ml^2, \qquad (2)$$

bunda: m – tayoqchaning massasi, l – uzunligi.

(1) ni (2) yordamida qayta yozamiz:

$$\text{M} = \frac{1}{12} ml^2 \varepsilon. \qquad (3)$$

(3) yordamida aylantiruvchi momentning (kuch momentining) birligini tekshirib ko'ramiz:

$$[M]=[m][l]^2[\varepsilon]=1\text{kg}\cdot 1\text{m}^2\cdot 1\frac{1}{\text{s}^2}=1\text{kg}\frac{\text{m}}{\text{s}^2}\cdot\text{m}=1\text{N}\cdot\text{m}$$

va uning to'g'riligiga ishonch hosil qilgach berilganlarni qo'yamiz:

$$M=\frac{1}{12}\cdot 0{,}4\cdot(0{,}5)^2\cdot 3\text{N}\cdot\text{m}=0{,}1\cdot 0{,}25\text{N}\cdot\text{m}=0{,}025\text{N}\cdot\text{m}.$$

Javob: $M=0{,}025\text{N}\cdot\text{m}$.

4-misol. Massasi 0,2 kg bo'lgan harakatsiz chig'ir orqali oshirib tashlangan chilvirning uchlariga, massalari 0,3 kg va 0,5 kg bo'lgan yuklar osilgan. Agar chig'irning massasi gardish bo'ylab tekis taqsimlangan bo'lsa, yuklar harakatlangan paytda chig'irning har ikkala tomonida ham chilvirning taranglik kuchlari T_1 va T_2 lar aniqlansin.

Berilgan:

$m=0{,}2\text{kg};$

$m_1=0{,}3\text{kg};$

$m_2=0{,}5\text{kg}.$

$T_1=?$

$T_2=?$

$$M=J\cdot\varepsilon, \tag{1}$$

bu yerda: $J=mr^2-$ massasi gardish bo'ylab tekis taqsimlangan chig'irning inersiya momenti,

$\varepsilon=\frac{a}{r}$ aylanma harakat burchak tezlanishi, a – yuklarning chiziqli tezlanishi, r – chig'irning radiusi. Demak,

Yechish: Aylanma harakat dinamikasining asosiy qonunini yozamiz:

$$M=mr^2\cdot\frac{a}{r}=mra, \tag{2}$$

Ikkinchi tomondan kuch momentining aniqlanish ta'rifidan

$$M=F\cdot r=(T_2'-T_1')r=(T_2-T_1)r, \tag{3}$$

bu yerda: $F=T_2'-T_1'=T_2-T_1, T_1=T_1', T_2=T_2'$ chilvirning taranglik kuchlari, (2) va (3) larni tenglashtirib olamiz

$$(T_2-T_1)r=mra \quad \text{yoki}$$

$$T_2-T_1=ma. \tag{4}$$

Endi yuklarning harakat qonunlarini yozamiz. Bunda $P_2 > P_1 (m_2 > m_1)$ ligini nazarda tutamiz:

$$\left.\begin{aligned} T_1 - m_1 g &= m_1 a, \\ m_2 g - T_2 &= m_2 a \end{aligned}\right\} \quad \text{yoki} \quad \left.\begin{aligned} T_1 &= m_1 a + m_1 g \\ T_2 &= m_2 g - m_2 a \end{aligned}\right\}. \qquad (5)$$

(5) ning ikkinchi tenglamasidan birinchisini ayirib olamiz:

$$T_2 - T_1 = m_2 (g - a) - m_1 (a + g) \qquad (6)$$

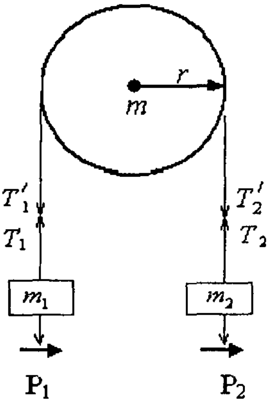
(4) va (6) larni tenglashtirib a ni topamiz:

$$a = \frac{(m_2 - m_1)}{m_1 + m_2 + m} g . \qquad (7)$$

(7) ni (5) ga qo'yib quyidagini olamiz:

$$T_1 = \frac{m_1 (m + 2m_2)}{m + m_1 + m_2} g , \qquad (8)$$

$$T_2 = \frac{m_2 (m + 2m_1)}{m + m_1 + m_2} g . \qquad (9)$$



17-rasm

T_1 va T_2 lar uchun topilgan ifodalardan kuch birliklari hosil bo'lishi ko'rinib turganligidan berilganlarni qo'yamiz:

$$T_1 = \frac{0{,}3(0{,}2 + 2 \cdot 0{,}5)}{0{,}2 + 0{,}3 + 0{,}5} \cdot 9{,}8\text{N} = 3{,}53\text{N},$$

$$T_2 = \frac{0{,}5(0{,}2 + 2 \cdot 0{,}3)}{0{,}+0{,}3 + 0{,}5} \cdot 9{,}8\text{N} = 3{,}92\text{N},$$

Javob: $T_1 = 3{,}53\text{N}$; $T_2 = 3{,}92\text{N}$.

5-misol. Jukovskiy kursisida turgan odam gorizontal yo'nalishda 20m/s tezlik bilan uchayotgan, massasi 0,4 kg bo'lgan to'pni qo'li bilan ushlab oladi. To'pning trayektoriyasi kursi aylanayotgan tik o'qdan 0,8 m masofadan o'tadi. Agar odam va kursining yig'indi inersiya momenti 6 kg m² ga teng bo'lsa, Jukovskiy kursisi to'pni ushlagan odam bilan birgalikda qanday ω burchak tezlik bilan aylana boshlaydi?

Berilgan:

$\omega_0 = 0;$

$\vartheta = 20\,\mathrm{m/s};$

$m = 0{,}4\,\mathrm{kg};$

$r = 0{,}8\,\mathrm{m};$

$J = 6\,\mathrm{kgm}^2.$

$\omega = ?$

Yechish: To'p va odam – kursi sistemasi uchun impuls momentining saqlanish qonunini yozamiz:

$$J\omega_0 + J_T\omega_T = J\omega + J_T\omega = (J + J_T)\omega. \quad (1)$$

Bu yerda: J – odam – kursining yig'indi inersiya momenti, ω_0 – boshlang'ich burchak tezlik, J_T – to'pning inersiya momenti, ω_T – to'pning burchak tezligi.

Agar to'pni moddiy nuqta sifatida qarasak:

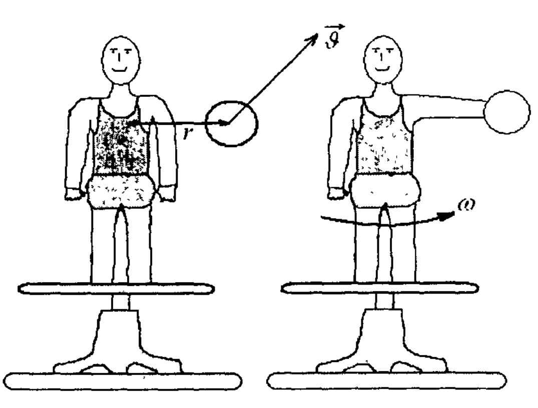
$$J_T = mr^2. \quad (2)$$

To'pning burchak tezligini esa chiziqli tezligi orqali ifodalaymiz:

$$\omega_T = \frac{\vartheta_T}{r}. \quad (3)$$

(2) va (3) larni (1) ga qo'ysak va $\omega_0 = 0$ ni nazarda tutsak, quyidagilarni olamiz:

$$\omega = \frac{m\vartheta_T r}{J + mr^2}. \quad (4)$$



18-rasm

(4) yordamida ω ning birligini tekshiraylik:

$$[\omega]=\frac{[m][\vartheta][r]}{[J]}=\frac{1\text{kg}\cdot 1\frac{\text{m}}{\text{s}}\cdot 1\text{m}}{1\text{kg}\cdot\text{m}^2}=1\text{s}^{-1}.$$

Burchak tezlik birligi hosil bo'lishiga ishonch hosil qilganimizdan keyin berilganlarni (4) ga qo'yamiz:

$$\omega=\frac{0{,}4\cdot 20\cdot 0{,}8}{6+0{,}4\cdot(0{,}8)^2}\text{s}^{-1}=\frac{6{,}4}{6+0{,}26}\text{s}^{-1}=\frac{6{,}4}{6{,}26}\text{s}^{-1}=1{,}02\text{s}^{-1}.$$

Javob: $\omega=1{,}02\text{s}^{-1}=1{,}02\frac{\text{rad}}{\text{s}}$.

6-misol. Radiusi 15 sm bo'lgan, 8 s^{-1} chastota bilan aylanayotgan chig'irni 6 s davomida to'xtatish uchun unga quyilishi kerak bo'lgan kuch momenti aniqlansin. Chig'irning 5 kg massasi gardishi bo'ylab tekis taqsimlangan deb hisoblansin.

Berilgan:

$r=15\text{sm}=0{,}15\text{m}$;

$n_1=8\text{s}^{-1}$;

$n_2=0$;

$\Delta t=6\text{s}$;

$m=5$ kg

M=?

Yechish: Kuch momenti va inersiya momenti o'zgarmas bo'lganda, qo'zg'almas o'qqa nisbatan qattiq jism aylanma harakat dinamikasining asosiy qonuni quyidagi ko'rinishda bo'ladi:

$$\vec{M}\cdot\Delta t=J\cdot\Delta\vec{\omega},$$

yoki skalyar ko'rinishda

$$M\cdot\Delta t=J\cdot\Delta\omega. \qquad (1)$$

Bu yerda: M – chig'irga qo'yilgan kuch momenti, Δt – uning ta'sir etish vaqti, J – chig'irning inersiya momenti, $\Delta\omega$ – burchak tezligining o'zgarishi. (1) dan:

$$M=\frac{J\cdot\Delta\omega}{\Delta t}. \qquad (2)$$

Massasi gardishi bo'ylab taqsimlangan chig'irning inersiya momenti

$$J = mr^2 \qquad (3)$$

ifoda bilan aniqlanadi. $\Delta\omega$ ni esa $\omega = 2\pi n$ ligidan foydalanib quyidagi ko'rinishda topamiz:

$$\Delta\omega = \omega_1 - \omega_2 = 2\pi n_1 - 2\pi n_2 = 2\pi(n_1 - n_2). \qquad (4)$$

(3) va (4) larni (2) ga qo'yib quyidagini olamiz:

$$M = \frac{2\pi(n_1 - n_2)mr}{\Delta t}. \qquad (5)$$

(5) asosida M ning birligini tekshirib ko'ramiz:

$$[M] = \frac{[n][m][r]^2}{[t]} = \frac{1\text{s}^{-1} \cdot 1\text{kg} \cdot 1\text{m}^2}{1\text{s}} = 1\text{kg}\frac{\text{m}^2}{\text{s}^2} = 1\text{N} \cdot \text{m},$$

va uning to'g'riligiga ishonch hosil qilgandan keyin berilganlarni qo'yamiz

$$M = \frac{2 \cdot 3{,}14(8-0)5 \cdot (0{,}15)^2}{6}\text{N} \cdot \text{m} \approx \frac{6{,}28 \cdot 40 \cdot 0{,}02}{6}\text{N} \cdot \text{m} = 0{,}84\text{N} \cdot \text{m}$$

Javob: $M = 0{,}84 N \cdot m$.

7-misol. Massasi 10 kg bo'lgan yaxlit silindr 10 m/s tezlik bilan sirpanishsiz dumalaydi. Silindrning kinetik energiyasi *T,* va unga *–50* N kuch ta'sir etsa qancha Δt vaqtdan keyin to'xtashi aniqlansin.

Berilgan:
$m = 10\text{kg};$
$\vartheta_0 = 10\,\text{m/s};$
$F = -50\text{N};$
$\vartheta = 0$

$T = ?$
$\Delta t = ?$

Yechish: Sirpanishsiz dumalayotgan yaxlit Silindrning kinetik energiyasi quyidagicha ifoda bilan aniqlanadi:

$$T = \frac{1}{2}m\vartheta^2 + \frac{1}{2}J\omega^2. \qquad (1)$$

Bu yerda $\frac{1}{2}m\vartheta^2$ – ilgarilanma harakat kinetik energiyasi, $\frac{1}{2}J\omega^2$ – aylanma harakat kinetik energiyasi. *R* radiusli yaxlit silindrning inersiya momenti $J = \frac{1}{2}mR^2$, burchakli va chiziqli tezliklari orasidagi munosabat $\omega = \frac{\vartheta}{R}$ ekanligini nazarda tutsak (1) quyidagi ko'rinishni oladi:

$$T = \frac{1}{2}m\vartheta^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\cdot(\frac{\vartheta}{R})^2 = \frac{1}{2}m\vartheta^2 + \frac{1}{4}m\vartheta^2 = \frac{3}{4}m\vartheta^2. \quad (2)$$

Nyutonning ikkinchi qonuniga muvofiq

$$a = \frac{F}{m}. \quad (3)$$

Tezlanishning ta'rifiga binoan

$$a = \frac{\Delta\vartheta}{\Delta t} = \frac{\vartheta - \vartheta_0}{\Delta t}. \quad (4)$$

(3) va (4) larni tenglashtirib undan Δt ni topsak:

$$\Delta t = \frac{m(\vartheta - \vartheta_0)}{F}. \quad (5)$$

(5) yordamida Δt ning birligini tekshiramiz:

$$[t] = \frac{[m][\vartheta]}{[F]} = \frac{1\text{kg}\cdot 1\frac{\text{m}}{\text{s}}}{1\text{kg}\cdot \text{m}/\text{s}^2} = 1\,\text{s}.$$

Berilganlarni (2) va (5) larga qo'yib olamiz:

$$T = \frac{3}{4}\cdot 10\cdot(10)^2\,\text{J} = 7{,}5\cdot 100\text{J} = 750\text{J} = 0{,}75\text{kJ}.$$

$$\Delta t = \frac{10(0-10)}{-50} = 2\text{s}.$$

Javob: $T = 0{,}75\text{kJ}; \Delta t = 2\text{s}$

8-misol. Uzunligi 1m ga teng ipning uchiga bog'langan 100 g massali sharcha gorizontal tekislikda sirpangancha 1 s^{-1} chastota bilan aylanadi. Ip qisqartiriladi va sharcha aylanish o'qiga 0,5 m masofagacha yaqinlashadi.

Shunda sharcha qanday n_2 chastota bilan aylanadi? Ipni qisqartirishda tashqi kuch qanday A ish bajaradi? Sharchaning tekislikka ishqalanishi inobatga olinmasin.

Berilgan:

$m = 100\text{g} = 0{,}1\text{kg};$

$l_1 = 1\text{m};$

$n_1 = 1\text{s}^{-1};$

$l_2 = 0{,}5\text{m}.$

$A=?$

$n_2 = ?$

Yechish: Impuls momentining saqlanish qonuniga muvofiq

$$J_1\omega_1 = J_2\omega_2, \qquad (1)$$

bunda $J_1 = ml_1^2$ – "ip-sharcha" sistemasining ip qisqarti-rilguncha inersiya momenti, $\omega_1 = 2\pi n_1$ – burchak tezligi, $J_2 = ml_2^2$ – "ip-sharcha" sistemasining ip qisqartirilgandan keyingi inersiya momenti, $\omega_2 = 2\pi n_2$ burchak tezligi.

Natijada quyidagini olamiz:

$$ml_1^2 \cdot 2\pi n_1 = ml_2^2 \cdot 2\pi n_2.$$

Bundan esa:

$$n_2 = \frac{(l_1)^2}{(l_2)^2} n_1. \qquad (2)$$

Tashqi kuchning ipni qisqartirish uchun bajargan ishi, "ip-sharcha" sistemasining oldingi va keyingi kinetik energiyalarining farqiga teng bo'ladi

$$A = T_2 - T_1.$$

Agar aylanma harakat kinetik energiyasi $T = \frac{1}{2}J\omega^2$ kabi aniqlanishini nazarda tutsak,

$$A = \frac{1}{2}J_2\omega_2^2 - \frac{1}{2}J_1\omega_1^2$$

yoki

$$A=\frac{1}{2}(ml_2^2)(2\pi n_2)^2-\frac{1}{2}(ml_1^2)(2\pi n_1)^2.$$

Agar (2) ni nazarda tutsak:

$$A=2\pi^2 mn_1^2\frac{(l_1)^2}{(l_2)}(l_1^2-l_2^2). \qquad (3)$$

(3) asosida A ning birligini tekshirib ko‘ramiz:

$$[A]=[m][n]^2[l]^2=1\text{kg}\cdot(1\text{s}^{-1})^2\cdot(1\text{m})^2=1\text{kg}\frac{\text{m}^2}{\text{s}^2}=1\text{J}.$$

Berilganlarni (2) va (3) larga qo‘yib olamiz:

$$n_2=\frac{(1)^2}{(0,5)}\cdot 1\text{s}^{-1}=4\text{s}^{-1},$$

$$A=2\cdot(3,14)^2\cdot 0,1\cdot(1)^2\frac{(1)^2}{(0,5)}\cdot[(1)^2-(0,5)^2]\text{J}=0,8\cdot 0,75\cdot 9,86\text{J}=5,92\text{J}.$$

Javob: n_2=4s^{-1}; A= 5,92 J.

Mustaqil yechish uchun masalalar

1. Uzunligi 60 sm va massasi 100 g bo‘lgan ingichka bir jinsli tayoqchaning, unga tik va tayoqchaning uchlaridan biridan 20 sm masofadagi nuqtasidan o‘tuvchi o‘qqa nisbatan inersiya momenti aniqlansin. [$4\cdot 10^{-3}$kg $\cdot$ m^2.]

2. Diskning diametri 20 sm, massasi 800 g. Diskning radiuslaridan birining markazidan, disk tekisligiga tik bo‘lib o‘tgan o‘qqa nisbatan inersiya momenti aniqlansin. [$6\cdot 10^{-3}$kg $\cdot$ m^2.]

3. 0,5m radiusli bir jinsli yaxlit diskning gardishiga o‘zgarmas urinma 100 N kuch qo‘yilgan. Disk aylanganda unga 2 N $\cdot$ m ishqalanish kuchi momenti ta’sir qiladi. Agar diskning burchakli tezlanishi o‘zgarmas va 12rad/s^2 ligi ma’lum bo‘lsa, diskning massasi aniqlansin. [32 kg.]

4. Massasi 100 kg radiusi 5 sm bo‘lgan val 8 s^{-1} chastota bilan aylangan. Valning silindrik sirtiga 40 N kuchga ega tormoz dastasining bosilishi ta’sirida 10 s dan keyin val to‘xtadi. Ishqalanish koeffitsienti aniqlansin. [0,31.]

5. 1 kg massali bir jinsli yaxlit silindr shaklidagi harakatsiz chig‘ir orqali, uchlariga 1kg va 2 kg massali yuklar bog‘langan vaznsiz ip tashlangan. Chig‘ir o‘qidagi ishqalanish hisobga olinmay:

1) yuklarning tezlanishi; 2) ip taranglik kuchlarining nisbati aniqlansin. [2,8 m/s^2; 1,11.]

6. Massasi 10 kg va radiusi 20 sm bo‘lgan shar markazdan o‘tuvchi o‘q atrofida aylanadi. Sharning aylanish tenglamasi $\varphi = A + Bt^2 + Ct^3$ ko‘rinishga ega. Bunda B=4 rad/s^2, C=−1 rad/s^3. Sharga ta’sir etayotgan kuch momentining o‘zgarish qonuni topilsin. Vaqtning 2 s onida kuch momenti aniqlansin. $[-0{,}64 N \cdot m.]$

7. Tormozlanish natijasida tekis sekinlanib aylanayotgan, inersiya momenti 2 kg · m^2 bo‘lgan g‘ildirakning aylanish tezligi 1 min vaqt davomida 300 ayl/min dan 180 ayl/min gacha kamaydi.

1) g‘altakning burchak tezlanishi; 2) tormozlanish kuch momenti;

3) tormozlanish kuchining ishi aniqlansin.

[0,21 rad/s^2; 2) 0,42 N · m; 3) 630 J.]

8. Radiusi 1 m bo‘lgan disk ko‘rinishdagi platforma inersiya bo‘yicha 6 min^{-1} chastota bilan aylanadi. Platforma chekkasida massasi 80 kg bo‘lgan odam turibdi. Agar odam markaziga o‘tsa, platforma qanday n_2 chastota bilan aylanadi? Platformaning inersiya momenti 120 kg · m^2 ga teng. Odamning inersiya momenti moddiy nuqtanikidek deb hisoblansin. [10 min^{-1}.]

9. G’ildirak $\varphi = A + Bt + Ct^2$ tenglamia bilan ifodalanuvchi qonun bo‘yicha aylanadi. Bunda A=2 rad; B=32 rad/s; C=−4rad/s^2. Agar g‘ildirakning inersiya momenti 100 kg · m^2 bo‘lsa g‘ildirakning aylanishdan to to‘xtagunigacha unga ta’sir etuvchi kuchlar erishadigan o‘rtacha quvvat topilsin. [12,8 kW.]

10. Massasi 80 kg va radiusi 30 m bo‘lgan disk ko‘rinishdagi g‘ildirak

harakatsiz holatda turibdi. G'ildirakka 10 s^{-1} chastota berish uchun qanday ish bajarmoq kerak? Agar disk shu massasi bilan kichikroq qalinlikka, lekin radiusi ikki marta katta bo'lganda qanday ish bajariladi? [7,11kJ; 28,4 kJ.]

11. Shar gorizontal sirt bo'ylab sirpanishsiz dumalaydi. Sharning to'la kinetik energiyasi 14 J ga teng. Sharning ilgarilanma T_1 va aylanma T_2 harakat kinetik energiyalari aniqlansin. [10J; 4J.]

5-§. Tortishish qonuni

Asosiy formulalar

Bir biridan r masofada joylashgan m_1 va m_2 massali moddiy nuqtalar orasidagi o'zaro tortishish kuchi:

$$F = G\frac{m_1 m_2}{r^2},$$

bunda: $G = 6{,}67 \cdot 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ tortishish doimiysi.

Tortishish maydonining kuchlanganligi moddiy nuqtaga ta'sir etayotgan F kuchining shu nuqta massasi m ga nisbati bilan aniqlanib, maydonning shu nuqtasining funksiyasidir:

$$g = \frac{F}{m}.$$

Massasi M sfero-simmetrik taqsimlangan jism tortishish maydonining kuchlanganligi:

$$g = G\frac{M}{r^2},$$

bunda: r — jism markazidan o'rganilayotgan nuqtagacha bo'lgan masofa.

Bir-biridan r masofada joylashgan m_1 va m_2 massali moddiy nuqtalarning (massalari sfero-simmetrik joylashgan jismlarning) o'zaro tortishish ta'sir potensial energiyasi:

$$P = -G\frac{m_1 m_2}{r}.$$

Tortishish maydonining potensiali $\varphi = \frac{P}{m}$,

bunda: P — maydonning ma'lum nuqtasida joylashtirilgan m massali moddiy nuqtaning potensial energiyasi. Massasi M sfero-simmetrik taqsimlangan jism tortishish maydonining potensiali

$$\varphi = -G\frac{M}{r}.$$

Yer sirtidan h balandlikdagi erkin tushish tezlanishi

$$g_h = \frac{g}{\left(1 + \frac{h}{r}\right)^2}.$$

Bunda R — Yerning radiusi; g — yer sirtidagi erkin tushish tezlanishi. Agar

$h << R$ bo‘lsa, unda

$$g_h = \left(1 - \frac{2h}{R}\right) g \quad \text{bo‘ladi.}$$

Masala yechishga doir misollar

1-misol. Har birining massasi 10 t dan bo‘lgan ikkita fazoviy kema 100 m masofagacha yaqinlashishsa, ularning o‘zaro tortishish kuchi F ning kattaligi qanday bo‘ladi?

Berilgan:

$m_1 = m_2 = m = 10t = 10^4 \text{kg}$;

$r = 100\text{m} = 10^2 \text{m}$;

$G = 6{,}67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$.

$F = ?$

Yechish: Fazoviy kemalarning o‘lchamlari ular orasidagi masofadan ko‘p kichik deb hisoblab, ularni moddiy nuqtalardek qaraymiz va quyidagi tortishish qonunidan foydalanamiz:

$$F = G\frac{m_1 m_2}{r^2}.$$

Agar $m_1 = m_2 = m$ ligini hisobga olsak,

$$F = G\frac{m_2}{r^2}.$$

Berilganlardan foydalanib, quyidagini topamiz:

$$F = 6{,}67 \cdot 10^{-11} \frac{(10^4)^2}{(10^2)^2} \text{N} = 6{,}67 \cdot 10^{-7} \text{N}.$$

Javob: $F = 6{,}67 \cdot 10^{-7}$N.

2-misol. Tortishish maydonining Yer sirtidan 1000 km balandlikdagi kuchlanganligi aniqlansin. Yerning radiusi va yer sirtidagi erkin tushish tezlanishi ma’lum deb topilsin.

Berilgan:

$h = 1000\text{km} = 10^6\text{m};$

$g_{Yer} = 9{,}8\,\text{m}/\text{s}^2;$

$R_{Yer} = 6{,}37\cdot10^6\text{m}.$

$g=?$

Yechish: Yer tortishish maydonining kuchlanganligi

$$g = \frac{F}{m} \qquad (1)$$

ifoda yordamida aniqlanadi. Bunda m – yer sirtidan h balandlikda joylashgan, sinalayotgan jismning massasi. Shu jism va Yer orasidagi tortish kuchi F esa quyidagicha aniqlanadi:

$$F = G\frac{m\cdot M_{Yer}}{(R_{Yer}+h)^2}. \qquad (2)$$

(2) ni (1) ga qo'yib quyidagini olamiz:

$$g = G\frac{M_{Yer}}{(R_{Yer}+h)^2} = (G\frac{M_{Yer}}{R_{Yer}^2})\cdot\frac{R_{Yer}^2}{(R_{Yer}+h)^2}. \qquad (3)$$

Yer sirtidagi erkin tushish tezlanishi

$$g_{Yer} = G\frac{M_{Yer}}{R_{Yer}^2}$$

ekanligini e'tiborga olsak, (3) quyidagi ko'rinishni oladi:

$$g = g_{Yer}\frac{R_{Yer}^2}{(R_{Yer}+h)^2}. \qquad (4)$$

(4) dan kuchlanganlikning birligi chiqishi ko'rinib turibdi. Berilganlardan foydalanib, quyidagini topamiz:

$$g = 9{,}8\frac{(6{,}37\cdot10^6)^2}{(6{,}37\cdot10^6+10^6)^2}\,\text{N}/\text{kg} = 9{,}8\left(\frac{6{,}37}{7{,}37}\right)^2\frac{\text{N}}{\text{kg}} = 7{,}3\,\text{N}/\text{kg}$$

Javob: $g = 7{,}3\frac{\text{N}}{\text{kg}}$.

3-misol. Yerning radiusi Oyning radiusidan 3,66 marta katta, Yerning o'rtacha zichligi Oyning o'rtacha zichligidan 1,66 marta katta. Agar Yer sirtidagi erkin tushish tezlanishi g ni ma'lum deb hisoblasak, Oy sirtidagi erkin tushish tezlanishi g_{Oy} aniqlansin.

Berilgan:

$m = \frac{R_{Yer}}{R_{Oy}} = 3{,}66;$

$k = \frac{\rho_{Yer}}{\rho_{oy}} = 1{,}66;$

$g = 9{,}8 m/s^2.$

$G_{oy} = ?$

Yechish: Ma'lumki Yer sirtidagi erkin tushish tezlanishi quyidagi ifoda yordamida aniqlanadi:

$$g = G\frac{M_{Yer}}{R_{Yer}^2} \qquad (1)$$

Shuningdek Oy sirtidagi erkin tushish tezlanishi

$$g_{Oy} = G\frac{M_{Oy}}{R_{Oy}^2} \qquad (2)$$

(2) ni (1) ga hadma-had bo'lib, quyidagini olamiz:

$$\frac{g_{Oy}}{g} = \frac{\frac{M_{Oy}}{R_{Oy}^2}}{\frac{M_{Yer}}{R_{Yer}^2}} = \frac{M_{Oy}}{M_{Yer}}\frac{R_{Yer}^2}{R_{Oy}^2}.$$

Agar $M = V\rho = -\frac{4}{3}\pi \cdot R^3 \cdot \rho$ ekanligini nazarda tutsak

$$\frac{g_{Oy}}{g} = \frac{\frac{4}{3}\cdot\pi\cdot R_{Oy}^3\cdot\rho_{Oy}}{\frac{4}{3}\cdot\pi\cdot R_{Yer}^3\cdot\rho_{Yer}}\cdot\frac{R_{Yer}^2}{R_{Oy}^2} = \frac{1}{\left(\frac{R_{Yer}}{R_{Oy}}\right)\left(\frac{\rho_{Yer}}{\rho_{Oy}}\right)} = \frac{1}{n\cdot k}.$$

Bundan

$$g_{Oy} = \frac{g}{n\cdot k}. \qquad (3)$$

Berilganlar yordamida hisoblaymiz:

$$g_{Oy} = \frac{9{,}8}{3{,}66\cdot 1{,}66}\frac{\mathrm{m}}{\mathrm{s}^2} = 1{,}61\mathrm{m/s}^2.$$

Javob: $g_{Oy} = 1{,}61\frac{\mathrm{m}}{\mathrm{s}^2}.$

4-misol. Yerning radiusiga teng balandlikda Yer tortishish maydonining kuchlanganligi va potensiali qanday o'zgaradi. Yer sirtida erkin tushish tezlanishi $g = 9{,}8\text{m/s}^2$ deb qabul qilinsin.

Berilgan:

$h = R = 6{,}37 \cdot 10^6\text{m};$

$g_{Yer} = 9{,}8\text{m/s}^2.$

$\Delta g = ?$

$\Delta u = ?$

Yechish: Yerning sirtida Yer tortishish maydonining kuchlanganligi

$$g_1 = G\frac{M}{R^2}. \qquad (1)$$

Yer sirtidan $h=R$ balandlikdagi kuchlanganlik esa

$$g_2 = G\frac{M}{(R+h)^2} = G\frac{M}{4R^2} \qquad (2)$$

ifodalar bilan aniqlanadi.

Kuchlanganlikning o'zgarishi esa

$$\Delta g = g_2 - g_1 = G\frac{M}{4R^2} - G\frac{M}{R^2} = G\frac{M}{R^2}\left(\frac{1}{4} - 1\right) = -\frac{3}{4}g_{Yer}. \qquad (3)$$

Bu yerda $g_{Yer} = G\frac{M}{R^2}$ hamda (1) va (2) lar hisobga olingan.

(3) dagi minus ishora Yer sirtidan uzoqlasha borilgan sari tortishish maydonining kuchlanganligi kamayib borishini ko'rsatadi.

Yerning sirtida Yer tortishish maydonining potensiali:

$$\varphi_1 = -G\frac{M}{R}. \qquad (4)$$

Yer sirtidan $h=R$ balandlikdagi potensiali esa

$$\varphi_2 = -G\frac{M}{(R+h)} = -G\frac{M}{2R} \qquad (5)$$

ifoda bilan aniqlanadi. Potensialning o'zgarishini topamiz:

$$\Delta\varphi = \varphi_2 - \varphi_1 = -G\frac{M}{2R} - \left(-G\frac{M}{R}\right) = G\frac{M}{R} - G\frac{M}{2R} = G\frac{M}{R^2}\cdot R\left(1 - \frac{1}{2}\right) = \frac{1}{2}gR. \qquad (6)$$

Berilganlarni (3) va (6) larga qo'yib, quyidagini topamiz:

$$\Delta g=-\frac{3}{4}9{,}8\frac{\text{N}}{\text{kg}}=-7{,}35\frac{\text{N}}{\text{kg}},$$

$$\Delta\varphi=\frac{1}{2}\cdot 9{,}8\cdot 6{,}37\cdot 10^6\frac{\text{J}}{\text{kg}}=31{,}36\cdot 10^6\frac{\text{J}}{\text{kg}}=31{,}36\frac{\text{MJ}}{\text{kg}}.$$

Javob: $\Delta g=-7{,}35\frac{\text{N}}{\text{kg}}$, $\Delta g=-7{,}35\frac{\text{N}}{\text{kg}}$.

5-misol. Agar fazoviy kema Yerdan 10 km/s boshlang'ich tezlik bilan uchirilgan bo'lsa, kemaning Yer radiusiga teng balandlikdagi tezligi qanday bo'ladi? Havoning qarshiligi inobatga olinmasin. Yerning radiusi R va Yer sirtidagi erkin tushish tezlanishi g ma'lum deb hisoblansin.

Berilgan:

$\vartheta_0=10\frac{\text{km}}{\text{s}}=10^4\frac{\text{m}}{\text{s}}$;

$h=R=637\cdot 10^6\text{m}$;

$g=9{,}8\,\text{m}/\text{s}^2$.

$\vartheta=?$

Yechish: Energiyaning saqlanish qonuniga binoan Yerdan uchirilgan fazoviy kemaning kinetik T_0 va potensial P_0 energiyalarining yig'indisi uning $h=R$ balandlikdagi kinetik T va potensial P energiyalarining yig'indisiga teng

$$T_0+P_0=T+P\ . \qquad (1)$$

Kinetik $T=\frac{m\vartheta^2}{2}$ va potensial $P=G\frac{\text{M}m}{(R+h)}$ energiyalarning aniqlanishidan olamiz ($h=R$ ligini hisobga olamiz)

$$-G\frac{\text{M}m}{R}+\frac{m\vartheta_0^2}{2}=\frac{m\vartheta^2}{2}-G\frac{\text{M}m}{(R+h)};\ \text{yoki}\ \frac{m\vartheta_0^2}{2}=\frac{m\vartheta^2}{2}+\left(G\frac{\text{M}}{R^2}\right)\frac{m}{2}R.$$

$G\cdot\frac{\text{M}}{R^2}=g$ ekanlgini e'tiborga olib va $\left(\frac{m}{2}\right)$ ga qisqartirib, quyidagini topamiz:

$$\vartheta_0^2=\vartheta^2+gR$$

yoki $\vartheta = \sqrt{\vartheta_0^2 - gR}$. (2)

Berilganlarni qo'yib hisoblaymiz:

$$\vartheta = \sqrt{(10^4)^2 - 9{,}8 \cdot 6{,}37 \cdot 10^6}\ \frac{\text{m}}{\text{s}} = 6{,}12 \cdot 10^3\ \frac{\text{m}}{\text{s}} = 6{,}12 \frac{\text{km}}{\text{s}};$$

Javob: $\vartheta = 6{,}12 \frac{\text{km}}{\text{s}}$.

6-misol. Yer sirtidan tikka uchiriladigan fazoviy kema Yer radiusiga teng masofagacha uzoqlasha olishi uchun qanday ϑ_1 tezlik bilan uchirilishi kerak. Yerning radiusi R va Yer sirtidagi erkin tushish tezlanishi g ma'lum deb hisoblansin.

Berilgan:

$h = R;$

$R = 6{,}37 \cdot 10^6 \text{m};$

$g = 9{,}8 \text{m}/\text{s}^2.$

$\vartheta_1 = ?$

Yechish: Yer-kema sistemasi uchun mexanik energiyaning saqlanish qonuniga muvofiq uchirilgan kemaning kinetik T_0 va potensial P_0 energiyalarining yig'indisi uning $h=R$ balandlikdagi potensial energiyasiga teng bo'lishi kerak ($h=R$ da kema tezligi nolga teng $\vartheta = 0$, demak, $T=0$):

$$T_0 + P_a = P. \quad (1)$$

Kinetik va potensial energiyalarning ifodalaridan

$$\frac{m\vartheta_1^2}{2} - G\frac{mM}{R} = -G\frac{mM}{(h+R)}.$$

$h=R$ va $G\frac{M}{R^2} = g$ dan foydalansak, quyidagi ifoda hosil bo'ladi:

$$\frac{m\vartheta_1^2}{2} = \frac{1}{2}\left(G\frac{M}{R^2}2mR - G\frac{M}{R^2}mR\right) = \frac{1}{2}mgR.$$

Bundan ϑ_1 ni aniqlasak, $\vartheta_1 = \sqrt{gR}$ ni olamiz. Bu birinchi kosmik tezlik uchun topilgan ifodadir.

Berilganlarni qo‘yib quyidagi olamiz:

$$\vartheta_1 = \sqrt{9{,}8 \cdot 6{,}37 \cdot 10^6}\,\frac{\text{m}}{\text{s}} = 7{,}9 \cdot 10^3\,\frac{\text{m}}{\text{s}} = 7{,}9\,\frac{\text{km}}{\text{s}}.$$

Javob: $\vartheta_1 = 7{,}9\,\dfrac{\text{km}}{\text{s}}$.

7-misol. Oy sirti yaqinidagi birinchi (aylanma) va ikkinchi (parabolik) kosmik tezliklarning qiymatlari hisoblansin.

Yechish: Oy uchun birinchi va ikkinchi kosmik tezliklarni topish uchun avvalgi misollardagi mulohazalarni Oy-kema sistemasi uchun takrorlashimiz kerak. Bunday takror bilan shug‘ullanmasdan hosil qilingan natijalardan foydalanamiz:

$$\vartheta_1^{Oy} = \sqrt{g_{Oy} R_{Oy}}\,, \qquad (1)$$

va $$\vartheta_2^{Oy} = \sqrt{2 g_{Oy} R_{Oy}}\,. \qquad (2)$$

4-misoldan ma’lumki,

$$R_{Oy} = \frac{R_{Yer}}{n}; \quad g_{Oy} = \frac{g}{n \cdot k}; \quad n = 3{,}66; \quad k = 1{,}66. \qquad (3)$$

(3) dan foydalanib (1) va (2) ni qayta yozamiz:

$$\vartheta_1^{Oy} = \sqrt{\frac{g}{n \cdot k} \cdot \frac{R_{Yer}}{n}} = \frac{\sqrt{gR_Y}}{n\sqrt{k}} = \frac{\vartheta_1^{Yer}}{n\sqrt{k}}; \qquad (4)$$

$$\vartheta_2^{Oy} = \sqrt{2\frac{g}{n \cdot k} \cdot \frac{R_{Yer}}{n}} = \frac{\sqrt{2gR_{Yer}}}{n\sqrt{k}} = \frac{\vartheta_2^{Yer}}{n\sqrt{k}}; \qquad (5)$$

$\vartheta_1^{Yer} = 7{,}9\,\text{km}/\text{s}$; $\vartheta_2^{Yer} = 11{,}2\,\text{km}/\text{s}$ ligidan

$$\vartheta_1^{Oy} = \frac{7{,}9}{3{,}66\sqrt{1{,}66}}\,\text{km}/\text{s} = 1{,}68\,\text{km}/\text{s}; \quad \vartheta_2^{Oy} = \frac{11{,}2}{3{,}66\sqrt{1{,}66}}\,\text{km}/\text{s} = 2{,}37\,\text{km}/\text{s};$$

Javob: $\vartheta_1^{Oy} = 1{,}68\,\text{km}/\text{s}$; $\vartheta_2^{Oy} = 2{,}37\,\text{km}/\text{s}$.

MUSTAQIL YECHISH UCHUN MISOLLAR

1. Tik yuqoriga qarab otilgan kosmik kema 3200 km balandlikka ko‘tarildi va so‘ngra tusha boshladi. Tushishning birinchi sekundida kema qanday yo‘lni o‘tadi? [S=2,18 m.]

2. Sun'iy yo'ldosh Yer atrofini 3,6 Mm balandlikda doiraviy orbita bo'ylab aylanadi. Yo'ldoshning chiziqli tezligi aniqlansin. Yerning radiusi va Yer sirtidagi erkin tushish tezlanishi ma'lum deb hisoblansin. $\left[\vartheta = 6{,}33\,{}^{km}\!/_{s}.\right]$

3. Mirrix sayyorasining Fobos va Deymos deb ataluvchi yo'ldoshlari mavjud. Birinchi sayyoradan 9500 km, ikkinchisi esa 2400 km masofada joylashgan. Bu yo'ldoshlarning Mirrix atrofida aylanish davri topilsin. [7,8 soat; [T=31,2 soat; T=31,2 soat.]

4. Sun'iy yo'ldosh aylana orbita bo'ylab Yer ekvatori tekisligida harakatlanadi va hamma vaqt Yer sirtidagi bitta joy ustida qoladi. Yo'ldoshning burchak tezligi va orbitasining radiusi aniqlansin. [$\omega = 7{,}27 \cdot 10^{-5}$ rad/s ; R=42,2 mm.]

5. Yer sirtidan qanday masofada erkin tushish tezlanishining qiymati $1\,\text{m/s}^2$ ga teng bo'ladi? [h=13,6 Mm.]

6. 1 kg. massali jism Yer sirtida turibdi. Quyidagi ikki hol uchun og'irlik kuchining o'zgarishi ΔP aniqlansin: 1) Jismni 5km balandlikka ko'targanda; 2) jismni chuqurligi 5 km bo'lgan shaxtaga tushirganda. Yer, 6,37 Mm radiusli va $5{,}5\,{}^{G}\!/_{sm^3}$ zichlikli bir jinsli kurra deb hisoblansin. [1) $\Delta\rho = 15{,}4$mN; 2) $\Delta\rho = 7{,}71 mN$.]

7. Yerning va Oyning markazlarini tutashtiruvchi to'g'ri chiziqning qaysi nuqtasida (Yerdan boshlab hisoblanganda) tortishish maydonining kuchlanganligi nolga teng bo'ladi? Yer va Oy markazlari orasidagi masofa R. Yerning massasi Oyning massasidan 81 marta katta. [l–0,9 *R*.]

8. Ikkita bir xil, bir jinsli va bir xil materialdan yasalgan sharlar bir-biriga tegib turibdi. Agar sharlarning massalari to'rt martadan ko'paytirilsa, ularning uzaro gravitatsion ta'sir potensial energiyasi qanday o'zgarishi aniqlansin. [n=14,6 marta oshdi.]

9. Bir hil massali ikkita yo'ldosh R_1 va R_2 radiusli aylanma orbitalar bo'ylab Yer atrofidan harakatlanadi. 1) Yo'ldoshlar to'la energiyalarining nisbati E_1/E_2; 2) ular impuls momentlarining nisbati L_1/L_2 – lar aniqlansin. [$E_1/E_2=R_1/R_2$.]

10. Quyosh sirti yaqinidagi birinchi va ikkinchi kosmik tezliklar topilsin. [$\vartheta_1 = 436$ km/s; $\vartheta_2 = 617$ km/s.]

6-§. DEFORMATSIYA. ELASTIKLIK KUCHLARI

Asosiy formulalar

Qattiq jismning bo'ylanma cho'zilishidagi yoki siqilishidagi nisbiy deformatsiya:

$$\varepsilon = \frac{x}{l},$$

bunda: ε – nisbiy cho'zilish (siqilish); x – absolyut cho'zilish (14-rasm); l – jismning boshlang'ich uzunligi. Siljishdagi nisbiy deformatsiya (kichik burchaklar uchun) $\operatorname{tg}\alpha = \frac{\Delta S}{h}$ formuladan aniqlanadi, bunda:

tgα –nisbiy siljish, ΔS – jism parallel qatlamlarining bir-biriga nisbatan absolyut siljishi (15-rasm), h – qatlamlar orasidagi masofa, γ – siljish burchagi. Buralishdagi nisbiy deformatsiya:

$$\varepsilon_{\varphi} = \frac{\varphi}{l}$$

bunda: j – buralish burchagi, l –jism uzunligi.

Bo'ylanma deformatsiyada hajmning nisbiy o'zgarishi:

$$\frac{\Delta V}{V} = \varepsilon(1 - 2\mu),$$

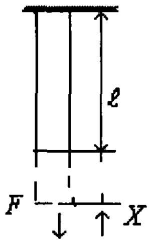
bunda: m-Puasson koeffitsienti bo'lib, nisbiy ko'ndalang deformatsiyaning nisbiy bo'ylanma deformatsiyaga nisbati bilan aniqlanadi:

$$\mu = \frac{\varepsilon_{\kappa}}{\varepsilon}.$$

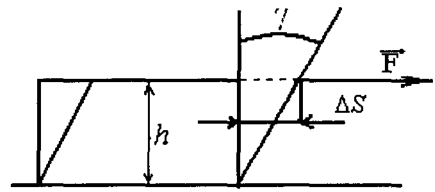
Normal kuchlanish $\sigma = \frac{F_{el}}{S}$,

bunda: F_{el} – jism ko'ndalang kesimiga tik yo'nalgan elastiklik kuchi; S – shu kesimning yuzasi.

Tangensial kuchlanish $\tau = \frac{F_{el}}{S}$,



19-rasm



20-rasm

bunda: F_{el} – jism qatlami bo'ylab ta'sir etuvchi elastiklik kuchi; S – shu qatlamning yuzasi.

Bo'ylanma cho'zilish yoki siljish uchun Guk qonuni:

$$F_{el} = -kx \text{ yoki } \quad \sigma = \varepsilon E,$$

bunda: k – elastiklik koeffitsienti (prujina holida qattilik, bikirlik), E – Yung moduli.

Siljish uchun Guk qonuni $\Delta S = \dfrac{F \cdot h}{G \cdot S}$ yoki $\tau = G\gamma$,

bunda: G – ko'ndalang elastiklik moduli (siljish moduli).

Bir jinsli yumaloq tayoqchani φ burchakka buraydigan moment:

$$M=C\varphi$$

bunda: C – buralish doimiysi.

Jismni deformatsiyalaganda bajariladigan ish: $A = \dfrac{kx^2}{2}$.

Cho'zilgan yoki siqilgan tayoqchaning potensial energiyasi:

$$P = \frac{kx^2}{2}, \text{ yoki } \quad P = \frac{\tau^2}{2E}V, \text{ yoki } \quad P = \frac{\mathrm{E}\varepsilon^2}{2}V,$$

bunda: V – jismning hajmi.

MASALA YECHISHGA MISOLLAR

1-misol. Diametri 5 sm, uzunligi 50 sm bo'lgan po'lat simga 2 kg massali yuk osilgan. 1) simning normal kuchlanishi; 2) simning absolyut va nisbiy cho'zilishlari topilsin.

Berilgan:

$d = 5\text{sm} = 5\cdot10^{-2}\text{m};$

$l = 50\text{sm} = 0{,}5\text{m};$

$m = 2\text{kg}.$

1) $G = ?$

2) $x = ?$

3) $\varepsilon = ?$

Yechish: Cho'zilgan simning normal kuchlanishi quyidagi formula bilan aniqlanadi:

$$\sigma = \frac{F}{S}. \qquad (1)$$

Agar $F=mg$, g – erkin tushish tezlanishi $g = 9{,}8\,\text{m}/\text{s}^2$,

$S = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{1}{4}\pi d^2$ simning ko'ndalang kesim yuzasi ekanligini nazarda tutsak:

$$\sigma = \frac{mg}{\frac{1}{4}\pi d^2} = \frac{4mg}{\pi d^2} \qquad (2)$$

ni olamiz.

(2) dan kuchlanishning birligi chiqishi ko'rinib turibdi.

Berilganlarni qo'yib hisoblaymiz:

$$G = \frac{4\cdot2\cdot9{,}8}{3{,}14\cdot(5\cdot10^{-2})^2}\frac{\text{N}}{\text{m}^2} = \frac{8\cdot9{,}8}{3{,}14\cdot25\cdot10^{-4}}\frac{\text{N}}{\text{m}^2} = \frac{78{,}4}{77{,}5}\cdot10^4\frac{\text{N}}{\text{m}^2} \approx$$

$$\approx 10^4\frac{\text{N}}{\text{m}^2} = 10\frac{\text{kN}}{\text{m}^2} = 10\text{kPa}$$

2) Absolyut cho'zilish

$$x = \frac{F\cdot l}{E\cdot S} \qquad (3)$$

formula bilan aniqlanadi. Bu yerda E – Yung moduli. Po'lat uchun E=2 10^{11} Pa. (3) ni o'zgartirib yozamiz:

$$x = \frac{mgl}{E\cdot\frac{1}{4}\pi d^2} = \frac{4mgl}{E\cdot\pi d^2} \qquad (4)$$

Simning nisbiy cho'zilishini

$$\varepsilon = \frac{x}{l} \tag{5}$$

formula yordamida aniqlaymiz.

Kattaliklarning son qiymatlarini (4) va (5) larga qo'yib hisoblaymiz:

$$x = \frac{4 - 2 \cdot 9{,}8 \cdot 0{,}5}{2 \cdot 10^{11} \cdot 314 \cdot (5 \cdot 10^{-2})^2}\ \text{m} = \frac{19{,}6 \cdot 10^{-7}}{3{,}14 \cdot 25}\ \text{m} = \frac{19{,}6}{77{,}5} \cdot 10^{-7}\ \text{m} \approx 0{,}25 \cdot 10^{-7}\ \text{m},$$

$$\varepsilon = \frac{0{,}25 \cdot 10^{-7}}{0{,}5} = 0{,}5 \cdot 10^{-7}.$$

Javob: 1) G=10 kPa; 2) $x = 0{,}25 \cdot 10^{-7}\ m$; 3) $\varepsilon = 0{,}5 \cdot 10^{-7}$.

2-misol. Simga bog'langan 10 kg massali qadoqtosh 2 s^{-1} chastota bilan gorizontal sirtda ishqalanishsiz sirpanganicha, simning uchidan o'tuvchi tik o'q atrofida aylanadi. Simning uzunligi 1,2 m va ko'ndalang kesimning yuzasi 2 mm^2 ga teng. Sim materialining kuchlanishi topilsin. Massasi inobatga olinmasin.

Berilgan:
m =10 kg;
n =2 s^{-1};
l =1,2 m;
S=2 mm^2= $2 \cdot 10^{-6}\ m^2$.
σ =?

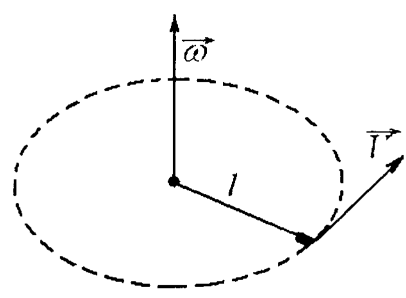
Yechish: Ta'riflanishiga binoan kuchlanish quyidagicha aniqlanadi:

$$\sigma = \frac{F}{S}. \tag{1}$$

Mazkur holda qadoqtoshga faqat markazga intilma kuch ta'sir qiladi:

$$F_{mi} = \frac{m\vartheta^2}{r}.$$

Agar bizning holimizda $r=l$ ekanligini va qadoqtoshning chiziqli ϑ va burchakli ω tezliklari orasidagi $\vartheta = \omega r$ munosabatni hisobga olsak, kuch uchun quyidagi ifodani topamiz:



21-rasm

$$F_{m_l} = \frac{m}{r}(\omega r)^2 = m\omega^2 r = m\omega^2 l. \qquad (2)$$

Burchakli tezlik ω va aylanish chastotasi n orasidagi

$$\omega = 2\pi n$$

bog'lanishdan foydalanamiz:

$$F_{m_l} = m(2\pi n)^2 l = 4\pi^2 n^2 ml. \qquad (3)$$

(3) ni (1) ga qo'yib, kuchlanish uchun

$$\sigma = \frac{4\pi^2 n^2 ml}{S} \qquad (4)$$

munosabatni olamiz. Endi (4) asosida σ ning birligini aniqlaymiz:

$$[\sigma] = \frac{[n]^2[m][l]}{[S]} = \frac{[1\text{s}^{-1}]^2 \cdot 1\text{kg} \cdot 1\text{m}}{1\text{m}^2} = \frac{1\frac{\text{kg}}{\text{s}^2}\cdot\text{m}}{1\text{m}^2} = 1\frac{\text{N}}{\text{m}^2} = 1\text{Pa}$$

va uning to'g'riligiga ishonch hosil qilgach, berilganlarni qo'yamiz

$$\sigma = \frac{4(3{,}14)^2(2)^2 \cdot 10 \cdot 1{,}2}{2 \cdot 10^{-6}}\text{Pa} = \frac{16 \cdot 12 \cdot 9{,}86}{2 \cdot 10^{-6}} \approx 948 \cdot 10^6\text{Pa} \approx 948\text{MPa}.$$

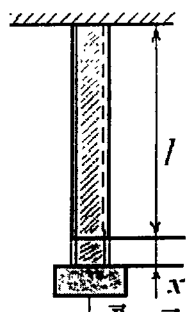
Javob: σ =948 MPa.

3-misol. Uzunligi 5 m va ko'ndalang kesimining yuzasi S=2 mm² bo'lgan tik simga m=5,1 kg massali yuk osilgan. Natijada sim x=0,6 mm ga uzaydi. Sim materiali uchun Yung moduli E topilsin.

Berilgan:
l=5 m;
S=2 mm² $= 2 \cdot 10^{-6}\text{m}^2$;
m=5,1 kg;
x=*0,6* mm $= 6 \cdot 10^{-4}$ m.

E =?

Yechish: Bo'ylanma cho'zilish uchun Guk qonunining $\sigma = \varepsilon E$ ifodasidan Yung moduli E ni aniqlab olamiz:



22-rasm

$$E = \frac{\sigma}{\varepsilon}. \qquad (1)$$

Sim ko'ndalang kesimiga tik yo'nalgan kuch yukning og'irlik kuchi $F=P=mg$ ekanligini nazarda tutib, normal kuchlanish uchun quyidagi ifodani topamiz:

$$\sigma=\frac{F}{S}=\frac{mg}{S}. \qquad (2)$$

Bo'ylama cho'zilish uchun nisbiy deformatsiya esa quyidagi formula bilan aniqlanadi:

$$\varepsilon=\frac{x}{l}. \qquad (3)$$

(2) va (3) larni (1) ga qo'yib quyidagini olamiz:

$$E=\frac{\left(\dfrac{mg}{S}\right)}{\left(\dfrac{x}{l}\right)}=\frac{mgl}{S\cdot x}. \qquad (4)$$

(4) dan E ning birligini tekshirib ko'ramiz:

$$[E]=\frac{[m][g][l]}{[S][x]}=\frac{1\text{kg}\cdot 1\frac{\text{m}}{\text{s}^2}\cdot 1\text{m}}{1\text{m}^2\cdot 1\text{m}}=\frac{1\text{N}}{1\text{m}^2}=1\text{Pa}$$

va kattaliklarning son qiymatlari yordamida E ning qiymatini topamiz:

$$E=\frac{5{,}1\cdot 9{,}8\cdot 5}{2\cdot 10^{-6}\cdot 6\cdot 10^{-4}}\text{Pa}=\frac{25{,}5\cdot 9{,}8}{12}\cdot 10^{10}\text{Pa}=20{,}8\cdot 10^{10}\text{Pa}=208\text{GPa}.$$

Javob: E= 208 GPa.

4-misol. Qattiqliklari $0{,}3\frac{\text{kN}}{\text{m}}$ va $0{,}5\frac{\text{kN}}{\text{m}}$ bo'lgan ikkita prujina ketma-ket biriktirilgan va shunday cho'zilganki, ikkinchi prujinaning absolyut deformatsiyasi 3 sm ga teng. Prujinalarni cho'zishda bajarilgan A ish hisoblansin.

Berilgan:

$$k_1=0{,}3\frac{\text{kN}}{\text{m}}=300\frac{\text{N}}{\text{m}};$$

$$k_2 = 0{,}5\frac{\text{kN}}{\text{m}} = 500\frac{\text{N}}{\text{m}};$$

$$x_2 = 3\text{sm} = 3\cdot 10^{-2}\text{m}.$$

$A=?$

Yechish: Prujina sistemasini cho'zishda bajarilgan ish A, har bir prujinani cho'zishda bajarilgan ishlar A_1 va A_2 larning yig'indisiga teng bo'ladi, ya'ni

$$A=A_1 + A_2. \qquad (1)$$

Agar jismni deformatsiyalaganda bajariladigan ish $A = \frac{kx^2}{r}$ dek aniqlanishini hisobga olsak,

$$A = \frac{k_1 x_1^2}{2} + \frac{k_2 x_2^2}{2} \qquad (2)$$

bo'ladi. Shu bilan birga prujinalar sistemasida vujudga keladigan elastiklik kuchlari uchun Nyutonning uchinchi qonunini qo'llaymiz:

$$F_1=F_2.$$

Cho'zilishda elastiklik kuchining $F=-kx$ dek aniqlanishidan

$$k_1x_1=k_2x_2$$

ni olamiz. Bu ifodadan x_1 ni aniqlaymiz:

$$x_1 = \frac{k_2}{k_2}x_2. \qquad (3)$$

(3) ni (2) ga qo'yib quyidagini olamiz:

$$A = \frac{k_1}{2}\left(\frac{k_2}{k_1}x_2\right)^2 + \frac{k_2 x_2^2}{2} = \frac{1}{2}\frac{k_2}{k_1}(k_2 + k_1)x_2^2. \qquad (4)$$

Hosil bo'lgan ifodadan ishning birligi chiqishi ko'rinib turganligidan uni tekshirib o'tirmasdan berilganlarni qo'yaveramiz:

$$A = \frac{1}{2}\frac{500}{300}(500+300)(3\cdot 10^{-2})^2\,\text{J} = \frac{40\cdot 9\cdot 10^{-2}}{6}\text{J} = 60\cdot 10^{-2}\,\text{J} = 0{,}6\text{J}.$$

Javob: A=0,6 J.

5-misol. 10 g massali o‘q, prujinasi 5 sm ga siqilgan prujinali to‘pponchadan qanday ϑ tezlik bilan uchib chiqadi? Prujinaning qattiqligi 200 N/m.

Berilgan:
$t = 10\text{g} = 10^{-2}$ kg;
$x = 5\text{sm} = 5\cdot 10^{-2}\text{m}$;
$k = 200\dfrac{\text{N}}{\text{m}}$.

$\vartheta = ?$

Yechish: Yer-o‘q sistemasi (pistolet bilan birgalikda) yopiq sistema bo‘lib unda konservativ kuchlar, ya’ni elastik kuchlar va tortishish kuchi ta’sir qiladi. Bunday sistema uchun o‘rinli bo‘lgan, mexanikada energiyaning saqlanish qonuniga binoan o‘q otilish jarayonida oldingi va keyingi to‘la energiyalar teng bo‘lmog‘i kerak:

$$E_1 = E_2 \quad \text{yoki} \quad T_1 + P_1 = T_2 + P_2 \qquad (1)$$

Otilguncha o‘q harakatsiz ($\vartheta_1 \neq 0$) va demak, uning kinetik energiyasi ham nolga teng. $T_l=0$. Shuningdek; Yer sirtini hisob boshi deb qabul qilsak ($h=0$), o‘q va to‘pponchaning Yer tortishish maydonidagi potensial energiyasi ham nolga teng bo‘ladi $P_2=0$.

Shunday qilib, o‘q otilishidan oldingi energiya siqilgan prujinaning potensial energiyasiga

$$P_1 = \frac{kx^2}{2}, \qquad (2)$$

otilishidan keyingi energiya esa to‘pponchadan chiqayotgan o‘qning kinetik energiyasiga

$$T_2 = \frac{m\vartheta^2}{2} \qquad (3)$$

teng bo‘ladi. Demak,

$$\frac{kx^2}{2} = \frac{m\vartheta^2}{2}.$$

Bundan

$$\vartheta = \sqrt{\frac{k}{m}}x. \qquad (4)$$

Birligini tekshiramiz:

$$[\vartheta]=\frac{[k]^{1/2}}{[m]^{1/2}}[x]=\frac{\left(1\,{}^{N}\!/_{m}\right)^{1/2}}{(1\,kg)^{1/2}}(1\,m)=\left(1\frac{kg\,{}^{m}\!/_{s^2}}{kg\cdot m}\right)^{1/2} m=1\frac{m}{s}$$

va berilganlarni qo'yib hisoblaymiz: $\vartheta=\frac{200}{10^{-2}}\cdot 5\cdot 10^{-2}\frac{m}{s}=7{,}1\,{}^{m}\!/_{s}$.

Javob: $\vartheta=7{,}1\,{}^{m}\!/_{s}$.

MUSTAQIL YECHISH UCHUN MASALALAR

1. Diametri 1 mm bo'lgan po'lat sim elastiklik chegarasi 294 MPa dan chiqmasdan ko'pi bilan qancha yukka chidashi mumkin? Shu yuk ta'sirida simning cho'zilishi uning boshlang'ich uzunligining qanday hissasini tashkil etadi? [231 N; $1{,}47\cdot 10^{-3}$.]

2. Uzunligi 1,2 m, ko'ndalang kesimining yuzasi 2 sm^2 va massasi 10 kg bo'lgan bir jinsli tayoqcha uchidan o'tuvchi, tik o'q atrofida 2 s^{-1} chastota bilan aylanadi. Shu aylanish chastotasi uchun tayoqcha materialining eng katta kuchlanishi topilsin. [4,74 MPa.]

3. Uzunligi 2 m va diametri 1mm bo'lgan sim amalda gorizontal tortilgan. Simning o'rtasiga 1 kg massali yuk osilganlarida, sim, yuk osilgan nuqta 4 sm gácha pasayadigan darajada cho'ziladi. Sim materialining Yung moduli aniqlansin. [196 GPa.]

4. Burama prujinaning yuqori uchida turgan taglik ustida qo'yilgan qadoqtosh prujinani 2 mm ga siqadi. Prujina uchiga 5 sm balandlikdan tushgan shu qadoqtoshning o'zi prujinani qanchagacha siqadi? [16,3 mm.]

5. Uzunligi 2 m va ko'ndalang kesim yuzasi 2 sm bo'lgan po'lat tayoqcha 10 kN kuch bilan cho'ziladi. Cho'zilgan tayoqchaning potensial energiyasi va energiyaning hajmiy zichligi ω topilsin. $\left[160J;\ 00,\frac{MJ}{m^3}.\right]$

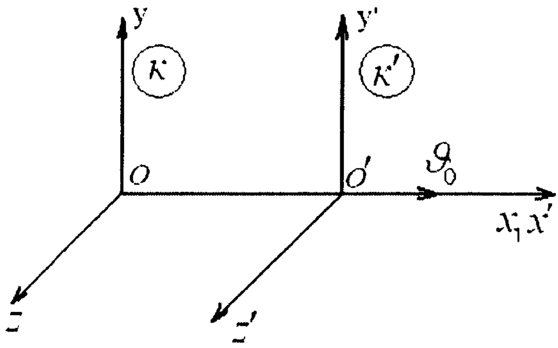
6. 12 t massali vagon 1m/s tezlik bilan harakatlangan. U prujinali buferga urilib, bufer prujinasini 10sm ga siqib, to'xtaydi. Prujinaning qattiqligi k topilsin. [1,2 MN/m.]

7. Agar alyuminiy tayoqchani cho'zish uchun 621 J ish bajarilgan bo'lsa, unda tayoqchaning nisbiy cho'zilishi ε aniqlansin. Tayoqchaning uzunligi 2 m, ko'ndalang kesimining yuzasi 1 mm^2 alyuminiy uchun Yung moduli 69 GPa [0,03.]

7-§. Relyativistik mexanika asoslari

Asosiy formulalar

Relyativistik mexanikada harakat tezligi, yorug'likning bo'shliqdagi tezligiga yaqin bo'lgan tezliklar bilan harakatlantiruvchi jismlarning harakati o'rganiladi. Maxsus nisbiylik nazariyasida (relyativistik mexanikadagi nisbiylik nazariyasida) faqat inersial sanoq sistemalarigina qaraladi.



23-rasm

Qulaylik uchun bu sanoq sistemalarining y, y, z, z' o'qlarining yo'nalishi mos keluvchi, sistemalarning o'zaro nisbiy tezligi umumiy x, x' o'qlar bo'yicha olamiz:

$$l = l_0\sqrt{1-\left(\vartheta/c\right)^2},$$

bunda: l_0 — tayoqchaning o'zi harakatsiz bo'lgan K' sanoq sistemasidagi uzunligi (xususiy uzunligi), l — tayoqchaning unga nisbatan ϑ tezlik bilan harakatlanayotgan sistemaga nisbatan (K — sistema) uzunligi. c — yorug'likning bo'shliqdagi tezligi.

Soat yurishining relyativistik sekinlashuvi:

$$\Delta t = \frac{\Delta t_o}{\sqrt{1-\left(\vartheta/c\right)^2}},$$

bunda: Δt_o – K' sistemadagi bir nuqtada ro'y beradigan, shu sistemaning soati bilan o'lchangan (harakatlanuvchi soatning xususiy vaqti) ikki hodisa orasidagi vaqt intervali; Δt – ikki hodisa orasidagi K sistemaning soati bilan o'lchangan vaqt intervali.

Tezliklarni relyativistik qo'shish:

$$\vartheta = \frac{\vartheta' + \vartheta_0}{1 + \vartheta_0 \vartheta' / c^2},$$

bunda: ϑ' – nisbiy tezlik (jismning K' sistemaga nisbatan tezligi);

ϑ_0 – ko'chirma tezlik (K' sistemaning K ga nisbatan tezligi);

ϑ – absolyut tezlik (jismning K sistemaga nisbatan tezligi).

Absolyut tezlik deb, jismning shartli ravishda harakatsiz deb qabul qilingan koordinata sistemasidagi tezligiga aytiladi.

Relyativistik massa:

$$m = \frac{m_o}{\sqrt{1-\beta^2}},$$

bunda: m_0 –tinchlikdagi massa; b – jismning, yorug'lik tezligi ulushlarida ifodalangan tezligi ($\beta = \vartheta / c$).

Relyativistik impuls:

$$P = m\vartheta = \frac{m_0 \vartheta}{\sqrt{1-\beta^2}} = m_0 c \frac{\beta}{\sqrt{1-\beta^2}}.$$

Relyativistik zarraning to'la energiyasi:

$$E = m \cdot c^2 = m_0 c^2 + T,$$

bunda: T – zarraning kinetik energiyasi; $m_0 c^2 = E_0$ – zarraning tinchlikdagi energiyasi. Agar zarraning tezligi yorug'lik tezligiga yaqin bo'lsa, zarraga relyativistik, agar $\vartheta << c$ bo'lsa, klassik deyiladi.

Relyativistik zarraning to'la energiyasi va impulsi orasidagi bog'lanish:

$$E^2 - P^2 c^2 = m_0^2 c^4.$$

Relyativistik zarraning kinetik energiyasi va impulsi orasidagi bog'lanish:

$$P^2 c^2 = T(T + 2m_0 c^2).$$

Masala yechishga misollar

1-misol. Tayoqcha inersial sanoq sistemasiga nisbatan o'zgarmas tezlik bilan bo'ylanma yo'nalishda harakatlanmoqda. Tezlikning qanday qiymatida tayoqchaning shu sistemadagi uzunligi tinch turgan tayoqcha uzunligidan 1% ga kam bo'ladi?

Berilgan:

$$\frac{l_o - l}{l_o} \cdot 100\% = 1\%;$$

$$\frac{l_o - l}{l_o} = 0{,}01.$$

$\vartheta = ?$

Yechish: Tayoqcha uzunligining relyativistik (Lorens) qisqarishi quyidagi formula bilan aniqlanadi:

$$l = l_o\sqrt{1-\beta^2}\,. \qquad (1)$$

Masalaning shartiga ko'ra esa

$$\frac{l_o - l}{l_o} = 0{,}01\,. \qquad (2)$$

(1) dagi l ning qiymatini (2) ga qo'yamiz:

$$\frac{l_0 - l_0\sqrt{1-\beta^2}}{l_0} = 0{,}01$$

yoki

$$1 - \sqrt{1-\beta^2} = 0{,}01\,.$$

Bu ifodadan $\beta = \vartheta/c$ ligini e'tiborga olib ϑ ni topamiz:

$$\sqrt{1-\beta^2} = 0{,}99, \qquad 1-\beta^2 = 0{,}98.$$

$$\beta^2 = 0{,}02, \qquad \vartheta = 0{,}141c.$$

Agar $c = 3\cdot10^8\ \text{m/s}$ ligini nazarda tutsak

$$\vartheta = 0{,}14\cdot3\cdot10^8\ \text{m/s} = 0{,}423\cdot10^8\ \text{m/s} = 42300\ \text{km/s}$$

Javob: $\vartheta = 0{,}423\cdot10^8\ \text{m/s} = 42300\ \text{km/s}$.

2-misol. Fazoviy kema – yo'ldosh ichida, uchishgacha Yerdagisi bilan tenglashtirilgan soat bor. Yo'ldoshning tezligi 7,9 km/s. Yerdagi kuzatuvchi o'z soati bilan 0,5 yilni o'lchasa, yo'ldoshdagi soat qancha orqada qoladi?

Berilgan:

$\vartheta_0 = 7{,}9\,\mathrm{km/s} = 7{,}9\cdot 10^3\,\mathrm{m/s}$;

$\tau_0 = 0{,}5\mathrm{y} = 1{,}58\cdot 10^7\mathrm{s}$;

$\beta = \dfrac{\vartheta_0}{c} = \dfrac{7{,}9\cdot 10^3}{3\cdot 10^8} = 2{,}43\cdot 10^{-5}$.

$\tau = \Delta\tau - \Delta\tau_0 = ?$

Yechish: Soat yurishining relyativistik sekinlashuvi quyidagi ifoda yordamida aniqlanadi:

$$\Delta\tau = \frac{\Delta\tau_0}{\sqrt{1-\beta^2}}. \qquad (1)$$

$\beta \ll 1$ ligini nazarda tutib $\dfrac{1}{\sqrt{1-\beta^2}}$ ni qatorga yoyish mumkin.

Dastlabki ikki had bilan chegaralanamiz

$$\frac{1}{\sqrt{1-\beta^2}} = 1 + \frac{1}{2}\beta^2. \qquad (2)$$

(2) ni (1) ga qo'yib, quyidagi olamiz:

$$\Delta\tau = \Delta\tau_0\left(1 + \frac{1}{2}\beta^2\right).$$

So'nggi ifodadan quyidagi munosabatni topamiz:

$$\tau = \Delta\tau - \Delta\tau_0 = \frac{1}{2}\beta^2; \quad \Delta\tau_0 = \frac{1}{2}\frac{\vartheta_0^2}{c^2}\cdot\Delta\tau_0.$$

Berilganlarni (3) ga qo'yib olamiz:

$$\tau = \frac{1}{2}(2{,}43\cdot 10^{-5})^2\cdot 1{,}58\cdot 10^7\mathrm{s} = 0{,}79\cdot(2{,}43)^2\cdot 10^{-3}\mathrm{s} = 5{,}7\cdot 10^{-3}\mathrm{s}.$$

Javob: $\tau = 5{,}7\cdot 10^{-3}\mathrm{s}$.

3-misol. Tayoqcha laboratoriya sanoq sistemasida (K – sistema) $\vartheta = 0{,}8c$ tezlik bilan harakatlanmoqda. K sistemada o'tkazilgan hisoblashlarga binoan uning uzunligi $l = 10$ m, x o'qi bilan hosil qilgan burchagi $\varphi = 30^0$ ga teng bo'lib chiqdi. Tayoqchaning o'zi bilan bog'langan K' – sistemadagi xususiy uzunligi l_0 va x' o'qi bilan hosil qilgan burchagi φ_0 aniqlansin.

Berilgan:

$\vartheta = 0{,}8$ c;

$l = 10$ m;

$\varphi = 30^0$;

$\beta = 0{,}8$.

$l_0 = ?$

$\varphi_0 = ?$

Yechish: K' – sistemada tayoqcha $x'o'y'$ tekisligida yotsin. 24 a-rasmdan ko'rinib turibdiki, tayoqchaning xususiy uzunligi l_0 va uning ox' o'qi bilan hosil qilgan burchagi φ_0:

$$l_0 = \sqrt{(\Delta x')^2 + (\Delta y')^2}, \quad \mathrm{tg}\varphi_0 = \frac{\Delta y'}{\Delta x'}. \tag{1}$$

Shu kattaliklarning o'zi K – sistemada (24 b-rasm)

$$l_0 = \sqrt{(\Delta x)^2 + (\Delta y)^2}, \quad \mathrm{tg}\varphi_0 = \frac{\Delta y}{\Delta x}. \tag{2}$$

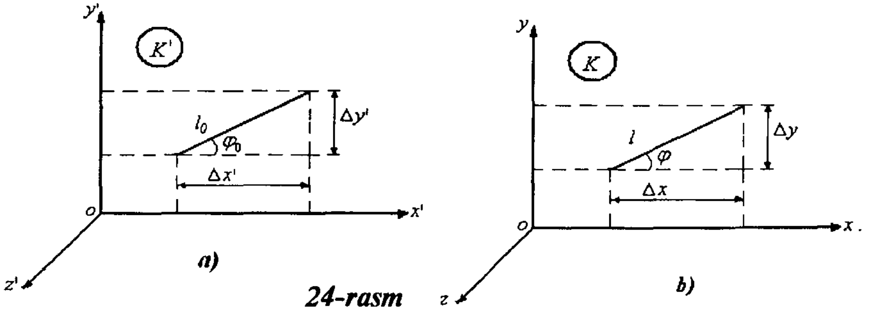
Shuni qayd etish lozimki, K' – sistemadan K ga o'tilganda tayoqchaning *oy* o'qi yo'nalishidagi o'lchamlari o'zgarmaydi. ox o'qi yo'nalishida esa relyativistik (Lorens) qisqarishiga uchraydi, ya'ni

$$\Delta y = \Delta y', \qquad \Delta x = \Delta x'\sqrt{1-\beta^2}. \tag{3}$$

Oxirgi munosabatlar hisobga olinganda tayoqchaning xususiy uzunligi quyidagi tenglik bilan ifodalanadi:

$$l_0 = \sqrt{\left(\frac{\Delta x}{\sqrt{1-\beta^2}}\right)^2 + (\Delta y)^2} = \frac{\sqrt{(\Delta x)^2 + (\Delta y)^2 - \beta^2(\Delta y)^2}}{\sqrt{1-\beta^2}},$$

yoki



24-rasm

$$l_0 = \frac{\sqrt{l^2 - \beta^2 (\Delta y)^2}}{\sqrt{1-\beta^2}}.$$

Agar $\Delta y = l \cdot \sin\varphi$ ekanligini nazarda tutsak (24 b-rasm) quyidagini olamiz:

$$l_0 = \frac{\sqrt{l^2 - \beta^2 l^2 \sin^2\varphi}}{\sqrt{1-\beta^2}} = \frac{l}{\sqrt{1-\beta^2}}\sqrt{1-\beta^2 \cdot \sin^2\varphi}. \qquad (4)$$

φ_0 ni topish uchun (2) va (3) lar yordamida (1) ni qayta yozamiz:

$$\mathrm{tg}\varphi_0 = \frac{\Delta y'}{\Delta x'} = \frac{\Delta y}{\Delta x}\sqrt{1-\beta^2} = \mathrm{tg}\varphi \cdot \sqrt{1-\beta^2},$$

bundan

$$\varphi_0 = \mathrm{arctg}\left(\mathrm{tg}\varphi \cdot \sqrt{1-\beta^2}\right). \qquad (5)$$

φ va β larning qiymatlarini (4) va (5)larga qo'yib va hisoblab topamiz:

$$l_0 = \frac{10}{\sqrt{1-(0{,}8)^2}}\sqrt{1-(0{,}8)^2 \cdot (\sin 30)^2}\,\mathrm{m} = \frac{10}{0{,}6}\sqrt{1-0{,}16}\,\mathrm{m} = \frac{\sqrt{0{,}84}}{0{,}6}10\mathrm{m} = 15{,}3\mathrm{m};$$

$$\varphi_0 = \mathrm{arctg}\left[\mathrm{tg}30^0 \cdot \sqrt{1-0{,}8^2}\right] = \mathrm{arctg}\left[\mathrm{tg}30^0 \cdot 0{,}6\right] = 19{,}1^0.$$

Javob: $l_0 = 15{,}3\mathrm{m}$; $\varphi_0 = 19{,}1^0$.

4-misol. Agar zarraning relyativistik massasi tinchlikdagi massasidan uch marta katta bo'lsa, zarra qanday ϑ tezlik bilan harakatlanadi?

Berilgan:

$\frac{m}{m_0}=3$.

$\vartheta=?$

Yechish: Relyativistik massa quyidagicha aniqlanadi:

$$m=\frac{m_0}{\sqrt{1-\beta^2}} \qquad (1)$$

bunda: m_0 – tinchlikdagi massa, $\beta=\frac{\vartheta}{c}$ – zarraning, yorug'likning tezligi ulushlarida ifodalangan tezligi.

(1) ni quyidagi ko'rinishda yozamiz:

$$\frac{m}{m_0}=\frac{1}{\sqrt{1-\beta^2}} \quad \text{yoki} \quad \sqrt{1-\beta^2}=\frac{1}{\left(\frac{m}{m_0}\right)},$$

bundan

$$\vartheta=c\sqrt{1-\frac{1}{\left(\frac{m}{m_0}\right)^2}}, \qquad (2)$$

$c=3\cdot10^8\,\frac{m}{s}$ – yorug'likning bo'shliqdagi tezligini hisobga olib (2) dan topamiz:

$$\vartheta=3\cdot10^8\cdot\sqrt{1-\frac{1}{3^2}}\cdot\frac{\text{m}}{\text{s}}=3\cdot10^8\sqrt{\frac{8}{9}}\frac{\text{m}}{\text{s}}=2{,}83\cdot10^8\,\frac{\text{m}}{\text{s}}.$$

Javob: $\vartheta=2{,}83\cdot10^8\,\frac{\text{m}}{\text{s}}$.

5-misol. Okeandagi suvning hajmi $1{,}37\cdot10^9\,\text{km}^3$ ga tengligi ma'lum. Agar suvning harorati 1 K ga ko'tarilsa, okeandagi suvning massasi qanchaga ortishi aniqlansin. Okeandagi suvning zichligi $1{,}03\cdot10^3\,\text{kg/m}^3$ ga teng deb qabul qilinsin.

Berilgan:

$V = 1{,}37 \cdot 10^9 \text{km}^3 = 1{,}37 \cdot 10^{18} \text{m}^3;$

$\Delta t = 1\text{K};$

$\rho = 1{,}03 \cdot 10^3 \text{kg}/\text{m}^3.$

$\Delta m = ?$

Yechish: Energiya va massa orasidagi bog'lanish formulasini yozamiz:

$$E = mc^2.$$

Bunda $c = 3 \cdot 10^8 \text{m}/\text{s}$ – yorug'likning bo'shliqdagi tezligi. Shuningdek, energiyaning ΔE o'zgarishi massaning Δm o'zgarishiga olib keladi:

$$\Delta E = \Delta m \cdot c^2 \quad \text{yoki} \quad \Delta m = \frac{\Delta E}{c^2} \qquad (1)$$

Bu yerda: ΔE – okeandagi suvning harorati Δt ga o'zgarishi natijasida olgan energiyasi, aniqrog'i, issiqlik miqdori:

$$\Delta E = Q = d \cdot m \cdot \Delta t = d \cdot \rho \cdot V \cdot \Delta t, \qquad (2)$$

bunda: $d = 4{,}13 \cdot 10^3 \dfrac{\text{J}}{\text{kg} \cdot \text{K}}$ – suvning solishtirma issiqlik sig'imi, shuningdek, $m = \rho \cdot V$ ligini inobatga olindi.

$$\Delta m = \frac{d \cdot \rho \cdot V \cdot \Delta t}{c^2}. \qquad (3)$$

(3) dan hosil qilinadigan Δm ning birligini tekshirib ko'ramiz:

$$[\Delta m] = \frac{[d] \cdot [\rho] \cdot [V] \cdot [\Delta t]}{[C]^2} = \frac{1\frac{\text{J}}{\text{kg} \cdot \text{K}} \cdot 1\frac{\text{kg}}{\text{m}^3} \cdot 1\text{m}^3 1\text{K}}{\left(1\frac{\text{m}}{\text{s}}\right)^2} = \frac{1\text{kg} \cdot \frac{\text{m}^2}{\text{s}^2}}{1\frac{\text{m}^2}{\text{s}^2}} = 1\text{kg}$$

va to'g'riligiga ishonch hosil qilgandan keyin kattaliklarning qiymatlarini qo'yamiz.

$$\Delta m = \frac{4{,}13 \cdot 10^3 \cdot 1{,}03 \cdot 10^3 \cdot 1{,}37 \cdot 10^{18} \cdot 1}{9 \cdot 10^{16}} \text{kg} = \frac{4{,}13 \cdot 1{,}03 \cdot 1{,}37}{9} \cdot 10^8 \text{kg} =$$

$$= 0{,}657 \cdot 10^8 \text{kg} = 6{,}57 \cdot 10^7 \text{kg}.$$

Javob: $\Delta m = 6{,}57 \cdot 10^7 \text{kg}.$

6-misol. Elektron 0,8c tezlik bilan harakatlanmoqda. Elektronning kinetik energiyasi T (mega elektron-voltlarda) aniqlansin.

Berilgan:	**Yechish:** Relyativistik zarraning to'la energiyasi
$\vartheta_e = 0{,}8c$;	uning tinchlikdagi energiyasi $E_0 = m_0c^2$
$\beta = \frac{\vartheta_e}{c} = 0{,}8.$	va kinetik energiyasi T larning yig'indisiga teng.
$T = ?$	Ya'ni $E = E_0 + T$.

Bundan:

$$T = E - E_0. \tag{1}$$

Elektron uchun $E_0 = 0{,}511 MeV$. Zarraning to'la energiyasi

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1-\beta^2}} = \frac{E_0}{\sqrt{1-\beta^2}}. \tag{2}$$

(2) ni (1) ga qo'yamiz:

$$T = \frac{E_0}{\sqrt{1-\beta^2}} - E_0 = E_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right). \tag{3}$$

Berilganlarni (3) ga qo'yib olamiz:

$$T = E_0\left(\frac{1}{\sqrt{1-(0{,}8)^2}} - 1\right) = E_0\left(\frac{1}{0{,}6} - 1\right) = 0{,}67 \cdot E_0 = .$$

$$= 0{,}67 \cdot 0{,}511 \text{MeV} = 0{,}34 \text{MeV}$$

Javob: T=0,34 MeV.

7-misol. Relyativistik zarraning kinetik energiyasi uning tinchlikdagi energiyasiga teng. Agar zarraning kinetik energiyasi 4 marta oshsa, impulsi necha marta ortadi?

Berilgan:

$T_0 = E_0$;

$\frac{T}{T_0} = n = 4.$

$\frac{P}{P_0} = ?$

Yechish: Energiyasi ortguncha zarra impulsi va kinetik energiyasi orasidagi munosabat

$$P_0^2c^2 = T_0(T_0 + 2E_0). \qquad (1)$$

Energiyasi ortgandan keyin zarra impulsi va kinetik energiyasi orasidagi munosabat:

$$P^2c^2 = T(T + 2E_0). \qquad (2)$$

(2) ni (1) ga hadma-had bo'lib quyidagini hosil qilamiz:

$$\left(\frac{P}{P_0}\right)^2 = \left(\frac{T}{T_0}\right)\frac{T + 2E_0}{T_0 + 2E_0}.$$

Agar $T_0 = E_0$ ni nazarda tutsak,

$$\left(\frac{P}{P_0}\right)^2 = \left(\frac{T}{T_0}\right)\frac{(T + 2E_0)}{3E_0} = \left(\frac{T}{T_0}\right)\left(\frac{T}{3T_0} + \frac{2E_0}{3T_0}\right) \quad \text{yoki}$$

$$\left(\frac{P}{P_0}\right)^2 = 4\left(\frac{4}{3} + \frac{2}{3}\right) = 8; \qquad \frac{P}{P_0} = \sqrt{8} = 2{,}82.$$

Javob: $\frac{P}{P_0} = 2{,}82.$

Mustaqil yechish uchun masalalar

1. Zarraning xususiy yashash vaqti harakatsiz soat bilan o'lchangan yashash vaqtidan 1,5% ga farq qiladi. $\beta = \frac{\vartheta}{c}$ aniqlansin. [0,172.]

2. K' sistemada tomonlari x' o'qiga parallel bo'lgan kvadrat turibdi. Agar K' sistema K nisbatan 0,95 c tezlik bilan harakatlanayotgan bo'lsa, K sistemada kvadrat diogonallari orasidagi burchak aniqlansin. $[72^0 66'.]$

3. Laboratoriya sanoq sistemasida (K-sistema) p-mezon hosil bo'lib, tug'ilganidan to parchalanganigacha 75 m masofani uchib o'tdi.

p-mezonning tezligi $\vartheta = 0,995$ c ga teng. Mezonning xususiy yashash vaqti aniqlansin. [25 ns].

4. K sistemaga nisbatan 200 Mm/s tezlik bilan harakatlanayotgan, K' sistemada tinchilikdagi massasi 2 kg bo'lgan jism, 200 Mm/s tezlik bilan harakatlanmoqda:

1) K sistemaga nisbatan jismning tezligi ϑ;

2) shu sistemada uning massasi m aniqlansin. [277 Mm/s; 5,2 kg].

5. Relyativistik impulsi klassik mexanikadagi impulsdan besh marta ko'p bo'lgan zarraning tezligi aniqlansin. [0,98c.]

6. Laboratoriya sanoq sistemasida ikkita zarra bor. Tinchlikdagi massasi m_0 bo'lgan birinchi zarra 0,6 c tezlik bilan harakatlanmoqda; massasi 2 m_0 bo'lgan boshqasi esa tinch turibdi. Zarralar sistemasi massa markazining tezligi ϑ_s aniqlansin. [0,231 c.]

7. Tinchlikdagi energiyalari teng bo'lgan bir xil kinetik energiyali (laboratoriya sanoq sistemasida) ikkita relyativistik zarra bir-biriga qarab harakatlanmoqda. 1) zarralarning laboratoriya sanoq sistemasidagi tezliklari; 2) zarralar yaqinlashuvining nisbiy tezligi (c – birliklarda); 3) zarralardan birining boshqa zarra bilan bog'langan sanoq sistemasiga nasbatan kinetik energiyasi (m_0c^2 – birliklarda) aniqlansin. [1) 0,866 c; 2) 0,9897 c; 3) 6 m_0c^2.]

8. Kinetik energiyasi tinchlikdagi energiyasiga teng bo'lgan zarraning impulsi P (m_0c birliklarda) aniqlansin. [1,79 m_0c.]

9. Impulsi m_0c ga teng bo'lgan relyativistik zarraning kinetik energiyasi T (m_0c^2 birliklari) aniqlansin. [0,412 m_0c^2.]

8-§. Mexanik tebranishlar va to'lqinlar. Akustika

Asosiy formulalar

Garmonik tebranishlar formulasi:

$$x = A\cos(\omega t + \varphi),$$

bunda: x – tebranayotgan nuqtaning muvozanat holatidan chetlanishi; t – vaqt, A, ω, t – mos ravishda amplituda, burchak tezlik (davriy chastota), tebranishning boshlang'ich fazasi.

Davriy chastota:

$$\omega = 2\pi\nu \quad \text{yoki} \quad \omega = 2\pi/T,$$

bunda: ν va T – tebranish chastotasi va davri.

Garmonik tebranayotgan nuqtaning tezligi:

$$\vartheta = \dot{x} = -A\omega\sin(\omega t + \varphi)$$

va tezlanishi:

$$a = \ddot{x} = -A\omega^2\cos(\omega t + \varphi),$$

to'la energiyasi

$$E = \frac{1}{2}m \cdot A^2 \cdot \omega^2 = \frac{1}{2}kA^2.$$

Garmonik tebranayotgan moddiy nuqtaning harakat tenglamasi

$$m \cdot \ddot{x} = -kx \quad \text{yoki} \quad \ddot{x} + \omega^2 x = 0,$$

bunda: $k = m \cdot \omega^2$ – kvazielastik koeffitsient.

Teng chastotali va bir to'g'ri chiziq bo'ylab ro'y beradigan ikki tebranishning qo'shilishi natijasida hosil bo'lgan tebranishning amplitudasi

$$A^2 = A_1^2 = A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1).$$

Natijaviy tebranishning boshlang'ich fazasi

$$\mathrm{tg}\varphi = \frac{a_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}.$$

Teng chastotali va o'zaro tik tebranishlarda ishtirok etadigan nuqta trayektoriyasining tenglamasi:

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1\cdot A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1).$$

Matematik mayatnikning tebranish davri $T = 2\pi\sqrt{\frac{l}{g}}$,

bunda: l – mayatnik uzunligi, g – erkin tushish tezlanishi.

Prujinali mayatnikning (prujinaga osilgan jism) tebranish davri:

$$T = 2\pi\sqrt{\frac{m}{k}},$$

bunda: m – jism massasi, k – prujinaning qattiqligi.

Fizik mayatnikning tebranish davri: $T = 2\pi\sqrt{\frac{L}{g}}$,

bunda: $L = \frac{J}{ma}$ fizik mayatnikning keltirilgan uzunligi, J – inersiya momenti, a – tebranish qo'yilgan nuqtadan mayatnik massasi markazigacha bo'lgan masofa.

So'nuvchi tebranishlarning tenglamasi:

$$\ddot{x}+2\delta\cdot\dot{x}+\omega_0^2x=0$$

bunda: $d{=}r/(2m)$ – so'nish koeffitsienti; r – qarshilik koeffitsienti,

$\omega_0 = \sqrt{\frac{k}{m}}$ – tebranishning xususiy davriy chastotasi.

So'nuvchi tebranishlarning burchak chastotasi: $\omega = \sqrt{\omega_0^2-\delta^2}$,

amplitudasi:

$$A(t) = A_0 l^{-\delta t}.$$

Logarifmik dekrementi: $O = \ln\frac{A(t)}{A(t+T)} = \delta T.$

Majburiy tebranishlarda rezonas chastota:

$$\omega_{rez} = \sqrt{\omega_0^2 - 2\delta^2}$$

va amplituda: $A_{rez} = f_0 / \left(2\delta\sqrt{\omega_0^2 + \delta^2}\right)$.

Bunda: $f_0 = F_0 / m$, F_0 – tashqi kuchning amplitudaviy qiymati.

Yassi to'lqin tenglamasi:

$$\xi(x,t) = A\cos(\omega t - kx),$$

bunda: $\xi\,(x,t)$ – x koordinatali muhit nuqtalarining t vaqtdagi siljishi;

$k = \dfrac{2\pi}{\lambda}$ – to'lqin soni; $\lambda = \vartheta T = \vartheta/\nu$ – to'lqin uzunligi.

Bo'ylanma to'lqinlarning elastik muhitdagi fazoviy tezligi:

$$\vartheta = \sqrt{E/\rho},$$

bunda: E – Yung moduli; ρ – moddaning zichligi; to'lqinning gazlarda tezligi:

$$\vartheta = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\alpha P/\rho},$$

bunda: γ – adiabata ko'rsatkichi; R – molyar gaz doimiysi; T – termodinamik harorat; M – molyar massa; P – gaz bosimi.
Akustikada Doppler effekti.

$$\nu = \frac{\vartheta + u_A}{\vartheta - u_M}\nu_0,$$

bunda: ν – harakatlanuvchi qurilma qabul qiladigan tovush chastotasi: ν_0 – manba chiqaradigan tovush chastotasi; ϑ – tovushning muhitdagi tezligi, u_m – manbaning muhitga nisbatan tezligi. u_a – asbobning muhitga nisbatan tezligi.

Tovush bosimining amplitudasi:

$$P_0 = 2\pi\nu\rho\vartheta A,$$

bunda A – muhit zarralarining tebranish amplitudasi.

Biror V hajmda mujassamlangan tovush maydonining energiyasi:

$$W = \langle \omega \rangle V = \frac{1}{2}\rho\xi_0^2 \cdot V = \frac{1}{2}\frac{P_0^2}{\rho\vartheta^2}V = \frac{1}{2}\rho\omega^2 A^2 .$$

Tovush energiyasining oqimi: $f = \dfrac{W}{t}$.

Tovush intensivligi: $I = f/S = \langle \omega \rangle \vartheta = \dfrac{N}{4\pi r^2}$,

bunda: N – manbaning quvvati.

Muhitning solishtirma akustik qarshiligi: $Z_S = \rho \cdot \vartheta$.

Akustik qarshilik: $Z_a = Z_s / S$.

S – akustik maydon kesimining yuzasi.

Tovush intensivligi darajasi:

$$L_P = 10\lg(I/I_0).$$

Bunda I_0 – intensivlikning nolinchi darajasi. (I_0=*1 QWt/m*^{2})

Masala yechishga misollar

1-misol. $x = A \cdot \sin\omega(t + \tau)$ tenglama bilan berilgan tebranishning davri T, chastotasi ν va boshlang'ich fazasi φ aniqlansin. Bunda $\omega = 2{,}5\pi\,\text{s}^{-1}$, $\tau = 0{,}4\,\text{s}$.

Berilgan:	Yechish:
$x = A \cdot \sin\omega(t + \tau)$;	**Yechish:** Berilgan tebranish tenglamasini garmonik tenglama
$\omega = 2{,}5\pi\text{s}^{-1}$;	$x = A\sin(\omega t + \varphi)$ (1)
$\tau = 0{,}4\,\text{s}$.	ni quyidagi ko'rinishiga keltiramiz:
$\text{T} = ?$	
$\nu = ?$	$x = A \cdot \sin(\omega t + \omega\tau)$. (2)
$\varphi = ?$	
	(1) va (2) larni solishtiramiz:

$$\varphi = \omega\tau , \tag{3}$$

$$\nu = \frac{\omega}{2\pi} , \tag{4}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{\nu} . \tag{5}$$

Berilganlarni (3), (4) va (5) larga qo'yamiz:

$$\varphi = 2{,}5\pi \cdot 0{,}4\text{rad} = \pi \text{ rad},$$

$$\nu = \frac{2{,}5\pi}{2\pi}\text{s}^{-1} = 1{,}25\text{s}^{-1} = 1{,}25\text{Hz},$$

$$T = \frac{2\pi}{2{,}5\pi}\text{s} = 0{,}8\text{s} .$$

Javob: T=0,8 s; $\nu = 1{,}25\text{Hz}$; $\varphi = \pi$ rad.

2-misol. 5 g massali moddiy nuqta 0,5 Hz chastota bilan garmonik tebranadi. Tebranish amplitudasi 3 sm:

1) siljish 1,5 sm bo'lganda nuqtaning tezligi;
2) nuqtaga ta'sir etuvchi maksimal kuch;
3) tebranayotgan nuqtaning to' la energiyasi aniqlansin.

Berilgan:

$m = 5\text{g} = 5 \cdot 10^{-3}\text{kg};$

$\nu = 0{,}5\text{Hz};$

$A = 3\text{sm} = 3 \cdot 10^{-2}\text{m};$

$x = 1{,}5\text{sm} = 1{,}5 \cdot 10^{-2}\text{m}.$

---

1) $\vartheta = ?$

2) $F_{max} = ?$

3) E=?

Yechish: 1) Garmonik tebranish tenglamasini keltiramiz:

$$x = A\cos(\omega t + \varphi) . \tag{1}$$

Siljishdan vaqt bo'yicha birinchi tartibli hosila olib tezlikni topamiz:

$$\vartheta = \dot{x} = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi) . \tag{2}$$

Tezlikni aniqlash uchun (2) dan vaqtni yo'qotish kerak. Buning uchun (1) va (2) larni kvadratga ko'tarib, birinchisini A ga, ikkinchisini $A^2\omega^2$ ga bo'lamiz va so'ngra qo'shamiz. Bunda $\sin^2(\omega t+\varphi)+\cos^2(\omega t+\varphi)=1$ ligini hisobga olamiz: $\frac{x^2}{A^2}+\frac{\vartheta^2}{A^2\omega^2}=1$.

Agar $\omega=2\pi\nu$ ni hisobga olsak, $\frac{x^2}{A^2}+\frac{\vartheta^2}{4\pi^2\cdot\nu^2\cdot A^2}=1$.

Oxirgi tenglamani ϑ ga nisbatan yechib, quyidagini topamiz:

$$\vartheta=\pm 2\pi\nu\sqrt{A^2-x^2}. \qquad (3)$$

Berilganlarni o'rniga qo'yib, hisoblaymiz:

$$\vartheta=\pm 2\cdot 3{,}14\cdot 0{,}5\sqrt{\left(3\cdot 10^{-2}\right)^2-\left(1{,}5\cdot 10^{-2}\right)^2}\,\frac{\text{m}}{\text{s}}=\pm 8{,}2\cdot 10^{-2}\,\frac{\text{m}}{\text{s}}.$$

2. Nuqtaga ta'sir etuvchi kuchni Nyutonning ikkinchi qonunidan foydalanib topamiz:

$$F=m\cdot a. \qquad (4)$$

Nuqtaning tezlanishi a ni tezlikdan ((2) ifoda bilan aniqlanadi) vaqt bo'yicha hosila olib topamiz:

$$a=\frac{d\vartheta}{dt}=\ddot{x}=-A\omega^2\cos(\omega t+\varphi)=-4\pi^2\cdot\nu^2 A\cos(\omega t+\varphi) \qquad (5)$$

(5) ni (4) ga qo'yib, quyidagini olamiz: $F=-4\pi^2\nu^2 A\cos(\omega t+\varphi)$.

$\cos(\omega t+\varphi)=-1$ bo'lganda kuch maksimal qiymatiga erishadi:

$$F_{\max}=4\pi^2\nu^2\cdot mA. \qquad (6)$$

(6) asosida kuchning birligini tekshiramiz

$$[F]=[\nu]^2\cdot[m]\,[A]=(1\text{Hz})^2\cdot 1\text{kg}\cdot 1\text{m}=1\text{kg}\cdot\text{m}\cdot\frac{1}{\text{s}^2}=1\text{N}.$$

Berilganlarni qo'yib hisoblaymiz:

$$F_{\max}=4\cdot(3{,}14)^2\cdot(0{,}5)^2\cdot 5\cdot 10^{-3}\cdot 3\cdot 10^{-2}\,\text{N}=1{,}49\cdot 10^{-3}\,\text{N}=1{,}43\text{mN}.$$

3. Tebranayotgan nuqtaning to'la energiyasi E maksimal kinetik energiya $T_{\max}$ ga tengdir:

$$E = T_{max} = \frac{1}{2} m \cdot \vartheta_{max}^2.$$

Maksimal tezlikni (2) dan $\sin(\omega t + \varphi) = -1$ ni qo'yib, quyidagini olamiz:

$$\vartheta_{max} = A\omega = 2\pi\nu A.$$

Unda to'la energiya uchun ifoda quyidagi ko'rinishni oladi:

$$E = 2\pi^2 m \nu^2 A^2. \qquad (7)$$

E ning birligini tekshiramiz:

$$[E] = [m]\,[\nu]^2 [A]^2 = 1\text{kg} \cdot 1\text{Hz}^2 \cdot 1\text{m}^2 = 1\text{kg}\frac{\text{m}^2}{\text{s}^2} = 1\text{J}$$

Kattaliklarning qiymatlarini qo'yamiz:

$$E = \cdot 2 \cdot (3{,}14)^2 \cdot 5 \cdot 10^{-3} \cdot (0{,}5)^2 \cdot (3 \cdot 10^{-2})^2 \text{J} = 22{,}1 \cdot 10^{-6}\text{J} = 22{,}1\text{mkJ}$$

Javob: 1) $\vartheta = \pm 8{,}2 \cdot 10^{-2}\frac{\text{m}}{\text{s}}$; 2) $F_{max} = 1{,}43\text{mN}$; 3) $E = 22{,}1\text{mkJ}$.

Eslatma: Garmonik tebranish tenglamasi $x = A\sin(\omega t + \varphi)$ ko'rinishda olingan holda ham natijalar shunday bo'ladi. Buni talabaning o'zi tekshirib ko'rishi mumkin.

3-misol. Nuqta ikkita bir xil yo'nalishli $x_1 = A_1 \sin\omega t$ va $x_2 = A_2 \cos\omega t$ tebranishlarda ishtirok etadi. Bunda A_1=1 sm, A_2=2 sm, $\omega = 1\text{s}^{-1}$. Natijaviy tebranishning amplitudasi A, uning chastotasi ν va boshlang'ich fazasi φ aniqlansin. Bu harakatning tenglamasi topilsin.

Berilgan:

$x_1 = A_1 \sin\omega t$;

$x_2 = A_2 \cos\omega t$;

$A_1 = 1\text{sm} = 10^{-2}\text{m}$;

$A_2 = 2\text{sm} = 2 \cdot 10^{-2}\text{m}$;

$\omega = 1\text{s}^{-1}$.

Yechish: Berilgan tenglamalarni bir xil trigonometrik funksiyalar orqali ifodalaymiz. Buning uchun $\sin x = \cos(x - \pi/2)$ ligidan foydalanamiz:

$$x_1 = A_1 \sin\omega t = A_1 \cos(\omega t - \frac{\pi}{2}),$$

$A = ?$ $\nu = ?$ $\varphi = ?$	$x_2 = A_2 \cos\omega t$. Harakat tenglamasining umumiy ko'rinishi $x = A\cos(\omega t + \varphi)$ (1) bo'ladi. Bu yerda

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1),$$

agar $\varphi_1 = -\frac{\pi}{2}$, $\varphi_0 = 0$ ekanini hisobga olsak,

$$\varphi_2 - \varphi_1 = 0 - \frac{\pi}{2}; \quad \cos(\varphi_2 - \varphi_1) = \cos(-\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0$$

va demak,

$$A^2 = A_1^2 + A_2^2 \qquad (2)$$

bo'ladi. Shuningdek

$$\sin\varphi_1 = \sin\left(-\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1.$$

$$\sin\varphi_2 = \sin 0 = 0;$$

$\cos\varphi_1 = \cos\left(-\frac{\pi}{2}\right) = -\cos\frac{\pi}{2} = 0;$ $\cos\varphi_2 = \cos 0 = 1$ dan foydalanib,

$$\operatorname{tg}\varphi = \frac{A_1(-1) + A_2 \cdot 0}{A_2 \cdot 0 + A_2 \cdot 1} = -\frac{A_1}{A_2},$$

$$\varphi = \operatorname{arctg}\left(-\frac{A_1}{A_2}\right) \qquad (3)$$

ni topamiz. Chastotani esa quyidagi munosabatdan aniqlaymiz:

$$\nu = \frac{\omega}{2\pi}. \qquad (4)$$

Berilganlarni (2), (3) va (4) larga qo'yib, quyidagini topamiz:

$$A^2(10^{-2}\text{m})^2 + (2\cdot 10^{-2}\text{m})^2 = 5\cdot 10^{-4}\text{m}^2;$$

$$A = \sqrt{5}\cdot 10^{-2}\text{m} = 2{,}24\cdot 10^{-2}\text{m};$$

$$\varphi = \text{arctg}(-\frac{10^{-2}}{2\cdot 10^{-2}}) = \text{arctg}(-\frac{1}{2}) = 0{,}353\pi\ \text{rad}.$$

$$\nu = \frac{1}{2\cdot 3{,}14}\text{Hz} = 0{,}159\text{Hz}.$$

Javob: $A = 2{,}24\cdot 10^{-2}\text{m}$; $\varphi = 0{,}353\pi\ \text{rad}$; $\nu = 0{,}159\text{Hz}$; $\omega = 1\text{s}^{-1}$; $x = A\cos(\omega t + \varphi)$.

4-misol. O'rama prujinaga yukcha osdilar. Buning natijasida prujina 9sm ga cho'zildi. Agar yuk pastga ozroq tortilsa va qo'yib yuborilsa, uning tebranish davri *T* qanday bo'ladi?

Berilgan:	Yechish:
$x = 9\text{sm} = 9\cdot 10^{-2}\text{m}.$	**Yechish:** Prujinali mayatnikning (prujinaga osilgan yukning) tebranish
T=?	davri quyidagi formula bilan aniqlanadi:

$$T = 2\pi\sqrt{\frac{m}{k}}, \qquad (1)$$

bunda: m – yukning massasi, k – prujinaning qattiqligi. Cho'zilgan prujinada vujudga keladigan elastiklik kuchning qiymatini

$$F = k\cdot x \qquad (2)$$

dan aniqlaymiz. O'z navbatida cho'zilishdan keyin

$$F=P=mg \qquad (3)$$

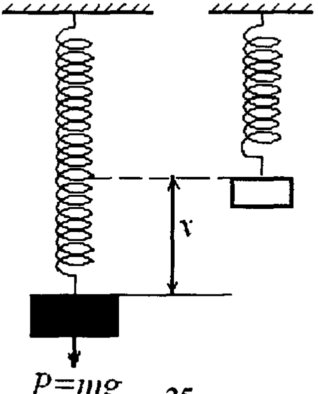
ya'ni $kx=mg,$

bundan $$m = \frac{kx}{g}. \qquad (4)$$

(4) ni (1) ga qo'yamiz $$T = 2P\sqrt{\frac{kx}{gk}} = 2P\sqrt{\frac{x}{g}}. \qquad (5)$$

(5) asosida T ning birligini tekshirib ko'ramiz:

$$[T]=\left(\left[\frac{x}{g}\right]\right)^{1/2}=\left(\frac{1\text{m}}{1\frac{\text{m}}{\text{s}^2}}\right)^{1/2}=1\text{s}$$



25-rasm

va to'g'riligiga ishonch hosil qilgandan keyin berilganlarni qo'yamiz:

$$T=2\cdot 3{,}14\sqrt{\frac{9\cdot 10^{-2}}{9{,}8}}\text{s}=6{,}28\cdot 0{,}096\text{s}=0{,}6\text{s}.$$

Javob: T= 0,6 s.

5-misol. Uzunligi 1m va massasi 400 g bo'lgan ingichka tayoqchaning uchlariga massalari 200 g va 300 g bo'lgan kichkina sharchalar mahkamlangan. Tayoqcha o'ziga tik va o'rtasidan o'tuvchi (26-rasmda 0 nuqta) gorizontal o'q atrofida tebranadi. Tayoqcha qilayotgan tebranishlar davri T aniqlansin.

Berilgan:
l =1 m;
m_3=400 g= 0,4 kg;
m_1=200 g= 0,2 kg;
m_2=300 g= 0,3 kg.

T=?

Yechish: Uchlariga sharchalar mahkamlangan tayoqchani fizik mayatnik sifatida qarash kerak. Fizik mayatnikning tebranish davri quyidagi ifoda yordamida aniqlanadi:

$$T=2\pi\sqrt{\frac{J}{mgl_c}}\ . \qquad (1)$$

bunda J – mayatnikning tebranish o'qiga nisbatan inersiya momenti, m – uning massasi $(m=m_1+m_2+m_3)$; l_c – mayatnikning massa markazidan o'qqacha bo'lgan masofa.

Mayatnikning inersiya momenti J sharchalarning J_1, J_2 va tayoqchaning J_3 inersiya momentlarining yig'indisiga teng.

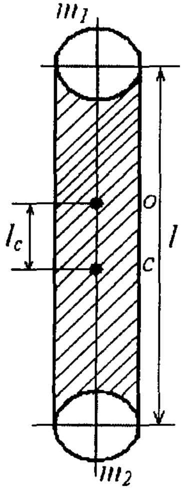
$$J=J_1+J_2+J_3. \qquad (2)$$

Sharchalarning inersiya momentlarini ularni moddiy nuqtalar sifatida qarab, quyidagi topamiz:

$$J_1 = m_1\left(\frac{l}{2}\right)^2; \quad J_2 = m_2\left(\frac{l}{2}\right)^2. \qquad (3)$$

Tayoqchaning o'rtasidan o'tgan o'qqa nisbatan inersiya momenti:

$$J_3 = \frac{1}{12} m_3 l^2. \qquad (4)$$



26-rasm

Tebranish o'qidan mayatnikning massa markazigacha bo'lgan l_c masofani quyidagi mulohazalarga asoslanib topamiz. Agar x o'qi tayoqcha bo'ylab yo'naltirilsa va koordinata o'qlarining boshi 0 nuqta bilan mos keltirilsa, unda izlanayotgan l_c masofa mayatnik massa markazining koordinatasiga teng bo'ladi, ya'ni

$$l_c = x_c = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1\left(-\frac{1}{2}\right) + m_2\left(\frac{l}{2}\right) + m_3 0}{m_1 + m_2 + m_3} = \frac{(m_2 - m_1)l}{2(m_1 + m_2 + m_3)} =$$

$$= \frac{(m_2 - m_1)l}{2m}.$$

$g = 9{,}8\frac{\text{m}}{\text{s}^2}$ — erkin tushish tezlanishi ekanligini nazarda tutsak topilgan ifodalarni (1) ga qo'yib quyidagini olamiz:

$$T = 2\pi\sqrt{\frac{m_1\left(\frac{l}{2}\right)^2 + m_2\left(\frac{l}{2}\right)^2 + \frac{1}{12}m_3 l^2}{(m_1 + m_2 + m_3)g\cdot\left(\frac{m_2 - m_1}{2m}\right)l}} = 2\pi\sqrt{l\left(\frac{3m_1 + 3m_2 + m_3}{6g(m_2 - m_1)}\right)}. \qquad (6)$$

(6) asosida T ning birligini tekshiramiz:

$$T=\left(\frac{[l]\cdot[m]}{[m][g]}\right)^{1/2}=\left(\frac{[l]}{[g]}\right)^{1/2}=\left(\frac{1\text{m}}{1\frac{\text{m}}{\text{s}^2}}\right)^{1/2}=(1\text{s}^2)^{1/2}=1\text{s}.$$

Kattaliklarning qiymatlarini qo'yib hisoblaymiz:

$$T=2\cdot3{,}14\sqrt{\frac{1\cdot(3\cdot0{,}2+3\cdot0{,}3+0{,}4)}{6\cdot(0{,}3-0{,}2)\cdot9{,}8}}\text{s}=6{,}28\sqrt{\frac{1{,}9}{5{,}88}}\text{s}=3{,}57\,\text{s}.$$

Javob: T=3,57s.

6-misol. Mayatnik tebranishning logarifmik dekrementi 0,003 ga teng. Mayatnikning amplitudasi ikki marta kamayishi uchun qilishi kerak bo'lgan to'la tebranishlar soni N aniqlansin.

Berilgan:
δ=0,003;
$\frac{A_0}{A(t)}=2$.
N=?

Yechish: So'nuvchi tebranishlarning amlitudasi quyidagi qonunga binoan o'zgaradi

$$A(t)=A_0e^{-\delta t}, \tag{1}$$

bunda δ – so'nish koeffitsenti, t – tebranish vaqti. Agar tebranish davrini T va amplitudasi ikki marotaba kamayishidagi tebranishlar sonini N bilan belgilasak, quyidagi munosabat o'rinlidir:

$$t=N\cdot T\cdot \tag{2}$$

(1) ni bir oz o'zgartirib va (2) ni quyib qayta yozamiz:

$$\frac{A_0}{A(t)}=e^{\delta t}\quad \text{yoki}\quad \frac{A_0}{A(t)}=e^{\delta NT}.$$

$\delta T=\theta$ – logarifmik dekrementga tengligidan

$$\frac{A_0}{A(t)}=e^{N\theta}. \tag{3}$$

(3) ni logarifmlab N ni topamiz:

$$N=\frac{1}{\theta}\ln\frac{A_0}{A(t)}. \tag{4}$$

Berilganlarni (4) ga qo'yib, hisoblaymiz:

$$N = \frac{1}{0{,}003}\ln 2 = \frac{0{,}693}{0{,}003} = 231.$$

Javob: N=231.

7-misol. Yassi to'lqin tenglamasi $\xi(x,t) = A\cos(\omega t - kx)$ berilgan, bunda $A = 0{,}5\text{sm}$, $\omega = 628\text{s}^{-1}$; $k = 2\text{m}^{-1}$. 1) tebranish chastotasi ν va to'lqin uzunligi λ; 2) fazoviy tezlik ϑ; 3) muhit zarralari tebranishlari tezligining $\dot{\xi}_{max}$ va tezlanishining $\ddot{\xi}_{max}$ maksimal qiymatlari aniqlansin.

Berilgan:

$\xi(x,t) = A\cos(\omega t - kx)$;

$A = 0{,}5\text{sm} = 5\cdot 10^{-3}\text{m}$;

$\omega = 628\text{s}^{-1}$;

$k = 2\text{m}^{-1}$.

1) $\nu = ?$

$\lambda = ?$

2) $\vartheta = ?$

3) $\dot{\xi}_{max} = ?$

$\ddot{\xi}_{max} = ?$

Yechish: 1. Tebranish chastotasi ν ni aniqlash uchun uning doiraviy chastota ω bilan bog'lanishini:

$$\nu = \frac{\omega}{2\pi}, \qquad (1)$$

shuningdek, to'lqin uzunligi λ ning to'lqin soni k bilan bog'lanishini yozamiz:

$$\lambda = \frac{2\pi}{k}, \qquad (2)$$

ω va k larning qiymatlarini qo'yib olamiz:

$$\nu = \frac{628\text{s}^{-1}}{2\cdot 3{,}14} = 100\text{s}^{-1} = 100\text{Hz}.$$

$$\lambda = \frac{2\cdot 3{,}14}{2\text{m}^{-1}} = 3{,}14\text{m}.$$

2. To'lqinning fazoviy tezligi ϑ hamda to'lqin uzunligi λ va chastota ν lar orasida quyidagi munosabat mavjud:

$$\vartheta = \lambda\nu. \qquad (3)$$

λ va ν larning qiymatlari yordamida quyidagini topamiz:

$$\vartheta = 3{,}14 \cdot 100\frac{\text{m}}{\text{s}} = 314\,{}^{\text{m}}\!/_{\text{s}};$$

3. Zarralar tebranishi tezligini topish uchun berilgan to‘lqin tenglamasidan vaqt bo‘yicha hosila olamiz: $\dot{\xi} = \frac{d\xi}{dt} = -A\omega\sin(\omega t - kx)$.

$\dot{\xi}$ o‘zining maksimal qiymatiga $\sin(\omega t - kx) = -1$ da erishadi. Unda

$$\dot{\xi}_{\max} = A\omega. \qquad (4)$$

Zarralar tebranishining tezlanishini topish uchun esa, tezlikdan vaqt bo‘yicha hosila olamiz:

$$\ddot{\xi} = \frac{d\dot{\xi}}{dt} = -A\omega^2\cos(\omega t - kx).$$

$\ddot{\xi}$ ning maksimal qiymati $\cos(\omega t - kx) = -1$ da bo‘ladi.

Ya’ni, $$\ddot{\xi}_{\max} = A\omega^2. \qquad (5)$$

Kattaliklarning qiymatlarini (4) va (5) larga qo‘yamiz:

$$\dot{\xi}_{\max} = 5 \cdot 10^{-3} \cdot 628\frac{\text{m}}{\text{s}} = 3{,}14\,{}^{\text{m}}\!/_{\text{s}};$$

$$\ddot{\xi}_{\max} = 5 \cdot 10^{-3} \cdot (628)^2\frac{\text{m}}{\text{s}^2} = 5 \cdot 10^{-3} \cdot 39{,}44 \cdot 10^4\frac{\text{m}}{\text{s}^2} = 1{,}971 \cdot 10^3\frac{\text{m}}{\text{s}^2}.$$

Javob: 1) $\nu = 100\text{Hz}$; $\lambda = 3{,}14\text{m}$; 2) $\vartheta = 314\,{}^{\text{m}}\!/_{\text{s}}$; 3) $\dot{\xi}_{\max} = 3{,}14\,{}^{\text{m}}\!/_{\text{s}}$; $\ddot{\xi}_{\max} = 1{,}971 \cdot 10^3\frac{\text{m}}{\text{s}^2}$.

8-misol. Tovush manbayidan 800 m masofada bo‘lgan odam, havodan kelgan tovushni suvdan kelgan tovushga nisbatan 1,78 s kech eshitadi. Agar havoning harorati 350K bo‘lsa, tovushning suvdagi tezligi ϑ_s topilsin.

Berilgan:	**Yechish:** Tezlikni aniqlash formulasidan foydalansak, tovushning suvdagi tezligini quyidagicha aniqlaymiz:
$l = 800\text{m};$	
$\Delta t = t_h - t_s = 1{,}78\text{s};$	
$T = 350\text{K}.$	$$\vartheta_c = \frac{l}{t_s}. \quad (1)$$
$\vartheta_s = ?$	Agar masalaning shartiga ko'ra

$$\Delta t = t_h - t_s, \text{ yoki } t_s = t_h - \Delta t$$

ekanligini nazarda tutsak, (1) quyidagi ko'rinishni oladi:

$$\vartheta_s = \frac{\ell}{t_h - \Delta t}. \quad (2)$$

Shuningdek, havo holida quyidagi tenglik o'rinli:

$$t_h = \frac{l}{\vartheta_h}. \quad (3)$$

Tovushning havodagi tezligini aniqlash formulasidan

$$\vartheta_h = \sqrt{\gamma \frac{RT}{M}} \quad (4)$$

ni olamiz. $t_h = \dfrac{l}{\sqrt{\gamma \frac{RT}{M}}} = l \cdot \sqrt{\dfrac{M}{\gamma RT}}.$ (5)

Bunda: M – molyar massa, γ – adiabata ko'rsatkichi, T – harorat. $R = 8{,}31 \dfrac{\text{J}}{\text{K} \cdot \text{mol}}$ – molyar gaz doimiysi. Havo uchun $M = 29 \cdot 10^{-3} \dfrac{\text{kg}}{\text{mol}}$; γ=1,4. (5) ni (2) ga qo'yib, quyidagini olamiz:

$$\vartheta_s = \frac{l}{l \cdot \sqrt{\frac{M}{\gamma RT}} - \Delta t}. \quad (6)$$

Hosil qilingan ifodaning to‘g‘riligini ϑ_s ning birligini aniqlash yordamida tekshirib ko‘ramiz:

$$[\vartheta]=\frac{[l]}{[l]\left(\frac{[M]}{[R][T]}\right)^{1/2}-[t]}=\frac{1\text{m}}{1\text{m}\left(\frac{1\,\text{kg}/\text{mol}}{1\frac{\text{J}}{\text{K}\cdot\text{mol}}\cdot 1\text{K}}\right)^{1/2}-1\text{s}}=\frac{1\text{m}}{1\text{m}\left(1\frac{\text{s}^2}{\text{m}^2}\right)^{1/2}-1\text{s}}=1\frac{\text{m}}{\text{s}},$$

va to‘g‘riligiga ishonch hosil qilganimizdan keyin kattaliklarning qiymatlarini qo‘yib hisoblaymiz:

$$\vartheta_c=\frac{800}{800\cdot\sqrt{\frac{29\cdot10^{-3}}{1{,}4\cdot 8{,}31\cdot 350}}-1{,}78}\frac{\text{m}}{\text{s}}=\frac{800}{2{,}95\cdot 0{,}8-1{,}78}\frac{\text{m}}{\text{s}}=\frac{800}{0{,}58}\frac{\text{m}}{\text{s}}\approx 1379\frac{\text{m}}{\text{s}}\approx 1{,}4\frac{\text{km}}{\text{s}};$$

Javob: $\vartheta_c \approx 1{,}4\frac{\text{km}}{\text{s}}$.

Mustaqil yechish uchun masalalar

1. 3 sm amplituda va $\frac{\pi}{2}\text{s}^{-1}$ burchak chastota bilan garmonik tebranayotgan nuqta tezligining va tezlanishning maksimal qiymatlari aniqlansin. [$\dot{x}_{max}$=4,71 sm/s; $\ddot{x}_{max}$=7,4 sm/s^2.]

2. Ikkita $x_1 = A_1 \sin\omega t$ va $x_2 = A_2 \sin\omega(t+\tau)$ bir xil yo‘nalishli va davrli tebranishlarning qo‘shilishi natijasida vujudga kelgan tebranishning amplitudasi va boshlang‘ich fazasi aniqlansin. Bunda $A_1 = A_2 = 1\text{sm}$; $\omega = \pi\,\text{s}^{-1}$; $\tau = 0{,}5\text{s}$. Natijaviy tebranishning tenglamasi topilsin.

$$\left[1{,}41\text{sm}; \frac{\pi}{4}\text{rad}; \cos(\omega t+\varphi).\right]$$

3. $x = A_1 \sin\omega t$ a $y = A_2 \cos\omega(t+\tau)$ tenglamalar bilan ifodalanuvchi ikkita o‘zaro tik tebranishlar ustma-ust tushmoqda. Bunda A_1=2 sm, A_2=1sm, $\omega = \pi s^{-1}$, $\tau = 0{,}5$. Trayektoriya tenglamasi topilsin va u nuqtaning harakat yo‘nalishi ko‘rsatilib tuzilsin. $\left[-\frac{1}{2}x.\right]$

4. Garmonik tebranayotgan nuqtaning to'la energiyasi 30 mkJ, nuqtaga ta'sir etayotgan maksimal kuch esa 1,5 mN. Agar tebranishlar davri 2 s, boshlang'ich fazasi esa $\pi/3$ bo'lsa, bu nuqtaning harakat tenglamasi yozilsin.

$\left[0{,}04\cos(\pi t+\frac{\pi}{3}).\right]$

5. Fizik mayatnik uzunligi 25 sm bo'lgan ingichka bir jinsli tayoqchadan iborat. Tebranish chastotasi maksimal bo'lishi uchun osilish nuqtasi massa markazidan qanday masofada bo'lishi kerakligi aniqlansin? [7,2 sm.]

6. Uzunliklari 16sm ga farq qiladigan ikkita matematik mayatnik bir xil vaqtda, biri 10, ikkinchisi 6 marta tebranadi. Mayatniklarning uzunliklari l_1 va l_2 lar aniqlansin. [9 sm; 25 sm.]

7. 500 g massali qadoqtosh qattiqligi 20 N/m bo'lgan burama prujinaga osilgan va qandaydir muhitda elastik tebranadi. Tebranishning logarifmik dekrementi 0,004. Qadoqtoshning tebranishlar amplitudasi 2 marta kamayishi uchun qilish kerak bo'lgan to'la tebranishlar soni aniqlansin. Bu kamayish qancha t vaqtda ro'y beradi. [173; 2 min 52 s.]

8. Yassi tovush to'lqinining davri 3 ms, amplitudasi 0,2 mm va to'lqin uzunligi 1,2 m ga teng. Tebranish manbaidan 2 m masofada bo'lgan muhit nuqtalari uchun:

1) 7 s da siljish $\xi(x,t)$; 2) Shu onning o'zi uchun tezlik ξ va tezlanish ξ – topilsin. Tebranishning boshlang'ich fazasi nolga teng deb qabul qilinsin. $\left[1) -0{,}1\,\text{mm};\quad 2)\ 0{,}363\,\text{m/s};\,0{,}439\,\text{m/s}^2.\right]$

9. Normal sharoitda, qandaydir gazda tovushning tezligi 308m/s. Gazning zichligi 1,78 kg/m^3. Berilgan gaz uchun adiabata ko'rsatkichi γ ning qiymati aniqlansin. [1,67.]

10. Agar turg'un to'lqinda: 1) birinchi va y ettinchi do'ngliklar; 2) birinchi va to'rtinchi tugunlar orasidagi masofa 15 sm bo'lsa, yugurma to'lqin uzunligi λ aniqlansin. [1) 5 sm. 2) 10 sm.]

11. Poyezd 120 km/soat tezlik bilan harakatlanmoqda. U, 5s davom etuvchi hushtak chaladi:

1) Poyezd yaqinlashayotgan bo'lsa;

2) Uzoqlashayotgan bo'lsa, hushtakning tuyulma davom etish vaqti qancha bo'ladi? Tovush tezligi 348 ms ga teng deb qabul qilinsin. [1) 4,5 s; 2) 5,5 s.]

12. 20 sm diametrli quvurdagi havoning 300K harorat va 200 kPa bosimdagi akustik qarshiligi Z_a aniqlansin. [2,57kPa·s/m^3.]

13. Chastotalar mos ravishda 50 Hz, 200 Hz va 1 kHz bo'lgan uchta tovush bir xil 40 dB intensivlik darajasiga ega. Shu tovushlarning qattiqlik darajasi L_N aniqlansin. [birinchisi eshitilmaydi:, 20: 40.]

14. Nuqtaviy izotop tovush manbayining quvvati 100 mkV, 500 Hz chastotada tovush manbaidan 10m masofada qattiqlik darajasi L_N topilsin. [50.]

II-BOB. MOLEKULYAR FIZIKA VA TERMODINAMIKA

9-§. Ideal gazlarning molekulyar-kinetik nazariyasi

Asosiy formulalar

Jismdagi yoki sistemadagi modda miqdori quyidagi munosabat yordamida aniqlanadi:

$$\nu = \frac{N}{N_A},$$

bunda: N – jismni yoki sistemani tashkil qiluvchi tarkibiy elementlar (molekulalar, atomlar, ionlar va hokazo) soni; N_A – Avogadro soni. Demak, modda miqdori – bu sistema yoki jismda mavjud bo'lgan tarkibiy elementlar soni bilan aniqlanib, mollarda ifodalanadi. Bir Mol – tarkibiy elementlarning soni 0,012 kg massali uglerod 12 da mavjud bo'lgan atomlar soniga teng sistemadagi modda miqdori.

Moddaning molyar massasi:

$$M = m/\nu,$$

bunda: m – bir jinsli jism (sistema) massasi; ν – shu jismdagi modda miqdori.

Jismning nisbiy molekulyar massasi:

$$M_r = \sum_i n_i A_{r,i},$$

bunda: n_i – mazkur modda molekulasining tarkibiga kiruvchi, i – kimyoviy elementning atomlar soni; $A_{r,i}$ – shu elementning nisbiy atom massasi bo'lib, D.I. Mendeleyevning elementlar davriy sistemasidan olinadi.

Moddaning molyar massasi M bilan nisbiy molekulyar massasi M_z orasidagi bog'lanish

$$M = M_z \cdot k,$$

bunda $k = 10^{-3}\,\text{kg/mol}$.

Gaz aralashmasining molyar massasi:

$$M_{AR} = \sum_{i=1}^{k} m_i \Big/ \sum_{i=1}^{k} \nu_i \,.$$

Bunda: m_i – aralashma i tarkibiy qismning massasi; ν – aralashma i – tarkibiy qismining modda miqdori; k – aralashma tarkibiy qismlarining soni.

ω_i gaz aralashmasi i-tarkibiy qismining massaviy ulushi bo'lib, i-tarkibiy qismi massasining aralashma massasiga nisbati bilan aniqlanadigan o'lchamsiz miqdordir

$$\omega_i = m_i / m,$$

bunda: m_i – aralashma i – tashkil etuvchisining massasi; m – aralashmaning massasi.

n xil idieal gaz aralashmasining bosimi uchun **Dalton qonuni:**

$$p = \sum_{i=1}^{n} p_i = p_1 + p_2 + \ldots + p_n,$$

bunda: p_i – aralashma i – tarkibiy qismining parsial bosimi.

***Boyl-Mariott qonuni*:**

T=const, m=const da $P \cdot V = \text{const}$,

bunda: P – ideal gaz bosimi, V – hajmi, T – termodinamik harorat, m – massasi.

Gey-Lyussak qonuni:

$P = \text{const}, m = \text{const}$ da $V = V_0(1 + \alpha t)$ yoki $V_1/V_2 = T_1/T_2$.

Sharl qonuni:

$V = \text{const}, m = \text{const}$ da $P = P_0(1 + \alpha t)$ yoki $P_1/P_2 = T_1/T_2$,

bunda: t – Selsiy shkalasi bo'yicha olingan harorat $T = t + 273$,

V_0 va P_0 – mos ravishda 0^0 dagi hajm va bosim, $\alpha = \frac{1}{273}\text{K}^{-1}$.

Ideal gazning holat tenglamasi *(Klapeyron-Mendeleyev tenglamasi):*

$$P \cdot V = \frac{m}{M} RT \quad \text{yoki} \quad P \cdot V = \nu \cdot RT,$$

bunda: m – massa, M – Molyar massa, R – gaz molyar doimiysi,

T – termodinamik harorat, ν – modda miqdori.

Ideal gaz kinetik nazariyasining asosiy tenglamasi:

$$P = \frac{3}{2} n < \varepsilon_u >,$$

bunda: $n = \frac{N}{v}$ – bir jinsli sistemasi molekulalarning konsenratsiyasi, $<\varepsilon_i>$ – molekulalar ilgarilanma harakatining o'rtacha kinetik energiyasi.

Molekulaning i ta erkinlik darajasiga mos keluvchi o'rtacha kinetik energiyasi (molekulaning to'la energiyasi):

$$< \varepsilon_i >= \frac{i}{2} kT;$$

molekulaning ilgarilanma harakati o'rtacha kinetik energiyasi:

$$< \varepsilon_i >= \frac{3}{2} kT;$$

molekulaning aylanma harakat o'rtacha kinetik energiyasi:

$$< \varepsilon_{ayl} >= \frac{i-3}{2} kT;$$

bu yerda $k = R/N_A$ – Bolsman doimiysi.

Gaz bosimining, molekula o'rtacha kvadrat tezligi $< V_{kv} >$ ga bog'liqligi:

$$P = \frac{1}{3} n m_0 < \vartheta_{k\vartheta} >^2,$$

yoki $PV = \frac{2}{3} N \left(\frac{m_0 < \vartheta_{k\vartheta} >^2}{2} \right) = \frac{2}{3} N\varepsilon = \frac{2}{3} E$,

yoki $PV = \frac{1}{3} N m_0 < \vartheta_{k\vartheta} >^2 = \frac{1}{3} m < \vartheta_{k\vartheta} >$,

bunda: m_0 – bitta molekulaning massasi, $E = N \cdot \varepsilon$ – barcha molekulalarning ilgarilanma harakat kinetik energiyalarining yig'indisi, N – gazdagi molekulalar soni, $m = N \cdot m_0$ – gaz massasi.

Gaz bosimining molekulalar konsentratsiyasi va haroratga bogliqligi

$$p = n \cdot k \cdot T.$$

Molekulalarning tezligi: o‘rtacha kvadratik

$$<\vartheta_{k\vartheta}> = \sqrt{\frac{3kT}{m_0}}, \quad \text{yoki} \quad <\vartheta_{k\vartheta}> = \sqrt{\frac{3RT}{M}};$$

o‘rtacha arifmetik

$$<\vartheta_{k\vartheta}> = \sqrt{8kT/(\pi m_0)}, \quad \text{yoki} \quad <\vartheta> = \sqrt{8\,RT/(\pi m)};$$

eng katta ehtimolli

$$\vartheta_e = \sqrt{2\,kT/m_1}, \quad \text{yoki} \quad \vartheta_e = \sqrt{8RT/(\pi m)}.$$

Masala yechishga misollar

1-misol. 0,2 kg massali azotning modda miqdori ν molekulalari soni N aniqlansin.

Berilgan:	**Yechish:**
$m = 0{,}2\,\text{kg}$ N_2 $\nu = ?$ $N = ?$	Azotning modda miqdorini $$\nu = \frac{m}{M} \qquad (1)$$ ifodadan aniqlaymiz. Bunda M – azotning molyar massasi. Agar $M = M_r \cdot k$ ekanidan foydalansak, $k = 10^{-3}$ kg/mol va azot uchun $M_r = 28$ (D.I. Mendeleyev davriy sistemasidan) ligidan

$$M = 28 \cdot 10^{-3}\,\text{kg/mol}.$$

Moddadagi molekulalar soni quyidagi ifoda yordamida aniqlanadi:

$$N = \nu \cdot N_A. \qquad (2)$$

Bu yerda $N_A = 6{,}02 \cdot 10^{23}\,\text{mol}^{-1}$ – Avogadro soni.

Bu sodda ifodalar uchun birliklarni tekshirib o‘tirmasdan, kattaliklarning qiymatlarini (1) va (2) larga qo‘yamiz:

$$\nu = \frac{0{,}2}{28 \cdot 10^{-3}} \text{mol} = 7{,}14 \text{mol};$$

$$N = 7{,}14 \cdot 6{,}02 \cdot 10^{23} \text{dona} = 4{,}30 \cdot 10^{24} \text{dona};$$

Javob: $\nu = 7{,}14$ mol, $N = 4{,}30 \cdot 10^{24}$ dona.

2-misol. Bitta osh tuzi molekulasining molyar massasi M va massasi m_0 topilsin.

Berilgan:	**Yechish:** Osh tuzi NaCl ning molyar massasini quyidagicha topamiz:
NaCl; $N = 1$.	$M = M_{Na} + M_{Cl}$.
$M = ?$ $m_0 = ?$	Natriyning molyar massasi $M_{Na} = 23 \cdot 10^{-3}\, \text{kg/mol}$ va xlorning molyar massasi $M_{Cl} = 35{,}5 \cdot 10^{-3}\, \text{kg/mol}$ ligidan olamiz.

$$M = 23 \cdot 10^{-3}\, \text{kg/mol} + 35{,}5 \cdot 10^{-3}\, \text{kg/mol} = 58{,}5 \cdot 10^{-3}\, \text{kg/mol}.$$

Bitta molekulaning massasi ushbu ifodadan topiladi:

$$m_0 = \frac{M}{N_A}$$

bunda $N_A = 6{,}02 \cdot 10^{23} \text{mol}$. Unda

$$m_0 = \frac{58{,}5 \cdot 10^{-3}}{6{,}02 \cdot 10^{23}} \text{kg} = 9{,}72 \cdot 10^{-26} \text{kg}.$$

Javob: $M = 58{,}5 \cdot 10^{-3}\, \text{kg/mol}$; $m_0 = 9{,}72 \cdot 10^{-26} \text{kg}$.

3-misol. Ballonda 2 mPa bosim ostida, 400 K haroratda turgan azotning zichligi ρ hisoblansin.

Berilgan:

N_2;

$P = 25mPa = 2 \cdot 10^{-3} Pa$;

$T = 400K$.

$\rho = ?$

Yechish: Ideal gazning holat tenglamasini yozamiz:

$$P \cdot V = \frac{m}{M} R \cdot T. \quad (1)$$

Ushbu ifodadan uncha qiyin bo‘lmagan o‘zgarishlardan keyin quyidagini olamiz:

$$\frac{m}{V} = \frac{P \cdot M}{R \cdot T}. \quad (2)$$

Agar zichlik $\rho = \frac{m}{V}$, ya’ni gaz massasining uning hajmiga nisbati kabi aniqlanishini nazarda tutsak,

$$\rho = \frac{P \cdot M}{R \cdot T} \quad (3)$$

ni hosil qilamiz. Bunda, M gazning molyar massasi, R — gaz molyar doimiysi

$$R = 8{,}31 \frac{J}{mol \cdot K}.$$

Azot uchun $M = 28 \cdot 10^{-3} \frac{kg}{mol}$.

(3) yordamida zichlikning birligi hosil bo‘lishini tekshirib ko‘ramiz:

$$[\rho] = \frac{[P][M]}{[R][T]} = \frac{1Pa \cdot 1 kg/mol}{1\frac{J}{mol \cdot K} \cdot 1K} = 1\frac{\frac{N}{m^2} \cdot kg}{N \cdot m} = 1\frac{kg}{m^3},$$

va uning to‘g‘riligiga ishonch hosil qilgandan keyin berilganlarni qo‘yamiz:

$$\rho = \frac{2 \cdot 10^{-3} \cdot 28 \cdot 10^{-3}}{8{,}31 \cdot 400} \frac{kg}{m^3} = \frac{14}{8{,}31} \cdot 10^{-8} \frac{kg}{m^3} = 1{,}68 \cdot 10^{-8} \frac{kg}{m^3}.$$

Javob: $\rho = 1{,}68 \cdot 10^{-8} \frac{kg}{m^3}$.

4-misol. Havo sharining qobig'i 1600 m^3 sig'imga ega. Bosimi 60 kPa va harorati 280K bo'lgan balandlikda, qobiqni to'ldirib turgan vodorod hosil qiladigan ko'tarish kuchi F topilsin. Shar ko'tarilishida vodorod sharning quyi qismidagi teshikdan chiqishi mumkin.

Berilgan:

$V = 1600\text{m}^3;$

$P = 60\text{kPa} = 6\cdot 10^4\,\text{Pa};$

$T = 280\text{K}.$

$F_K = ?$

Yechish: Vodorodning ko'tarish kuchi

$$F_K = (M_2 - M_1)g\frac{PV}{RT} \qquad (1)$$

formula yordamida anilanadi.

Bu yerda $g = 9{,}8\frac{\text{m}}{\text{s}^2}$ – erkin tushish tezlanishi, $R = 8{,}31\frac{\text{J}}{\text{mol}\cdot\text{K}}$ – gaz molyar doimiysi.

(1) formulada $M_2 = M_x = \rho_x V_m$; $M_1 = M_B = \rho_B V_m$.

Shunday qilib,

$$F_K = (\rho_x - \rho_B)\frac{g\cdot P\cdot V\cdot V_m}{RT}, \qquad (2)$$

bunda: $V_m = 22_04\cdot 10^{-3}\frac{\text{m}^3}{\text{mol}}$ – gazning molyar hajmi; $\rho_x = 1{,}29\frac{\text{kg}}{\text{m}^3}$ – havoning, $\rho_B = 0{,}09\,{}^{\text{kg}}\!/_{\text{m}^3}$ – vodorodning zichliklari. Kattaliklarning birliklari yordamida (2) ni tekshirib ko'ramiz:

$$[F] = \frac{[\rho][g][P][V][V_m]}{[R]\cdot[T]} = \frac{1\frac{\text{kg}}{\text{m}^3}\cdot 1\frac{\text{m}}{\text{s}^2}\cdot 1\text{Pa}\cdot 1\text{m}^3\cdot 1\frac{\text{m}^3}{\text{mol}}}{1\frac{\text{J}}{\text{mol}\cdot\text{K}}\cdot 1\text{K}} = 1\frac{\text{kg}\cdot\text{m}\cdot\text{N}\cdot\text{m}^3}{\text{s}^2\text{m}^3} = 1\text{N}$$

va to'g'riligiga ishonch hosil qilganimizdan keyin kattaliklarning qiymatlarini qo'yamiz:

$$F_K = (1{,}29 - 0{,}09)\frac{9{,}8\cdot 6\cdot 10^4\cdot 1600\cdot 22{,}4\cdot 10^{-3}}{8{,}31\cdot 280}\text{N} =$$

$$= \frac{1,2 \cdot 0,98 \cdot 6 \cdot 1,6 \cdot 22,4}{8,31 \cdot 2,8} \cdot 10^3 \mathrm{N} = 10,9 \cdot 10^3 \mathrm{N} = 10,9 \mathrm{kN}.$$

Javob: $F_K = 10,9\mathrm{kN}$.

5-misol. Sig'imlari 20 *l* va 44 *l* bo'lgan ballonlarda gaz saqlanadi. Birinchi ballondagi bosim 2,4 MPa, ikkinchisidagi 1,6 MPa. Agar gazning harorati oldingidek qolsa, ballonlar ulangandan keyin umumiy bosim P va parsial bosimlar P_1' va P_2' lar aniqlansin.

Berilgan:

$V_1 = 20l = 2 \cdot 10^{-2}\mathrm{m}^3;$

$V_2 = 44l = 4,4 \cdot 10^{-2}\mathrm{m}^3;$

$P_1 = 2,4\mathrm{MPa} = 2,4 \cdot 10^6\mathrm{Pa};$

$P_2 = 1,6\mathrm{MPa} = 1,6 \cdot 10^6\mathrm{Pa};$

$T_1 = T_2 = T;$

$V = V_1 + V_2.$

$P = ?$

$P_1 = ?$

$P_2 = ?$

Yechish: Birinchi ballondagi gaz uchun Klapeyron-Mendeleyev tenglamasini yozamiz:

$$P_1V_1 = \nu_1 RT. \quad (1)$$

Shuningdek, ikkinchi ballondagi gaz uchun:

$$P_2V_2 = \nu_2 RT. \quad (2)$$

Ballonlar bir-biriga ulangandan keyin dastlab birinchi ballonda bo'lgan gaz uchun:

$$P_1'(V_1 + V_2) = \nu_1 RT. \quad (3)$$

Shuningdek, dastlab ikkinchi ballonda bo'lgan gaz uchun:

$$P_2'(V_1 + V_2) = \nu_2 RT. \quad (4)$$

(1) ni (3) ga bo'lamiz:

$$\frac{P_1V_1}{P_1'(V_1 + V_2)} = \frac{\nu_1 RT}{\nu_1 RT} = 1 \quad \text{yoki} \quad P_1' = \frac{P_1V_1}{V_1 + V_2}. \quad (5)$$

Shuningdek (2) ni (4) ga bo'lamiz:

$$\frac{P_2V_2}{P_2' \cdot (V_1 + V_2)} = \frac{V_2 RT}{V_2 RT} = 1, \quad \text{yoki} \quad P_2' = \frac{P_2V_2}{V_1 + V_2}. \quad (6)$$

Umumiy bosim ularning yig'indisidan iborat:

$$P = P_1' + P_2'. \qquad (7)$$

Kattaliklarning qiymatlarini (5), (6) va (7) larga qo'yamiz:

$$P_1' = \frac{2{,}4 \cdot 10^6 \cdot 2 \cdot 10^{-2}}{2 \cdot 10^{-2} + 4{,}4 \cdot 10^{-2}} \text{Pa} = \frac{4{,}8 \cdot 10^6}{6{,}4} \text{Pa} = 0{,}76 \cdot 10^6 \text{Pa} = 0{,}76 \text{MPa}$$

$$P_2' = \frac{1{,}6 \cdot 10^6 \cdot 4{,}4 \cdot 10^{-2}}{2 \cdot 10^{-2} + 4{,}4 \cdot 10^{-2}} \text{Pa} = \frac{7{,}04 \cdot 10^6}{6{,}4} \text{Pa} = 1{,}12 \text{MPa},$$

$$P = 0{,}76 \text{MPa} + 1{,}12 \text{MPa} = 1{,}88 \text{MPa}.$$

Javob: $P_1' = 0{,}76$MPa; $P_2' = 1{,}12$MPa; $P = 1{,}88$MPa.

6-misol. Ballonda, 1MPa bosim ostida kislorod va azotdan iborat gaz aralashmasi turibdi. Agar aralashmada kislorodning massasviy ulushi 0,2 bo'lsa, kislorodning P_1 va azotning P_2 parsial bosimlari aniqlansin.

Berilgan:

O_2, N_2;

$P = 1\text{MPa} = 10^6 \text{Pa}$;

$\omega_1 = 0{,}2$.

$P_1 = ?$

$P_2 = ?$

Yechish: Kislorod (O_2)uchun gaz holati tenglamasini yozamiz:

$$P_1 V = \frac{m_1}{M_1} RT. \qquad (1)$$

Shuningdek, azot (N_2)

$$P_2 V = \frac{m_2}{M_2} RT \qquad (2)$$

va gaz aralashmasi uchun

$$PV = \frac{m}{M} RT. \qquad (3)$$

Ushbu ifodalarda harorat va hajm bir xilligi nazarga olindi, aralashma parametrlari ko'rsatgichsiz yozildi. Shu bilan birga kelgusida $m = m_1 + m_2$,

$\omega_1 = \frac{m_1}{m}$, $\omega_2 = \frac{m_2}{m}$ va $\omega_1 + \omega_2 = 1$ ligini e'tiborga olamiz.

(1) ni (3) ga bo'lamiz:

$$\frac{P_1V}{PV}=\frac{m_1\cdot M}{M_1\cdot m} \quad \text{yoki } P_1=\frac{Pm_1\cdot M}{m\cdot M_1}=\frac{PM\omega_1}{M_1}. \qquad (4)$$

Shuningdek, (2) ni (3) ga bo'lib, quyidagini olamiz:

$$\frac{P_2V}{PV}=\frac{m_2\cdot M}{M_2\cdot m}, \text{ yoki } P_2=\frac{P\cdot M}{M_2}\frac{m_2}{m}=\frac{P\cdot M}{M_2}\omega_2=\frac{PM}{M_2}(1-\omega_1) \qquad (5)$$

Endi (1) va (2) ni qo'shib Dalton qonuni asosida olamiz:

$$(P_1+P_2)V=PV=\left(\frac{m_1}{M_1}+\frac{m_2}{M_2}\right)RT. \qquad (6)$$

Natijani (3) bilan solishtiramiz:

$$\frac{m}{M}=\frac{m_1}{M_1}+\frac{m_2}{M_2}, \text{ yoki bundan } M=\frac{M_1\cdot M_2}{\omega_1M_2+(1-\omega_1)M_1} \qquad (7)$$

(7) ni (4) va (5) ga qo'yish quyidagi natijani beradi:

$$P_1=\frac{\omega_1M_2\cdot P}{\omega_1M_2+(1-\omega_1)M_1}, \qquad (8)$$

$$P_2=\frac{(1-\omega_1)M_1P}{\omega_1M_2+(1-\omega_1)M_1}. \qquad (9)$$

Hosil qilingan ifodalardan zarur birliklar chiqishi ko'rinib turganligidan birdaniga berilganlarni o'rniga qo'yamiz:

$$(M_1=32\cdot10^{-3}\frac{\text{kg}}{\text{mol}}, \quad M_2=28\cdot10^{-3}\text{kg/mol})$$

$$P_1=\frac{0{,}2\cdot28\cdot10^{-3}\cdot10^6}{0{,}2\cdot28\cdot10^{-3}+(1-0{,}2)\cdot32\cdot10^{-3}}\text{Pa}\frac{5{,}6\cdot10^6}{5{,}6\cdot25{,}6}\text{Pa}=0{,}18\cdot10^6\text{Pa}=0{,}18\text{MPa};$$

$$P_2=\frac{(1-0{,}2)\cdot32\cdot10^{-3}\cdot10^6}{0{,}2\cdot28\cdot10^{-3}+(1-0{,}2)32\cdot10^{-3}}\text{Pa}=\frac{25{,}6\cdot10^6}{5{,}6\cdot25{,}6}\text{Pa}=0{,}82\cdot10^6\text{Pa}=0{,}8\text{MPa}.$$

Javob: $P_1=0{,}18\text{MPa}$; $P_2=0{,}82\text{MPa}$.

7-misol. Qanday T haroratda geliy atomining o'rtacha kvadratik tezligi ikkinchi kosmik tezlik $11,2\,\mathrm{km/s}$ ga teng bo'ladi?

Berilgan:

He;

$<\vartheta_{k\vartheta}> = 11,2\frac{\mathrm{km}}{\mathrm{s}} = 11,2\cdot10^3\frac{\mathrm{m}}{\mathrm{s}}$

$T = ?$

Yechish: Molekulaning o'rtacha kvadratik tezligi quyidagi formula bilan aniqlanadi:

$$<\vartheta_{k\vartheta}> = \sqrt{\frac{3kT}{m_0}}.$$

Bundan
$$T = \frac{m_0 <\vartheta_{k\vartheta}>^2}{3k}. \qquad (1)$$

Bu yerda: $k = 1,38\cdot10^{-23}\frac{\mathrm{J}}{\mathrm{K}}$ – Bolsman doimiysi.

m_0 – geliy atomining massasi.

$$m_0 = 4\text{a.m.b.} = 4\cdot1,66\cdot10^{-27}\,\mathrm{kg} = 6,64\cdot10^{-27}\,\mathrm{kg}.$$

Berilganlarni (1) ga qo'yib hisoblaymiz:

$$T = \frac{6,64\cdot10^{-27}\cdot(11,2\cdot10^3)^2}{3\cdot1,38\cdot10^{-23}}\mathrm{K} = 20,1\cdot10^3\mathrm{K} = 20,1\mathrm{kK}.$$

Javob: $T = 20,1\mathrm{kK}$.

Mustaqil yechish uchun masalalar

1. Oltinda: 1) modda miqdori 0,5 mol; 2) massasi 3 g bo'lsa, nechta atom bor? [1) $3,01\cdot10^{23}\,\mathrm{mol}^{-1}$; 2) $9,11\cdot10^{+21}$.]

2. 0,1 mol vodoroddagi atomlar soni N va vodorod bitta atomining massasi aniqlansin. [12,04 · 1022; $1,66\cdot10^{-27}$ kg.]

3. 4 l hajmli idishda 0,3mol azot gazi saqlanadi. Gazning zichligi aniqlansin. [2,1 $\mathrm{kg/m^3}$.]

4. Ma'lum miqdordagi suv 4^0C haroratda 0,5 $\mathrm{dm^3}$ hajmni egallaydi. Undagi modda miqdori ν va molekulalar soni N aniqlansin. [2,78 mol; $165,24\cdot10^{23}$.]

5. Sig'imi 10 l bo'lgan yopiq idishda massalari mos ravishda 4 g va 12 g bo'lgan kislorod va azot gazlarining aralashmasi saqlanadi. Agar

aralashmaning harorati 280 K bo'lsa, unda aralashmaning bosimi p va molyar massasi M_{ar} topilsin. [128,8 kPa; $27 \cdot 10^{-3}$ kg/mol.]

6. Massaviy ulushlari mos ravishda 3/4 va 1/4 bo'lgan vodorod va geliy gazlarining aralashmasi 4Pa bosim ostida 320 K haroratda saqlanadi. Shunday gaz aralashmasining zichligi topilsin. [$4{,}33 \cdot 10^{-6}$ kg/m^3.]

7. 30 l sig'imli ballonda 300 K haroratda va 828 kPa bosimda vodorod va geliy aralashmasi saqlanadi. Aralashmaning massasi 24 g. Vodorodning m_1 va geliyning m_2 massalari aniqlansin. [16g; 8g.]

8. Sig'imi 20 l bo'lgan idishda, 200 kPa bosim ostida 300 K haroratli kislorod saqlanadi. Ballondagi gazning 16g qismi sarflangandan keyin uning harorati 290 K gacha pasaydi. Ballonda qolgan kislorodning bosimi aniqlansin. [131 kPa.]

9. 7 g massali azot 0,1mPa bosim va 290K harorat ostida turibdi. Izobark ravishda qizdirilishi natijasida azot 10l hajmni egalladi.

1) Gazning kengayishigacha bo'lgan hajmi V_1; 2) gazning kengayishidan keyingi harorati T_2;

3) gazning kengayishigacha va kengayishidan keyingi zichliklari ρ_1 va ρ_2 lar aniqlansin. [1)$6{,}03 \cdot 10^{-3}$m^3; 2)481K; 3) $1{,}16\frac{kg}{m^3}$; $0{,}7\,kg/m^3$.]

10. 300 K haroratda havodagi to'yingan suv bug'larining zichligi g aniqlansin. Shu haroratda to'yingan suv bug'uning bosimi 3,55 kPa ga teng. [$2{,}56 \cdot 10^{-2}$ kg/m^3.]

11. Bir xil hajmli ikkita idishda bir xil gaz bor. Birinchi idishdagi bosim 1mPa va gazning harorati 700K. Boshqasida esa bosim 2.11mPa va gazning harorati 200K. Idishlar tutashtirildi va ulardagi gazning harorati yana 200K gacha pasaytirildi. Idishlardagi bosim P aniqlansin. [0,89 mPa.]

12. 0,3l sig'imli idishda 290 K haroratli gaz bor. Agar idishdan 10^9 ta molekula chiqib ketsa, idishdagi gazning bosimi qanchaga kamayadi?[133Pa]

13. Harorati 300K bo'lgan kislorod molekulasining o'rtacha kinetik energiyasi $<\varepsilon>$ aniqlansin. [$1{,}24 \cdot 10^{20}$ J.]

14. 300 K haroratli azot molekulasining; 1)eng katta ehtimolli $\vartheta_э$; 2) o'rtacha arifmetik $<\vartheta>$; 3) o'rtacha kvadratik ϑ_{kv} tezliklari aniqlansin. [1)$422\frac{m}{s}$; 2) $476\frac{m}{s}$; 3) $517\frac{m}{s}$.]

10-§. Statistik fizika elementlari

Asosiy formulalar

Ideal gazning tezliklari ϑ dan $\vartheta + d\vartheta$ gacha oraliqda bo'lgan molekulalarning soni:

$$dN(\vartheta) = Nf(\vartheta)d\vartheta .$$

Bunda: $f(\vartheta)$ – molekulalarning tezlik modullari bo'yicha taqsimot funksiyasi bo'lib, quyidagicha aniqlanadi:

$$f(\vartheta) = 4\pi\left(\frac{m_0}{2kT}\right)^{3/2} \exp - \mu_0\vartheta^2 /(2kT) \, \vartheta^2$$

N– molekulalarning umumiy soni; m_0– molekulalarning massasi. Nisbiy tezliklari u dan $u+du$ gacha oraliqda joylashgan molekulalar soni

$$dN(u) = N \cdot f(u)du .$$

Bunda $f(u)$ – nisbiy tezliklar bo'yicha taqsimot funksiyasi bo'lib,

$$f(u) = \frac{4}{\sqrt{\pi}} e^{-u^2} u^2$$

kabi aniqlanadi; $u = \vartheta/\vartheta_e$ – nisbiy tezlik, ϑ_e – eng katta ehtimolli tezlik.

Ideal gaz molekulalarining tezliklari bo'yicha taqsimot funksiyalariga Maksvell taqsimotlari deyiladi.

Ideal gazning, impulslari P dan $P+dP$ gacha oraliqda joylashgan molekulalarining soni:

$$dN(P) = Nf(P)dP ,$$

bunda

$$f(P) = 4\pi\left(\frac{1}{2\pi m_0 kT}\right)^{3/2} e^{-p/(2m_0kT)} p^2$$

– impulslar bo'yicha taqsimot funksiyasi.

Ideal gazning energiyalari ε dan $\varepsilon + d\varepsilon$ gacha oraliqda joylashgan molekulalarining soni

$$dN(\varepsilon)=Nf(\varepsilon)d\varepsilon,$$

bunda $$f(\varepsilon)=\frac{2}{\sqrt{\pi}}\frac{e^{-E/(kT)}}{(kT)^{3/2}}\varepsilon^{1/2}$$

— energiyalar bo‘yicha taqsimot funksiyasi.

Zarralarning (molekulalarning) tashqi potensial maydondagi taqsimoti — Bolsman taqsimoti quyidagicha yoziladi:

$$n=n_0e^{-m_0gh/(kT)} \quad \text{yoki} \quad n=n_0e^{-h/(kT)}$$

bunda n va n_0 — zarralarning mos ravishda h va $h=0$ balandlikdagi konsentratsiyasi. $P=m_0gh$ — zarralarning potensial energiyasi.

$$P=P_0e^{-m_0g(h-h_0)/(kT)}$$

bunda: P va P_0 mos ravishda h va h_0 balandliklardagi bosim.

Keltirilgan taqsimot funksiyalaridan foydalanib ko‘pincha fizik kattaliklarning o‘rtacha qiymatlari topiladi. Taqsimot funksiyasi birga normallashtirilgan istalgan fizik kattaliklarning o‘rtacha qiymati

$$<x>=\int xf(x)dx$$

kabi aniqlanadi.

Vaqt birligida gazning bitta molekulasiga urilishlarning o‘rtacha soni

$$<Z>=\sqrt{2}\pi d^2\cdot n<\vartheta>,$$

bunda: d — molekulalarning effektiv diametri; p — molekulalar konsentratsiyasi; $<\vartheta>$ — molekulalarning o‘rtacha arifmetik tezligi.

Gaz molekulasi erkin yugurish yo‘lining o‘rtacha uzunligi:

$$<l>=1/\left(\sqrt{2}\pi d^2n\right).$$

Molekulalarning, gaz qatlamining bir sirt elementidan boshqasiga ko‘chiradigan impulsi (harakat miqdori) $dp=\eta\frac{d\vartheta}{dz}\Delta zdt$,

bunda: η — gazning dinamik qovushqoqligi; $\frac{d\vartheta}{dz}$ — tezlik gradiyenti, ΔS — sirt elementining yuzasi; dt — ko‘chirish vaqti.

Dinamik qovushoqlik:

$$\eta = \frac{1}{3}\rho < \vartheta >< l >,$$

bunda: ρ – gazning (suyuqlikning) zichligi; $< \vartheta >$ – molekulalarning betartib harakat o'rtacha tezligi; $< l >$ – ular erkin yugurish yo'lining o'rtacha uzunligi.

Nyuton qonuni: $F = \frac{dp}{dt} = \eta \frac{d\vartheta}{dz} \Delta S$,

bunda: F – harakatlanuvchi gaz qatlamlari orasidagi ichki ishqalanish kuchi.

Fure qonuni: $\Delta Q = -\lambda \frac{dT}{dx} S \Delta t$,

bunda: ΔQ – issiqlik o'tkazuvchanlik natijasida S – yuzali ko'ndalang kesim orqali, Δt vaqtda oqib o'tgan issiqlik miqdori; λ – issiqlik o'tkazuvchanlik koeffitsienti; $\frac{dT}{dx}$ – harorat gradienti. Gazning issiqlik o'tkazuvchanlik koeffitsienti quyidagicha aniqlanadi:

$$\lambda = \frac{1}{3} C \rho < \vartheta >< l >, \text{ yoki } \lambda = \frac{1}{6} kn < \vartheta >< l >.$$

bunda: C_v – gazning o'zgarmas hajmdagi solishtirma issiqlik sig'imi.

Fik qonuni: $\Delta m = -D \frac{dn}{dx} m_0 S \cdot \Delta t$,

bunda: Δm – diffuziya natijasida S yuzali sirt orqali Δt vaqtda ko'chiriladigan gaz massasi, D – diffuziya koeffitsienti, $\frac{dn}{dx}$ – molekulalar konsentratsiyasining gradiyenti; m_0 – bitta molekulaning massasi. Diffuziya koeffitsienti quyidagicha aniqlanadi:

$$D = \frac{1}{3} < \vartheta >< l >.$$

Masala yechishga misollar

1-misol. Bir xil ideal gaz molekulalari ilgarilanma harakat o'rtacha kinetik energiyasi $<\varepsilon_i>$, ilgarilanma harakat kinetik energiyasining eng katta ehtimolli qiymati ε_e dan necha marta farq qilishi aniqlansin.

Berilgan:
$T = \text{const}$.

$\dfrac{<\varepsilon_i>}{\varepsilon_e} = ?$

Yechish: Molekula ilgarilanma harakatining o'rtacha kinetik energiyasi quyidagi formulaga muvofiq aniqlanadi:

$$<\varepsilon_i> = \int_0^\infty \varepsilon F(\varepsilon)d\varepsilon . \qquad (1)$$

Bu yerda

$$f(\varepsilon) = \frac{2}{\sqrt{\pi}} \frac{e^{-\frac{\varepsilon}{kT}}}{(kT)^{3/2}} \varepsilon^{1/2} \qquad (2)$$

— energiya bo'yicha taqsimot funksiyasi. Unda

$$<\varepsilon_i> = \frac{2}{\sqrt{\pi}(kT)^{3/2}} \int_0^\infty \varepsilon^{3/2} e^{-\frac{\varepsilon}{kT}} d\varepsilon . \qquad (3)$$

Integral ostidagi ifodani jadvalda beriladigan

$$\int_0^\infty x^{3/2} e^{-ax} dx = \frac{3}{4}\sqrt{\pi} a^{-5/2} \qquad (4)$$

integraldan foydalanib bajaramiz:

$$<\varepsilon_i> = \frac{2}{\sqrt{\pi}(kT)^{3/2}} \frac{3}{4} \sqrt{\pi} \left(\frac{1}{kT}\right)^{-5/2} = \frac{3}{2}kT . \qquad (5)$$

Molekulaning ilgarilanma harakat kinetik energiyasining eng katta ehtimolli qiymati ε_e quyidagi ifoda yordamida aniqlanadi:

$$\varepsilon_e = \frac{1}{2}kT . \qquad (6)$$

(5) va (6) ifodalar yordamida so‘ralgan ifodani tuzamiz:

$$\frac{<\varepsilon>}{\varepsilon_e}=\frac{\frac{3}{2}kT}{\frac{1}{2}kT}=3.$$

Javob: $\frac{<\varepsilon>}{\varepsilon_e}=3$.

2-misol. Havoda muallaq turgan zarralarning har birining massasi 10^{-18} g ga teng. Agar balandlik 10 m ga orsa, zarralarning konsentratsiyasi necha marta kamayadi? Havoning harorati 300 K.

Berilgan:	Yechish:
$m=10^{-18}$ g$=10^{-21}$ kg; $\Delta h=h_2-h_1=10$ m; $T=300$ K; $\frac{P_1}{P_2}=?$	**Yechish:** Zarralarning tashqi potensial maydondagi taqsimoti, Bolsman taqsimotidan foydalanib zarralarning dastlabki h_1 balandlikdagi konsentratsiyasini aniqlaymiz:

$$n_1=n_0e^{-mgh_1/(kT)}. \qquad (1)$$

Xuddi shuningdek h_2 balandlikda zarralar konsentratsiyasi:

$$n_2=n_0e^{-mgh_2/(kT)}, \qquad (2)$$

bunda: n_0 – zarralarning $h=0$ balandlikdagi (Yer sirtidagi) konsentratsiyasi.

(1) va (2) yordamida $\frac{n_1}{n_2}$ nisbatini aniqlaymiz:

$$\frac{n_1}{n_2}=\frac{n_0e^{-mgh_1/(kT)}}{n_0e^{-mgh_2/(kT)}}=e^{mg(h_2-h_1)/(kT)}=e^{mg\Delta h/(kT)}. \qquad (3)$$

(3) ga berilganlarni va Bolsman doimiysining qiymati $k = 1{,}38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$ ni qo'yib, quyidagini olamiz:

$$\frac{n_1}{n_2} = \exp\left(10^{-21} \cdot 9{,}8 \cdot 10/(1{,}38 \cdot 10^{-23} \cdot 300)\right) \approx \exp\left(10^{-19}/(4{,}14 \cdot 10^{-21})\right) = e^{21{,}15} = 1{,}5 \cdot 10^9.$$

Javob: $\frac{n_1}{n_2} = 1{,}5 \cdot 10^9$ marta.

3-misol. Harorati 310 K bo'lgan azot 80 mkPa bosim ostida turibdi. Shunday sharoitda saqlanadigan azot molekulasining o'rtacha erkin yugurish yo'li $< l >$ aniqlansin.

Berilgan:
T=310 K;
P =80 mkPa=8 · 10^{-5}Pa.
$< l >$=?

Yechish: Gaz molekulasi erkin yugurishining o'rtacha uzunligi

$$< l > = \frac{1}{\sqrt{2}\pi d^2 n} \qquad (1)$$

ifoda yordamida aniqlanadi. Bu yerda: d – molekulaning effektiv diametri, azot uchun $d = 0{,}38 \cdot 10^{-9}$m, n – hajm birligidagi molekulalar soni, ya'ni

$$n = \frac{N}{V}. \qquad (2)$$

Molekulalarning umumiy soni N ni topish uchun gaz holati tenglamasidan foydalanamiz:

$$PV = \frac{m}{M} RT = \frac{m}{M} k \cdot N_A T = \frac{mN_A}{M} kT = NkT. \qquad (3)$$

Ushbu almashtirishda $R = k \cdot N_A$ va $N = \frac{mN_A}{M}$ dan foydalandik. N_A – Avogadro soni, $k = 1{,}38 \cdot 10^{-23} J/K$ – Bolsman doimiysi. (3) dan N ni topib $N = \frac{P \cdot V}{kT}$ (2) ga qo'yamiz:

$$n = \frac{P}{kT} \qquad (4)$$

va molekulalarning konsentratsiyasi uchun ifodani topamiz. (4) ni (1) ga qo‘yib,

$$< l >= \frac{kT}{\sqrt{2}\pi d^2 P} \qquad (5)$$

ni olamiz. (5) ni kattaliklarning o‘lchamlari yordamida tekshiramiz:

$$[l] = \frac{[k][T]}{[d^2][P]} = \frac{1\frac{\text{J}}{\text{K}}\cdot 1\text{K}}{1\text{m}^2\cdot 1\text{Pa}} = 1\frac{\text{N}\cdot\text{m}}{\text{m}^2\frac{\text{N}}{\text{m}^2}} = 1\text{m}.$$

Kattaliklarning qiymatini (5) ga qo‘yib olamiz:

$$< l >= \frac{1{,}38\cdot 10^{-23}\cdot 310}{\sqrt{2}\cdot 3{,}14\cdot(0{,}38\cdot 10^{-9})^2\cdot 8\cdot 10^{-5}}\text{m} = \frac{4{,}3\cdot 10^2}{6{,}9}\text{m} = 62{,}3\text{m}.$$

Javob: $< l >= 62{,}3\text{m}$.

4-misol. Agar vodorod molekulasi erkin yugurish yo‘lining o‘rtacha uzunligi 1sm bo‘lsa, siyraklashgan vodorodning zichligi aniqlansin.

Berilgan:	Yechish:
$< l >= 1$ sm=10^{-2}m.	**Yechish:** Ma’lumki, zichlik quyidagi ifoda yordamida aniqlanadi:
$\rho = ?$	

$$\rho = \frac{m}{V}. \qquad (1)$$

Gaz egallagan hajmni molekulalar konsentratsiyasini aniqlash formulasi $n = \frac{N}{V}$ dan topsak,

$$V = \frac{N}{n}. \qquad (2)$$

Agar o‘z navbatida $N=\nu N_A$ va $\nu = \frac{\text{n}}{\text{M}}$ ligini nazarda tutsak, hajm uchun quyidagi ifodani olamiz:

$$V = \frac{\nu N_A}{n} = \frac{m \cdot N_A}{Mn}. \tag{3}$$

(3) ni (1) ga qo‘yib olamiz:

$$\rho = \frac{m}{\left(\dfrac{m \cdot N_A}{M \cdot n}\right)} = \frac{n \cdot M}{N_A}, \tag{4}$$

bu yerda: $M=2 \cdot 10^{-3}\ \frac{\text{kg}}{\text{mol}}$ vodorod molyar massasi, $N_A = 6{,}02 \cdot 10^{23}\,\text{mol}^{-1}$ – Avogadro soni. (4) ifodadagi konsentratsiya n ni masala shartida berilgan erkin yugurish yo‘lining o‘rtacha uzunligi $<l>$ orqali ifodalaymiz:

$$< l > = \frac{1}{\sqrt{2}\pi d^2 n}.$$

Bundan $$n = \frac{1}{\sqrt{2}\pi d^2 < l >}. \tag{5}$$

(5) ni (4) ga qo‘yamiz:

$$\rho = \frac{M}{\sqrt{2}\pi d^2 < l > N_A}, \tag{6}$$

bu yerda $d = 0{,}28 \cdot 10^{-9}\,m$ – vodorod molekulasining effektiv diametri. (6) ning to‘g‘riligini o‘lchamliklar yordamida tekshirib ko‘ramiz:

$$[\rho] = \frac{[M]}{[d^2][l][N_A]} = \frac{1\dfrac{\text{kg}}{\text{mol}}}{1\text{m}^2 \cdot 1\text{m} \cdot 1\text{mol}^{-1}} = 1\frac{\text{kg}}{\text{m}^3}$$

va to‘g‘riligiga ishonch hosil qilgach, kattaliklarning son qiymatlarini unga qo‘yamiz:

$$\rho = \frac{2 \cdot 10^{-3}}{\sqrt{2} \cdot 3{,}14 \cdot \left(0{,}28 \cdot 10^{-9}\right) \cdot 10^{-2} \cdot 6{,}02 \cdot 10^{23}} \frac{\text{kg}}{\text{m}^3} = 1{,}55 \cdot 10^{-6}\frac{\text{kg}}{\text{m}^3} = 1{,}55\frac{\mu\text{g}}{\text{m}^3}.$$

Javob: $\rho = 1{,}55\frac{\mu g}{m^3}$.

5-misol. Agar havoning zichligi 0,05kg/m^3 bo'lsa, 280K haroratda 1sm^3 hajmdagi havo molekulalarining, 5s davomida o'zaro to'qnashishlarning o'rtacha soni aniqlansin.

Berilgan:

ρ=0,05 kg/m^3;

T =280 K;

V =1 sm^3=10^{-6}m^3;

t =5 s.

$Z = ?$

Yechish: Biror V hajmdagi molekulalarning t vaqt davomida to'qnashishlarining o'rtacha soni quyidagicha aniqlanadi:

$$Z = \frac{1}{2} < z > \cdot n \cdot V \cdot t. \qquad (1)$$

Bu yerda: $< z >$ – vaqt birligida gazning bitta molekulasiga urilishlarining o'rtacha soni bo'lib,

$$< Z > = \frac{< \vartheta >}{< \ell >} \qquad (2)$$

ifoda yordamida aniqlanadi. $< \vartheta >$ molekulaning o'rtacha arifmetik tezligi

$$< \vartheta > = \sqrt{\frac{2kT}{\pi m}} = \sqrt{\frac{2kT}{\pi \rho_x \cdot V_m \cdot \nu}}. \qquad (3)$$

Ushbu ifodani olishda $m = \rho_x \cdot V_m \cdot \nu$ munosabatdan foydalandik:

ρ_x – havoning zichligi, $V_m = 22{,}4 \cdot 10^{-3}\frac{m^3}{mol}$ – bir mol gazning hajmi,

ν – gazdagi mollar soni; $k = 1{,}38 \cdot 10^{-23}\frac{J}{K}$ – Bolsman doimiysi,

$< l >$ – erkin yugurish yo'lining o'rtacha uzunligi. Uni dinamik qovushqoqlik ifodasi

$$\eta = \frac{1}{3}\rho_x < \vartheta >< l >$$

dan aniqlaymiz. Havo uchun $\eta_x = 17{,}2 \cdot 10^{-6} Pa \cdot s$.

$$<l> = \frac{3\eta}{\rho_x <\vartheta>}. \qquad (4)$$

(4) va (3) ni (2) ga qo'yib quyidagini olamiz:

$$<Z> = \frac{\rho_x <\vartheta>^2}{3\eta} = \frac{\rho_x}{3\eta}\frac{2kT}{\pi\rho_x V_m \nu} = \frac{2}{3}\frac{kT}{\pi\eta V_m \nu}. \qquad (5)$$

Molekulalarning konsentratsiyasini esa quyidagi munosabat yordamida aniqlaymiz:

$$n = \frac{N}{V} = \frac{\nu N_A}{V}, \qquad (6)$$

bu yerda: $N = \nu N_A$ ligidan foydalandik, $N_A = 6{,}02 \cdot 10^{23} \text{mol}^{-1}$ – Avogadro soni.

(5) va (6) larni (1) ga qo'yib Z uchun quyidagi ifodani hosil qilamiz:

$$Z = \frac{1}{2} \cdot \frac{2}{3}\frac{kT}{\pi\eta V_m \nu}\frac{\nu N_A}{V} V \cdot t = \frac{N_A \cdot k \cdot T \cdot t}{3\pi\eta V_m}. \qquad (7)$$

Topilgan ifoda yordamida Z ning o'lchov birligi hosil bo'lishini tekshirib ko'ramiz:

$$[Z] = \frac{[N_A][k][T][t]}{[\eta][V_m]} = \frac{1\, mol^{-1} \cdot 1\frac{J}{K} \cdot 1\, K \cdot 1\text{s}}{1\text{Pa} \cdot s \cdot \frac{\text{m}^3}{\text{mol}}} = 1\frac{\text{N} \cdot \text{m}}{\frac{\text{N}}{\text{m}^2} \cdot \text{m}^3} = 1$$

va kattaliklarning son qiymatlarini (7) ga qo'yamiz:

$$Z = \frac{6{,}02 \cdot 10^{23} \cdot 1{,}38 \cdot 10^{-23} \cdot 280 \cdot 5}{3 \cdot 3{,}14 \cdot 17{,}2 \cdot 10^{-6} \cdot 22{,}4 \cdot 10^{-3}} = 3{,}15 \cdot 10^{-9} \text{marta.}$$

Javob: $Z = 3{,}15 \cdot 10^{-9}$ marta.

6-misol. Azot diffuziyasi D: 1) normal sharoitda; 2) P=100Pa bosim va T=300K haroratda hisoblansin.

Berilgan:
1) P=101325 Pa;
T=273,15 K;

2) P=100 Pa;

T=300K.

D=?

Yechish: Diffuziya (diffuziya koeffitsienti) quyidagi formula yordamida aniqlanadi:

$$D=\frac{1}{3}<\vartheta><l> \quad (1)$$

bunda: $<\vartheta>$ – o'rtacha arifmetik tezlik

$$<\vartheta>=\sqrt{\frac{8kT}{\pi m}} \quad (2)$$

va $<l>$ erkin yugurish yo'lining o'rtacha uzunligi

$$<l>=\frac{1}{\sqrt{2}\pi d^2 n}. \quad (3)$$

(2) va (3) ni (1) ga qo'yib, quyidagini olamiz:

$$D=\frac{1}{3}\sqrt{\frac{8kT}{\pi m}}\frac{1}{\sqrt{2}\pi d^2 n}. \quad (4)$$

Bu yerda: $k=1{,}38\cdot10^{-23}\,\frac{\text{J}}{\text{K}}$ – Bolsman doimiysi, $m=46{,}48\cdot10^{-27}$kg – azot molekulasining massasi, d=0,38 · 10^{-9}m – azot molekulasining effektiv diametri, n – molekulalar konsentratsiyasi bo'lib, oldingi masalada ko'rsatilganidek aniqlanadi

$$n=\frac{N}{V}=\frac{PN_A}{RT}. \quad (5)$$

(5) ni (4) ga qo'yib, quyidagi olamiz:

$$D=\frac{2}{3}\sqrt{\frac{kT}{\pi m}}\frac{RT}{\pi d^2 PN_A}, \quad (6)$$

bu yerda: $N_A=6{,}02\cdot10^{23}\,\text{mol}^{-1}$ – Avogadro soni, $R=8{,}31\frac{\text{J}}{K\cdot\text{mol}}$.

Kattaliklarning o'lchamlari yordamida (6) ni tekshiramiz:

$$[D] = \frac{[k]^{1/2}[T]^{3/2}[R]}{[m]^{1/2}[d^2][P][N_A]} = \frac{\left[1\frac{J}{K}\right]^{1/2}[1K]^{3/2}1\frac{J}{K\cdot mol}}{[1kg]^{1/2}\cdot 1m^2\cdot 1Pa\cdot mol^{-1}} = \frac{[1J]^{1/2}1Jm}{[1kg]^{1/2}\cdot 1H\cdot m} = 1\frac{m^2}{s}$$

va topilgan ifodaning to'g'riligiga ishonch hosil qilingach, kattaliklarning qiymatlarini qo'yamiz:

$$1)\ D = \frac{2}{3}\sqrt{\frac{1,38\cdot 10^{-23}\cdot 273,15}{3,14\cdot 46,48\cdot 10^{-27}}}\cdot\frac{8,31\cdot 273,15}{3,14(0,38\cdot 10^{-9})^2\cdot 101325\cdot 6,02\cdot 10^{23}} = 90\cdot 10^5\frac{m^2}{s};$$

$$2)\ D = \frac{2}{3}\sqrt{\frac{1,38\cdot 10^{-23}\cdot 300}{3,14\cdot 46,48\cdot 10^{-27}}}\cdot\frac{8,31\cdot 300}{3,14(0,38\cdot 10^{-9})^2\cdot 100\cdot 6,02\cdot 10^{23}} = 0,061\frac{m^2}{s}.$$

Javob: 1) $D = 90\cdot 10^5\frac{m^2}{s}$; 2) $D = 0,061\frac{m^2}{s}$.

Mustaqil yechish uchun masalalar

1. Ideal gaz molekulalarining tezliklari bo'yicha taqsimot qonunidan foydalanib, molekulalarning o'rtacha arifmetik tezligi $<\vartheta>$ aniqlansin. $\left[\sqrt{8kT/(nm_0)}.\right]$

2. Tezliklari noldan to eng katta ehtimolli tezlik ϑ_e ning yuzdan bir bo'lagigacha oralig'da joylashgan ideal gaz molekulalarining nisbiy soni ω aniqlansin. $[7,52\cdot 10^{-7}.]$

3. Harorat bir foizga o'zgarganda ideal gaz molekulasi impulsining eng katta ehtimolli qiymati P_e necha foizga o'zgaradi. [0,5 foizga.]

4. Ideal gaz molekulalarining energiyalar bo'yicha taqsimot funksiyasidan foydalanib, molekulalar energiyasining eng katta ehtimolli qiymati ε_e topilsin. [1/2 kT.]

5. Energiyalari 0 da 0,01 gacha oraliqda joylashgan molekulalarning massasi m aniqlansin.

6. Havoda muallaq turgan chang zarralarining har birining massasi 10^{-18}g. 1 m balandlikdagi chang zarralari konsentratsiyasi n_1 ning, uning 0 dagi konsentratsiyasi n_0 nisbati 0,787 ga teng. Havoning harorati 300 K.

Shu berilganlar bo'yicha Avogadro doimiysi N_A ning qiymati topilsin. $5{,}97 \cdot 10^{23} mol^{-1}.]$

7. Uchayotgan vertolyot kabinasidagi barometrning ko'rsatishi, uchish maydonchasidagi barometr ko'rsatishining 0,9 qismiga teng. Havoning harorati 290 K va balandlikka bog'liq emas, deb hisoblash mumkin bo'lsa, vertolyot qanday balandlikda uchayotganligi aniqlansin. [885 m].

8. Hajmi 5l bo'lgan idishda massasi 0,5g azot gazi bor. Azot molekulalari erkin yugurish yo'lining o'rtacha uzunligi $< l >$ topilsin. [1,16 mkm].

9. Normal sharoitda 1s vaqt davomida kislorod molekulasiga urilishlarning o'rtacha soni <Z> topilsin. $[3{,}7 \cdot 10^9 s^{-1}.]$

10. Harorati 300 K bo'lgan va 5 kPa bosim ostida turgan vodorod molekulalari erkin yugurishining o'rtacha davom etish vaqti $< \tau >$ aniqlansin. Vodorod molekulasining diametri 0,28 nm ga teng deb qabul qilinsin. [13,3 ns.]

11. Normal sharoitdagi kislorodning diffuziya koeffitsienti D aniqlansin. $[9{,}18 \cdot 10^{-6} m^3.]$

12. Normal sharoitda geliyning issiqlik o'tkazuvchanligi λ aniqlansin. [38,6MW/(m · K).]

13. Agar ma'lum sharoitda azotning dinamik yopishqoqlik koeffitsienti 10mkPa · s bo'lsa, shu sharoitda azotning issiqlik o'tkazuvchanlik koeffitsienti λ aniqlansin. [7,42MW/(m · K).]

14. Quyidagi jarayonlarda diffuziya D ning, dinamik qovushqoqlik η ning va issiqlik o'tkazuvchanlik λ ning harorat T ga bog'liqligi aniqlansin: 1) izobara, 2) izoxora. Bu bog'lanishlar grafiklarda tasvirlansin.

15. Quyidagi jarayonlarda diffuziya D ning, dinamik qovushqoqlik η ning va issiqlik o'tkazuvchanlik λ ning bosim R ga bog'liqligi aniqlansin: 1) izoterma, 2) izoxora. Bu bog'lanishlar grafiklarda tasvirlansin.

11-§. Termodinamika asoslari

Asosiy formulalar

Gazning o'zgarmas hajmdagi va o'zgarmas bosimdagi molyar issiqlik sig'imlari:

$$C_v = \frac{i}{2}R, \qquad C_p = \frac{i+2}{2}R.$$

Gazning molyar (C_m) va solishtirma (c) issiqlik sig'imlari orasidagi munosabat:

$$C_m = C = cM,$$

bu yerda: i – gaz molekulasining to'la erkinlik darajasi, M – gazning molyar massasi, R – universal gaz doimiysi.

Ideal gazning ichki energiyasi:

$$U = N < \varepsilon > \text{ yoki}$$

$$U = \nu \cdot C_v \cdot T = \nu \cdot \frac{1}{2}RT = \frac{m}{M}\frac{1}{2}RT.$$

Bunda: $\langle \varepsilon \rangle$ – molekulaning o'rtacha kinetik energiyasi, N – gaz molekulalarining soni, ν – modda miqdori, T – harorat.

Adiabatik jarayonda gaz holati tenglamasi (Puasson tenglamasi):

$$PV^{\gamma} = \text{const},$$

bu yerda: $\gamma = \dfrac{C_p}{C_V}$, yoki $\gamma = \dfrac{i+2}{i}$ – adiabata ko'rsatkichi.

Mayer formulasi: $C_p - C_V = R$.

Adiabatik jarayonda gaz holati parametrlarining boshlang'ich va oxirgi qiymatlari orasidagi munosabatlar:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}; \qquad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}; \qquad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}.$$

Gaz hajmining o'zgarishida bajarilgan ish:

$$A = \int_{V_1}^{V_2} PdV,$$

bunda V_1 va V_2 mos ravishda gazning boshlang‘ich va oxirgi hajmlari.

Gazning ishi:

a) Izobarik jarayonda (P=const);

$$A = p(V_2 - V_1) = p\Delta V, \text{ yoki } A = \nu R(T_2 - T_1),$$

bunda T_1 va T_2 gazning boshlang‘ch va oxirgi haroratlari;

b) Izotermik jarayonda (T=const)

$$A = \nu RT \ln\frac{V_2}{V_1}, \text{ yoki } A = \nu RT \ln\frac{P_1}{P_2},$$

P_1 va P_2 gazning boshlang‘ich va oxirgi bosimlari;

d) Adiabatik jarayonda

$$A = \nu C_v(T_1 - T_2) = \nu\frac{i}{2}R(T_1 - T_2), \text{ yoki}$$

$$A = \frac{RT}{\gamma - 1}\nu\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right] = \frac{P_1V_1}{\gamma - 1}\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right].$$

Termodinamikaning birinchi qonuni umumiy holda quyidagicha yoziladi

$$Q = \Delta U + A$$

Bunda: Q – sistemaga berilgan issiqlik miqdori, ΔU – uning ichki energiyasining o‘zgarishi; A – tashqi kuchlarga qarshi bajarilgan ish.

a) Izobarik jarayonda $Q = \Delta U + A = \nu C_V \Delta t + \nu R \cdot \Delta T = \nu C_P \Delta T$;

b) izoxorik jarayonda $(A = 0)$ $Q = \Delta U = \nu C_V \cdot \Delta T$;

d) izotermik jarayonda $(\Delta U = 0)$, $Q = A = \nu RT \ln\frac{V_2}{V_1} = \nu RT \ln\frac{P_1}{P_2}$;

e) adiabatik jarayonda $(Q = 0)$, $A = -\Delta U = -\nu C_V \cdot \Delta T$.

Umumiy holda siklning foydali ish koeffitsienti (FIK)

$$\eta = \frac{A}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

bunda: A — sikl davomida bajarilgan ish, Q_1 — sistema olgan, Q_2 — sistema bergan issiqlik miqdori.

Karno siklining FIK $\eta = \frac{T_1 - T_2}{T_1}$

bunda: T_1 — isitgichning, T_2 — sovutgichning haroratlari.

Muvozanatli o'tishda sistema entropiyasining o'zgarishi

$$\Delta S = \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{dU + dA}{T}.$$

Bolsman formulasi: $S = k \ln \omega$,

bunda k — Bolsman doimiysi, ω — sistema holatlarining termodinamik ehtimoli.

Masala yechishga misollar

1-misol. Tarkibida 5 *l* vodorod va 3 l geliy bo'lgan gaz aralashmasining solishtirma isssiqlik sig'imi C_V aniqlansin. Gazlar bir xil sharoitda turibdi.

Berilgan:
$V_1 = 5\ l = 5 \cdot 10^{-3} m^3$;
$V_2 = 5\ l = 3 \cdot 10^{-3} m^3$.

$C_V = ?$

Yechish: Aralashmaning haroratini ΔT ga o'zgartirish uchun kerak bo'ladigan issiqlik miqdori Q ni ikki usulda aniqlash mumkin:

$$Q = C_V (m_1 + m_2) \cdot \Delta T \quad (1)$$

$$Q = C_{V1} \cdot m_1 \cdot \Delta T + C_{V2} \cdot m_2 \cdot \Delta T = (C_{V1} \cdot m_1 + C_{V2} \cdot m_2)\Delta T, \quad (2)$$

bu yerda: C_V — aralashmaning solishtirma issiqlik sig'imi;

C_{V1} — vodorodning, C_{V2} — geliyning solishtirma issiqlik sig'imlari;

m_1 va m_2 — ularning massalari.

(2) va (3) larni tenglashtirib olamiz:

$$C_V(m_1+m_2)\cdot\Delta T=(C_{V1}m_1+C_{V2}m_2)\Delta T \qquad \text{yoki}$$

$$C_V(m_1+m_2)=C_{V1}\cdot m_1+C_{V2}\cdot m_2 .$$

Bundan esa

$$C_V=C_{V1}\frac{m_1}{m_1+m_2}+C_{V2}\frac{m_2}{m_1+m_2}. \qquad (3)$$

Gazlarning massalarini quyidagi munosabatlardan aniqlaymiz:

$$m_1=\rho_m V_1,\quad m_2=\rho_{He}\cdot V_2,\quad \rho_H=0{,}99\frac{\text{kg}}{\text{m}^3}\ \text{va}\ \rho_{He}=0{,}18\frac{\text{kg}}{\text{m}^3}$$

mos ravishda vodorodning va geliyning zichliklari. Gazlarning solishtirma issiqlik sig'imlari esa quyidagicha topiladi:

$$C_{V1}=\frac{5}{2}\frac{R}{M_1} \qquad \text{va} \qquad C_{V2}=\frac{3}{2}\frac{R}{M_2}, \qquad (4)$$

bu yerda: H_2 uchun $i=5$ va He uchun $i=3$ ekanligi hisobga olingan, $M_1=2\cdot10^{-3}\dfrac{\text{kg}}{\text{mol}}$ va $M_2=4\cdot10^{-3}\dfrac{\text{kg}}{\text{mol}}$ mos ravishda vodorodning va geliyning molyar massalari, R=8,31J/mol – gaz molyar doimiysi.

(4) ni (3) ga qo'yib, quyidagini olamiz:

$$C_V=\frac{5}{2}\frac{R}{M_1}\left(\frac{\rho_H\cdot V_1}{\rho_H V_1+\rho_{He}V_2}\right)+\frac{3}{2}\frac{R}{M_2}\left(\frac{\rho_{He}\cdot V_2}{\rho_H V_1+\rho_{He}V_2}\right) \qquad (5)$$

hosil bo'lgan ifodadan solishtirmma issiqlik sig'imining birligi chiqishi ko'rinib turganligidan kattaliklarning qiymatlarini qo'yamiz:

$$C_V=\frac{5\cdot8{,}31}{2\cdot2\cdot10^{-3}}\left(\frac{0{,}09\cdot5\cdot10^{-3}}{0{,}09\cdot5\cdot10^{-3}+0{,}18\cdot3\cdot10^{-3}}\right)+\frac{3\cdot8{,}31}{2\cdot4\cdot10^{-3}}\left(\frac{0{,}18\cdot3\cdot10^{-3}}{0{,}09\cdot5\cdot10^{-3}+0{,}18\cdot3\cdot10^{-3}}\right)=$$

$$=10{,}39\cdot10^3\left(\frac{0{,}45}{0{,}45+0{,}54}\right)+3{,}12\cdot10^3\left(\frac{0{,}54}{0{,}45+0{,}54}\right)=4{,}53\cdot10^3\frac{\text{J}}{\text{kg}\cdot\text{K}}=4{,}53\frac{\text{kJ}}{\text{kg}\cdot\text{K}}.$$

Javob: $C_V=4{,}53\dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}$.

2-misol. Yopiq idishda, massasi 56 g bo'lgan azot va massasi 64 g bo'lgan kislorod gazlarning aralashmasi bor. Agar harorat 20^0 ga pasaytirilgan bo'lsa, aralashma ichki energiyasining o'zgarishi aniqlansin.

Berilgan:

1) N_2;

$m_1 = 56\text{g} = 56\cdot10^{-3}\text{kg}$;

2) O_2;

$m_2 = 64\text{g} = 64\cdot10^{-3}\text{kg}$;

$\Delta T = 20^0$.

$\Delta U = ?$

Yechish: Aralashma ichki energiyasining o'zgarishi azot va kislorod gazlari ichki energiyalarining o'zgarishiga teng, ya'ni

$$\Delta U = \Delta U_1 + \Delta U_2. \qquad (1)$$

Ichki energiyaning aniqlanish ifodasiga muvofiq

$$\Delta U_1 = \frac{m_1}{M_2}\frac{i_1}{2}R\Delta T \quad \text{va}$$

$$\Delta U_1 = \frac{m_2}{M_2}\frac{i_2}{2}R\Delta T. \qquad (2)$$

Agar kislorod ham, azot ham ikki atomli gaz ekanligini nazarda tutsak $i_1 = i_2 = 5$. (2) ni (1) ga qo'yib, quyidagini olamiz:

$$\Delta U = \frac{m_1}{M_1}\frac{5}{2}R\Delta T + \frac{m_2}{M_2}\frac{5}{2}R\Delta T = \frac{5}{2}R\Delta T\left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right), \qquad (3)$$

bu yerda: $M_1 = 28\cdot10^{-3}\,\frac{\text{kg}}{\text{mol}}$ – azotning, $M_2 = 32\cdot10^{-3}\,\frac{\text{kg}}{\text{mol}}$ – kislorodning molyar massalari .

Kattaliklarning son qiymatlarini (3) ga qo'yamiz:

$$\Delta U = \frac{5}{2}8{,}31\cdot20\left(\frac{56\cdot10^{-3}}{28\cdot10^{-3}} + \frac{64\cdot10^{-3}}{32\cdot10^{-3}}\right)\text{J} = 5\cdot83{,}1\cdot4\text{J} = 1662\text{J} = 1{,}66\text{kJ}.$$

Javob: $\Delta U = 1{,}66\text{kJ}$.

3-misol. Gazni adiabatik siqishda uning hajmi 10 marta kamaydi, bosimi esa 21,4 marta oshdi. Gaz solishtirma issiqlik sig'imlarining nisbati C_P/C_V topilsin.

Berilgan:

$n=\frac{V_1}{V_2}=10;$

$k=\frac{P_2}{P_1}=21{,}4;$

$Q=0.$

$\gamma=\frac{C_P}{C_V}=?$

Yechish: Adiabatik jarayonda gaz hajmining va bosimining boshlang'ich va oxirgi qiymatlari orasida quyidagi munosabat mavjud:

$$\frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^{\gamma}, \text{ yoki } k=n^{\gamma}. \qquad (1)$$

(1) tenglikni logarifmlab, quyidagini olamiz:

$$\ln(k)=\gamma\cdot\ln(n).$$

Bundan

$$C=\frac{C_P}{C_V}=\frac{\ln k}{\ln n}. \qquad (2)$$

Kattaliklarning son qiymatlarini qo'yib topamiz

$$\gamma=\frac{\ln 21{,}4}{\ln 10}=\frac{3{,}06}{2{,}30}=1{,}33.$$

Javob: $\gamma=1{,}33$.

4-misol. Ballonda 145 K haroratli kislorod 2 mPa bosim ostida saqlanadi. Ballondagi gazning yarmi tezgina chiqarib yuborilgandan keyin qolgan kislorodning harorati T_2 va bosimi P_2 aniqlansin.

Berilgan:

$T_1=145\text{K};$

$P_1=2\text{mPa}=2\cdot10^{-3}\text{Pa};$

$\frac{m_1}{m_2}=2.$

$P_2=?$

$T_2=?$

Yechish: Agar gazning tezgina chiqarib yuborilishini nazarda tutsak, unda kengiyishini adiabatik jarayon sifatida qarash mumkin. Adiabatik jarayonda gaz holati parametrlarining boshlang'ich va oxirgi qiymatlari orasida quyidagi munosabat mavjud:

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}. \qquad (1)$$

bu yerda: $\gamma=\frac{C_P}{C_V}=\frac{i+2}{i}$, i – gaz molekulasining erkinlik jarajasi. O_2 uchun i=5.

(1) dagi noma'lum parametrlar sonini kamaytirish maqsadida har ikkala holat uchun ham gaz holati tenglamasini yozamiz:

$$P_1V = \frac{m_1}{M}RT_1 \qquad \text{va} \qquad P_2V = \frac{m_2}{M}RT_2.$$

Tenglamalarni hadma-had bo'lamiz:

$$\frac{P_1}{P_2} = \frac{m_1}{m_2}\frac{T_1}{T_2}.$$

Bundan $\frac{T_2}{T_1} = \frac{m_1}{m_2}\frac{P_2}{P_1}$

yoki $\frac{m_1}{m_2} = 2$ ni nazarda tutsak,

$$\frac{T_2}{T_1} = 2\frac{P_2}{P_1} \qquad (2)$$

(2) ni (1) ga qo'yib va i=5 ni nazarga tutib, quyidagini topamiz:

$$2\frac{P_2}{P_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{yoki} \quad 2\frac{P_2}{P_1} = \left(\frac{P_2}{P_1}\right)^{\frac{2}{7}}.$$

Oxirgi ifodadan P_2 ni aniqlasak,

$$P_2 = \frac{P_1}{2^{7/5}} = \frac{P_1}{\sqrt[5]{2^7}}. \qquad (3)$$

Endi (2) ifodadan T_2 ni topamiz va (3) ni nazarda tutib qayta yozamiz:

$$T_2 = 2\frac{T_1}{P_1}P_2 = \frac{T_1}{2^{2/5}} = \frac{T_1}{\sqrt[5]{2^2}}. \qquad (4)$$

Kattaliklarning son qiymatlarini (3) va (4) larga qo'yib, quyidagini olamiz:

$$P_2 = \frac{2\cdot10^{-3}}{\sqrt[5]{2^7}}\,\text{Pa} = \frac{2\cdot10^{-3}}{2{,}55}\,\text{Pa} = 0{,}78\cdot10^{-3}\,\text{Pa} = 0{,}78\text{mPa};$$

$$T_2 = 2\frac{145}{2\cdot10^{-3}}\cdot0{,}78\cdot10^{-3}\,\text{K} = 113{,}1\text{K}.$$

Javob: $P_2 = 0{,}78\text{mPa}; \quad T_2 = 113{,}1\text{K}.$

5-misol. Adiabatik siqilishda havoning bosimi 50 kPa dan 500 kPa gacha ortdi. So'ngra esa, hajm o'zgartirilmasdan gazning harorati dastlabgi haroratgacha pasaytirildi. Jarayon so'ngida gazning bosimi P_3 qancha bo'ladi? Adiabata ko'rsatkichi 1,4 deb olinsin.

Berilgan:

$1) Q = 0,$

$P_1 = 50\text{kPa} = 5 \cdot 10^4 \text{ Pa},$

$P_2 = 500\text{kPa} = 5 \cdot 10^5 \text{ Pa}.$

$2) V = \text{const},$

$\gamma = 1,4.$

$P_3 = ?$

Yechish: Ikkinchi jarayon izoxorik (V=const) bo'lganligidan

$$\frac{P_2}{T_2} = \frac{P_3}{T_1} \qquad (1)$$

munosabat o'rinli bo'ladi. Bu yerda $T_3 = T_1$ e'tiborga olingan. (1) dan olamiz:

$$P_3 = \left(\frac{T_1}{T_2}\right) \cdot P_2. \qquad (2)$$

(2) tenglikdagi $\left(\frac{T_1}{T_2}\right)$ munosabatni topish uchun birinchi jarayonning adiabatikligidan foydalanamiz, ya'ni quyidagi munosabat o'rinli:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}, \quad \text{yoki} \quad \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}. \qquad (3)$$

(3) ni (2) ga qo'yamiz:

$$P_3 = P_2 \cdot \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}. \qquad (4)$$

Kattaliklarining qiymatlarini (4) ga qo'yib, quyidagini olamiz:

$$P_3 = 5 \cdot 10^5 \left(\frac{5 \cdot 10^4}{5 \cdot 10^5}\right)^{\frac{1,4-1}{1,4}} \text{Pa} = 5 \cdot 10^5 (10^{-1})^{\frac{2}{7}} \text{Pa} = 5 \cdot 10^5 \cdot 10^{-2/7} \text{Pa} =$$

$$= 5 \cdot 0,52 \cdot 10^5 \text{Pa} = 2,59 \cdot 10^5 \text{Pa} = 259\text{kPa}.$$

Javob: $P_3 = 259\text{kPa}.$

6-misol. a) izotermik jarayonda, b) izobarik jarayonda, – 10 J ish bajarish uchun kislorodga qancha issiqlik miqdori berilishi kerak?

Berilgan:

$A = 10\text{J}$;

$a) T = \text{const}$;

$b) P = \text{const}$.

$Q = ?$

Yechish: Termodinamikaning birinchi qonunini umumiy ko'rinishda yozamiz:

$$Q = A + \Delta U \qquad (1)$$

va uni alohida jarayonlarga tadbiq etamiz: a) izotermik jarayonda $\Delta U = 0$. Demak, $Q = A$ (2)

b) Izobarik jarayonda

$$Q = A + \Delta U. \qquad (3)$$

Ichki energiyaning o'zgarishi quyidagicha topiladi:

$$\Delta U = \frac{i}{2}\frac{m}{M}R\Delta T = \frac{i}{2}\frac{m}{M}R(T_2 - T_1) \qquad (4)$$

bu yerda: i – molekulaning erkinlik darajasi, O_2 uchun i=5,m – gaz massasi, M – molyar massasi, R – gaz molyar doimiysi. T_1 va T_2 – dastlabki va oxirgi haroratlar. Dastlabki va oxirgi holatlar uchun gaz holati tenglamasini yozamiz:

$$PV_1 = \frac{m}{M}RT_1 \quad \text{va} \quad PV_2 = \frac{m}{M}RT_2.$$

Ikkinchisidan birinchisini ayirib olamiz:

$$P(V_2 - V_1) = \frac{m}{M}R(\text{T}_2 - \text{T}_1) = \frac{m}{M}R\Delta\text{T}. \qquad (5)$$

Oxirgi ifodadan quyidagi munosabatni topamiz:

$$\frac{m}{M}\cdot\Delta\text{T} = \frac{P(V_2 - V_1)}{R} = \frac{P\cdot\Delta V}{R} = \frac{A}{R}. \qquad (6)$$

(6) ni (4) ga quysak,

$$\Delta U = \frac{i}{2}R\frac{\text{P}\cdot\Delta V}{r} = \frac{i}{2}R\cdot\frac{A}{R} = \frac{i}{2}A. \qquad (7)$$

(7) ni (3) ga qo'yib Q uchun quyidagini olamiz:

$$Q = A + \frac{i}{2}A = A\left(\frac{2+i}{2}\right). \qquad (8)$$

Shunday qilib,

$$Q = \frac{2+5}{2} \cdot 10\text{J} = 3{,}5 \cdot 10\text{J} = 35\text{J}.$$

Javob: a) $Q = 10\text{J}$; b) $Q = 35\text{J}$.

7-misol. Izotermik kengayish paytida 800 J issiqlik miqdori olsa, 0,4mol vodorodning hajmi necha marta ortadi? Gazning harorati 300 K.

Berilgan:

$T = \text{const}$,

$\nu = 0{,}4\text{mol}$;

$Q = 800\text{J}$,

$T = 300\text{K}$.

$\frac{V_2}{V_1} = ?$

Yechish: Izotermik jarayon ($\Delta U = 0$) uchun termodinamikaning birinchi qonuni quyidagi ko'rinishga ega:

$$Q = A = \nu RT \ln\frac{V_2}{V_1}, \qquad (1)$$

bu yerda: $R = 8{,}31\frac{\text{J}}{\text{K}\cdot\text{mol}}$ – gaz molyar doimiysi.

(1) dan so'ralgan nisbatni topamiz:

$$\ln\frac{V_2}{V_1} = \frac{Q}{\nu RT}, \quad \text{yoki} \quad \frac{V_2}{V_1} = e^{\frac{Q}{\nu RT}}. \qquad (2)$$

Kattaliklarning qiymatini (2) ga qo'yamiz:

$$\frac{V_2}{V_1} = \exp\left(\frac{800}{0{,}4 \cdot 8{,}31 \cdot 300}\right) = \exp\left(\frac{8}{9{,}97}\right) = e^{0{,}8} = 2{,}22\text{marta}$$

Javob: $\frac{V_2}{V_1} = 2{,}22\text{marta}$.

8-misol. 5 kg massali azot hajmi o'zgarmasdan 150 K ga qizdirildi. Gazga berilgan issiqlik miqdori Q, ichki energiyaning o'zgarishi ΔU, gaz bajargan ish A topilsin.

Berilgan:

$m = 5\text{kg};$

$\Delta T = 150\text{K},$

$V = \text{const}$

$Q = ?$

$\Delta U = ?$

$A = ?$

Yechish: Gazning tashqi kuchlarga qarshi bajargan ishi quyidagicha aniqlanadi:

$$A = \rho \cdot \Delta V.$$

V=const bo'lganidan $\Delta V = 0$ va demak,

$$A = 0. \qquad (1)$$

Ichki energiyaning o'zgarishi ΔU ni esa quyidagidek topamiz:

$$\Delta U = \frac{i}{2}\frac{m}{M} R \cdot \Delta T, \qquad (2)$$

bu yerda: i — molekulalarning erkinlik darajasi, N_2 uchun i=5; $M = 28 \cdot 10^{-3}\,\frac{\text{kg}}{\text{mol}}$ — azotning molyar massasi, $R = 8{,}31\,\frac{\text{J}}{\text{K} \cdot \text{mol}}$ — gaz molyar doimiysi.

Jarayon uchun termodinamikaning birinchi qonunini yozamiz:

$$Q = \Delta U,$$

chunki $A = 0$.

Kattaliklarning son qiymatlarini o'rniga qo'yib hisoblaymiz:

$$\Delta U = \frac{5}{2}\frac{5}{28 \cdot 10^{-3}} \cdot 8{,}31 \cdot 150\text{J} = \frac{25 \cdot 15 \cdot 8{,}31}{56} \cdot 10^4\,\text{J} = 55{,}65 \cdot 10^4\,\text{J} = 556{,}5\text{kJ}.$$

$Q = 556{,}5\text{kJ}$

Javob: $A=0$; $\Delta U = 556{,}5\text{J}$; $Q = 556{,}5\text{kJ}$.

9-misol. Massasi 1kg, harorati 300 K bo'lgan azot bosimi o'n marta ortgunicha: a) izotermik ravishda; b) adiabatik ravishda siqilgan. Har ikkala hol uchun gazni siqishda bajarilgan ish aniqlansin.

Berilgan:

m=1 kg;

T_1=300 K;

$\frac{P_2}{P_1} = 10;$

a) T=const;

b) Q=0

$A=?$

Yechish: a) Gazning izotermik ravishda siqilishida bajarilgan ish quyidagicha aniqlanadi:

$$A = \frac{m}{M} R T_1 \ln\frac{P_1}{P_2}, \qquad (1)$$

bu yerda: $M = 28 \cdot 10^{-3}\,\frac{\text{kg}}{\text{mol}}$ — azotning molyar massasi,

$R = 8{,}31 \frac{\text{J}}{\text{K}\cdot\text{mol}}$ – gaz molyar doimiysi, V_1 va V_2 – gazning dastlabki va oxirgi hajmlari.

b) Gazning adiabatik ravishda siqilishida bajarilgan ish quyidagicha aniqlanadi:

$$A = \frac{RT_1}{\gamma - 1}\frac{m}{M}\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right], \qquad (2)$$

bunda: $\gamma = \frac{C_P}{C_V} = \frac{i+2}{2}$ – adiabata ko'rsatkichi, i – molekulaning erkinlik darajasi. Azot uchun $i = 5$, $\gamma = 1{,}4$.

Adiabatik jarayon uchun quyidagi munosabat ham o'rinli:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}, \qquad \text{yoki} \qquad \left(\frac{P_2}{P_1}\right)^{1/\gamma} = \left(\frac{V_1}{V_2}\right) \qquad (3)$$

(3) ni (2) ga qo'ysak,

$$A = \frac{RT_1}{\gamma - 1}\frac{m}{M}\left[1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad (4)$$

hosil bo'ladi.

Kattaliklarning qiymatlarini (1) va (4) larga qo'yib olamiz:

a) $A = \frac{1}{28\cdot 10^{-3}}\cdot 8{,}31\cdot 300\cdot \ln\left(\frac{1}{10}\right)\text{J} = \frac{24{,}93}{28}\cdot 10^5\cdot \ln(0{,}1)\text{J} =$

$= -2{,}05\cdot 10^5\,\text{J} = -205\text{kJ};$

b) $A = \frac{8{,}31\cdot 300}{1{,}4-1}\frac{1}{2{,}8\cdot 10^{-3}}\left[1 - (10)^{\frac{0{,}4}{1{,}4}}\right]\text{J} = \frac{24{,}93\cdot 10^5}{11{,}2}(1 - 1{,}93)\text{J} =$

$= -2{,}07\cdot 10^5\,\text{J} = -207\text{kJ}.$

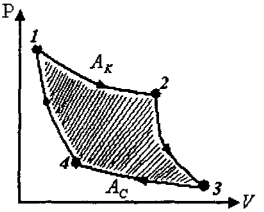
Har ikkila holda ham ishning manfiy chiqishi ish tashqi kuchlar tomonidan bajarilganligini ko'rsatadi.

Javob: $A=-205$ kJ; $A=-207$ kJ.

10-misol. Foydali ish koeffitsienti 0,4 ga teng bo'lgan Karno siklida, gazning izotermik ravishda kengayishida bajarilgan ish 8 J bo'lsa, gazning izotermik ravishda siqilishidagi A_S ish aniqlansin.

Berilgan:	**Yechish:**
$\eta = 0,4$; $A_K = 8\text{J}$; $T=$ const.	Siklning $P-V$ diagrammasini tuzamiz. 1–2 o'tish gazning izotermik kengayishini, 3–4 o'tish esa izotermik siqilishni ko'rsatadi. Karno siklining FIK quyidagicha aniqlanadi:
$A_S = ?$	

$$\eta = \frac{Q_1 - Q_2}{Q_1},$$



27-rasm

bu yerda: Q_1 – gaz isitgichdan olgan issiqlik miqdori, Q_2 – gaz sovutgichga bergan issiqlik miqdori.

Izotermik kengayishda bajarilgan A_k ish gaz isitgichdan olgan Q_1 issiqlik miqdoriga, izotermik siqilishdagi A_S ish esa, gaz sovutgichga bergan Q_2 issiqlik miqdoriga teng bo'ladi, ya'ni

$$Q_1 = A_k; \quad Q_2 = A_S . \tag{2}$$

(2) ni (1) ga qo'yamiz va undan A_S ni aniqlaymiz

$$\eta = \frac{A_k - A_S}{A_k}; \quad A_S = (1-\eta)A_k . \tag{3}$$

Berilganlarni (3) ga qo'yib hisoblaymiz:

$$A_S = (1-0,4)8\text{J} = 0,6 \cdot 8\text{J} = 4,8\text{J} .$$

Javob: $A_S = 4,8\text{J}$.

11-misol. Karno sikliga muvofiq ishlaydigan ideal issiqlik mashinasi 1 soat davomida sovitgichga 270 MJ issiqlik miqdori beradi. Agar isitgichning harorati 550 K, sovitgichniki esa 300 K bo'lsa, qurilmaning quvvati aniqlansin.

Berilgan:

$t = 1\text{soat} = 3600\text{s};$

$Q_2 = 270\text{MJ} = 27 \cdot 10^7\ \text{J};$

$T_1 = 550\text{K};$

$T_2 = 300\text{K}.$

$N=?$

Yechish: Qurilmaning quvvati

$$N = \frac{A}{T} \qquad (1)$$

ifoda yordamida aniqlanadi. A qurilmaning t vaqtdan bajargan ishi. Bu ishni

$$A = Q_1\left(\frac{T_1 - T_2}{T_1}\right) \qquad (2)$$

ifoda yordamida aniqlaymiz. (2) ni esa

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}, \quad \text{yoki} \quad Q_1 = Q_2\frac{T_1}{T_2} \qquad (3)$$

munosabat yordamida o'zgartirib yozamiz,

$$A = Q_2\frac{T_1}{T_2}\left(\frac{T_1 - T_2}{T_1}\right) = Q_2\left(\frac{T_1 - T_2}{T_2}\right). \qquad (4)$$

A uchun olingan ifodani (1) ga qo'ysak,

$$N = \frac{Q_2}{t}\left(\frac{T_1 - T_2}{T_2}\right) \qquad (5)$$

hosil bo'ladi.

Kattaliklarning qiymatlarini (5) ga qo'yamiz:

$$N = \frac{27 \cdot 10^7}{3600}\left(\frac{550 - 300}{300}\right)\text{W} = \frac{3}{4} \cdot \frac{5}{6} \cdot 10^5\text{W} - 0{,}625 \cdot 10^5\text{W} = 62{,}5\text{kW}.$$

Javob: $N = 62{,}5\text{kW}$.

12-misol. Ikki atomli ikki mol gaz vositasida ishlaydigan issiqlik mashinasi izoxora, izoterma va izobaradan iborat siklni amalga oshiradi.

Gaz egallagan maksimal hajm minimal hajmdan uch marta katta. Izotermik jarayon esa 450 K haroratda ro'y beradi. Bir sikl davomida bajariladigan ish va siklning FIK topilsin.

Berilgan:

$\nu = 2\text{mol};$

$i = 5;$

$\frac{V_{max}}{V_{min}} = 3;$

1) $V = \text{const};$

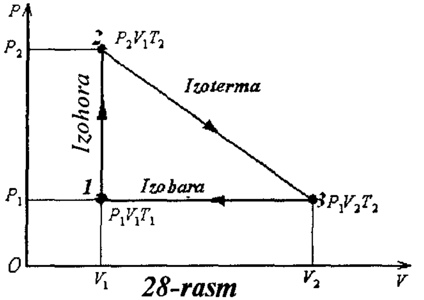
2) $T = 450\text{K} = \text{const};$

3) $P = \text{const}.$

$A = ?$

$\eta = ?$

Yechish: Siklning diagrammasini tuzamiz:



28-rasm

$V_2 = V_{max},$

$V_1 = V_{min},$

$T_2 = T = \text{const}.$

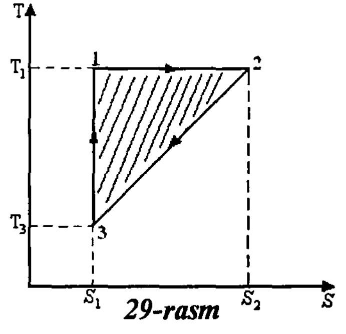
Izobarik jarayonda

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

ekanligini nazarda tutsak,

$$T_1 = T_2 \cdot \frac{V_1}{V_2} = T_2\left(\frac{V_{min}}{V_{max}}\right) = \frac{T_2}{3} \qquad (1)$$

ni hosil qilamiz.



29-rasm

Siklning FIK quyidagicha aniqlanadi:

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{A}{Q_1}, \qquad (2)$$

bu yerda: $Q_1 -$ isitgichdan olingan issiqlik miqdori, $Q_2 -$ sovitgichga berilgan issiqlik miqdori, $A = Q_1 - Q_2$ bir sikl davomida bajarilgan ish .

Sikl davomida olingan issiqlik miqdori, izoxorik va izotermik jarayonlarda olingan issiqlik miqdorlari yiqindisiga teng, ya'ni

$$Q_1 = Q_{12} + Q_{23}, \qquad (1)$$

Izoxorada olingan issiqlik miqdori ($1 \to 2$)

$$Q_{12} = \frac{i}{2} R\nu(T_2 - T_1) \qquad (2)$$

va izotermada olingan issiqlik miqdori ($2 \to 3$)

$$Q_{23} = \nu RT_2 \ln\frac{V_2}{V_1} = \nu RT_2 \ln\frac{V_{max}}{V_{min}} \qquad (3)$$

kabi aniqlanadi.

Berilgan issiqlik miqdori esa izobarada ($3 \to 1$) berilgan issiqlik miqdori bilan aniqlanadi.

$$Q_2 = Q_{31} = \frac{i+2}{2} R\nu(T_2 - T_1). \qquad (4)$$

Endi (1) ni nazarda tutib, A va η lar uchun ifodalarni topamiz

$$A = \frac{i}{2} RT_2\nu\left(1 - \frac{1}{3}\right) + \nu RT_2 \ln\frac{V_{max}}{V_{min}} - \frac{i+2}{2} R\nu T_2\left(1 - \frac{1}{3}\right) =$$

$$= \nu RT_2 \ln\frac{V_{max}}{V_{min}} - \nu RT_2 \cdot \frac{2}{3} = \nu RT_2\left(\ln\frac{V_{max}}{V_{min}} - \frac{2}{3}\right), \qquad (5)$$

$$\eta = \frac{\nu RT_2\left(\ln\frac{V_{max}}{V_{min}} - \frac{2}{3}\right)}{\nu RT_2\left(\frac{i}{3} + \ln\frac{V_{max}}{V_{min}}\right)} = \frac{\ln\frac{V_{max}}{V_{min}} - \frac{2}{3}}{\frac{i}{3} + \ln\frac{V_{max}}{V_{min}}} \qquad (6)$$

Berilganlarni (5) va (6) larga qo'yib hisoblaymiz: ($R - 8{,}31\frac{\text{J}}{\text{K} \cdot \text{mol}}$ – gaz molyar doimiysi)

$$A = 2\cdot 8{,}31\cdot 450(\ln 3 - \frac{2}{3})J = 7479(1{,}1 - 0{,}67)J = 3215{,}97J \approx 3{,}2kJ,$$

$$\eta = \frac{\ln 3 - \frac{2}{3}}{\frac{5}{3} + \ln 3} = \frac{1{,}1 - 0{,}67}{1{,}67 + 1{,}1} = \frac{0{,}43}{2{,}77} = 0{,}16.$$

Javob: A=3,2kJ; η=0,16.

Mustaqil yechish uchun masalalar

1. Gaz aralashmasi, bir xil sharoitlarda va teng hajmlarda olingan xlor va kripton gazlaridan iborat. Aralashmaning solishtirma issiqlik sig'imi C_p aniqlansin. [417 J/kg · K.]

2. 4 g massali karbonat angidrid va 6 g massali kislorod gazlaridan iborat aralashmaning solishtirma issiqlik sig'imlari *C* va *c* lar aniqlansin.

$$\left[839{,}3\frac{J}{kg\cdot K};\quad 1146{,}8\frac{J}{kg\cdot K}.\right]$$

3. Agar aralashmadagi har ikkala gazning ham massaviy ulushlari bir xil va 0,5 ga teng bo'lsa, vodorod va neon aralashmasining adiabata ko'rsatkichi topilsin. [1,42.]

4. Berk idishda 56 g massali azot va 64 g massali kislorod gazlarining aralashmasi bor. Harorati 200 K ga pasaytirilsa, aralashmaning ichki energiyasiga qancha o'zgaradi. [1,66 kJ.]

5. 300 K haroratli, 8 g kislorod egallab turgan hajm izotermik ravishda 4 marta orttirilgan bo'lsa, qanday A ish bajarilgan? [865 J.]

6. 15 l hajmni egallab turgan ikki atomli gaz izoxorik ravishda qizitilishi natijasida, bosimi 50 kPa ga oshdi. Gaz olgan issiqlik miqdori aniqlansin. [1875.]

7. Massasi 0,25 kg bo'lgan 200 K haroratli kislorodni adiabatik ravishda siqish uchun 0,25 kJ ish bajarilgan. Kislorodning keyingi harorati aniqlansin. [354 K.]

8. Vodorod 10 kPa bosimda 10 m^3 hajmni egallaydi. Gazni o'zgarmas hamda 300 kPa bosimgacha qizdirdilar. 1) gazning ichki energiyasining o'zgarishi; 2) gaz bajargan ish; 3) gazgacha berilgan issiqlik miqdori aniqlansin. [1) 0,4MJ; 2) 0; 3) 5MJ.]

9. $8\cdot 10^4$ Pa bosim ostidagi kislorod, izobarik ravishda hajmi uch marta ortganicha qizdirildi. 1) kislorod ichki energiyasining ortishi; 2) kengayishida gaz bajargan ish; 3) gazga berilgan issiqlik miqdori aniqlansin.

[1) 0,4Mт; 2) 160 kJ; 3) 560 kJ].

10. Izobarik jarayonda ideal gazga berilgan issiqlik miqdorining qanday ulushi gaz ichki energiyasining ortishiga va qanday ulushi kengayish ishiga sarflanadi? Uch xil, agar gaz: 1) bir atomli; 2) ikki atomli; 3) uch atomli, bo'lgan hollar qaralsin. [1) 0,6; 0,4 2)071; 0,29 3) 0,75; 0,25.]

11. 2 mol ideal gazni izobarik ravishda 90 K ga qizdirish uchun 2,1 kJ issiqlik miqdori sarflangan. 1) gaz bajargan ish; 2) gaz ichki energiyasining o'zgarishi; 3) shu gaz uchun adiabata ko'rsatkichi aniqlansin. [1) 1,5kJ; 2) 0,6kJ; 3) 1,4.]

12. Adiabatik siqilish natijasida 20 g massali kislorodning ichki energiyasi 8 kJ ga ortdi va harorati 900 K gacha ko'tarildi. 1) haroratning ortishi; 2) agar boshlang'ich bosim 200 K Pa bo'lsa, gazning oxirgi bosimi topilsin. [1) 616K; 2) 11,4 MPa.]

13. Harorati 300K va massasi 1 kg bo'lgan azot $0,5m^3$ hajmni egallaydi. Adiabatik siqilish natijasida gazning bosimi 3 marta ortadi. 1) gazning oxirgi hajmi; 2) uning keyingi harorati, 3) ichki energiyasining o'zgarishi aniqlansin. [1) $0,228m^3$; 2) 411K; 3) 82,4 kJ.]

14. 400 K haroratli azotni adiabatik ravishda kengaytirdilar. Natijada gazning hajmi 5 marta ortdi, ichki energiyasi esa 4 kJ ga kamaydi. Azotning massasi aniqlansin. [28g.]

15. Karno siklida gaz sovitgichga 14 kJ issiqlik miqdori berdi. Agar sovitgichning harorati 280 K bo'lganda, sikldagi foydali ish 6 kJ bo'lsa, isitgichning harorati aniqlansin. [400K.]

16. Ko'p atomli ideal gaz, ikkita izoxora va ikkita izobaradan iborat siklni bajaradi. Shu bilan birga gazning eng katta bosimi eng kichik bosimidan ikki, eng katta hajmi esa eng kichik hajmidan to'rt marta katta. Siklning FIK aniqlansin. [0,11.]

17. Ideal gaz Karno siklini bajaradi. Isitgichning harorati sovitgichning haroratidan to'rt marta yuqori. Gaz bir sikl davomida isitgichdan olinadigan issiqlik miqdorining qanday ulushining sovitgichga beradi? [0,25.]

18. 4 g massali azotning 5 *l* hajmdan to 9 l hajmgacha izobarik kengayishida entropiyaning o'zgarishi topilsin. [2,43J/K.]

19. Ikki atomli 3 mol ideal gazni termodinamik harorati ikki marta ortguncha qizitdilar. Agar qizitish: 1) izoxorik ravishda 2) izobarik ravishda amalga oshirilsa, entropiyaning o'zgarishi qanday bo'ladi?

$$\left[1)28,8\frac{J}{K};\quad 2)40,3\frac{J}{K}.\right]$$

20. 2 mol kislorodni, oldingi hajmi ikki marta ortguncha izobarik ravishda qizitdilar, so'ngra esa bosimi ikki marta kamayguncha izoxorik ravishda sovutdilar. Shu jarayonlar davomida entropiyaning o'zgarishi aniqlansin. [11,5 J/K.]

12-§. Real gazlar, suyuqliklar va qattiq jismlar

Asosiy formulalar

Bir mol real gaz uchun holat tenglamasi – Van-der-Vaals tenglamasi:

$$\left(P+\frac{a}{V_m^2}\right)(V_m-b)=RT.$$

Ixtiyoriy ν mol gaz uchun: $\left(P+\frac{\nu^2 a}{V^2}\right)(V-\nu b)=\nu RT$.

Bu yerda: a va b – Van-der-Vaals doimiylari (bir mol gazga mo'ljallangan), P – gazning idish devorlariga bosimi, V_m – molyar hajmi, V – gaz egallagan hajm.

Molekulalarning o'zaro ta'sir kuchlari vujudga keltiradigan ichki bosim:

$$P'=\frac{a}{V_m^2}, \quad \text{yoki} \quad P'=\nu^2\frac{a}{V^2}.$$

Gazning kritik parametlari – hajm, bosim va haroratning Van-der-Vaals doimiylari a va b lar bilan munosabati:

$$V_{mkr}=3b; \qquad P_{kr}=\frac{a}{27b^2}; \qquad T_{kr}=\frac{8a}{27Rb}.$$

Real gazning ichki energiyasi: $U=\nu(c_V T-\frac{a}{V_m})$,

bunda: C_V – gazning o'zgarmas hajmdagi molyar issiqlik sig'imi.

Sirt tarangligi: $\sigma=\frac{F}{l}$, yoki $\sigma=\frac{\Delta E}{\Delta S}$,

bunda: F – suyuqlik sirt o'rab turgan l konturga ta'sir etadigan sirt taranglik kuchi, ΔE – suyuqlik pardasining yuzasi ΔS ning o'zgarishiga bog'liq bo'lgan parda sirti ichki energiyasining o'zgarishi.

Suyuqlikning sirti ikki xil egrilik radiusiga ega bo'lishi tufayli vujudga keladigan qo'shimcha bosim (Laplas formulasi):

$$P=\sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right),$$

R_1 va R_2 – suyuqlik sirtining ikkita o'zaro tik kesimlarining egrilik radiuslari.

Sferik sirt holida: $P=\frac{2\sigma}{R}$.

Suyuqlikning kapillyar nayda ko'tarilish balandligi: $h=\frac{2\sigma\cdot\cos\theta}{\rho\cdot g\cdot R}$,

bunda: θ – chegaraviy burchak, R – kapillyarning radiusi, ρ – suyuqlikning zichligi, g – erkin tushish tezlanishi.

Kimyoviy sodda bir xil atomlardan tashkil topgan qattiq jismlarning molyar ichki energiyasi:

$$U_m=3RT,$$

bu yerda: R – molyar gaz doimiysi, T – termodinamik harorat.

Sistemaning (jismning) o'zgarmas hajmdagi issiqlik sig'imi:

$$C=\frac{dU}{dT},$$

bunda U – sistemaning (jismning) ichki energiyasi.

Kimyoviy sodda jismlarning molyar issiqlik sig'imi. Dyulong-Pti qonuni

$$C_m=3R.$$

Masala yechishga misollar

1-misol. 10 l sig'imli idishda 0,25 kg massali azot bor:
1) gazning ichki bosimi;
2) molekulalarning xususiy hajmi aniqlansin.

Berilgan:

$V=10l=10^{-2}\text{m}^3$;
$m=0,25\text{kg}$.

$P=?$
$V'=?$

Yechish: Gazning ichki bosimi

$$P'=\nu^2\frac{a}{V^2}=\left(\frac{m}{M}\right)^2\frac{a}{V^2} \quad (1)$$

ifoda yordamida aniqlanadi.

Molekulalarning xususiy hajmini topish uchun, bir moldagi molekulalarning hajmi Van-der-Vaals doimiysi b ning to‘rtdan bir qismiga tengligidan foydalanamiz:

$$V' = \nu\frac{b}{4} = \frac{mb}{4M}. \qquad (2)$$

Azot uchun Van-der-Vaals doimiylari $a = 0{,}135\frac{\text{N}\cdot\text{m}^4}{\text{mol}^2}$, $b = 3{,}86\cdot10^5\frac{\text{m}^3}{\text{mol}}$, $M = 28\cdot10^{-3}\frac{\text{kg}}{\text{mol}}$ – azotning molyar massasi.

Kattaliklarning qiymatlarini (1) va (2) larga qo‘yib hisoblaymiz:

$$\text{P}' = \left(\frac{0{,}25}{28\cdot10^{-3}}\right)^2\frac{0{,}135}{(10^{-2})}\text{Pa} = 79{,}72\cdot0{,}135\cdot10^4\,\text{Pa} = 107{,}6\cdot10^3\,\text{Pa} = 107{,}6\text{kPa}.$$

$$V' = \frac{0{,}25\cdot3{,}86\cdot10^{-5}}{4\cdot28\cdot10^{-3}}\text{m}^3 = 8{,}62\cdot10^{-5}\,\text{m}^3 = 86{,}2\text{sm}^3.$$

Javob: $\text{P}' = 107{,}6\text{kPa}$; $V' = 86{,}2\text{sm}^3$.

2-misol. Modda miqdori 1 mol bo‘lgan kislorod 300 K haroratda hajmni egallagan bo‘lsa, u ko‘rsatadigan bosim aniqlansin. Olingan natija Mendeleyev-Klapeyron formulasi bo‘yicha hisoblangan bosim bilan solishtirilsin.

Berilgan:

$\nu = 1\text{mol}$;

$V = 0{,}5l = 5\cdot10^{-4}\,\text{m}^3$;

$\text{T} = 300\text{K}$.

$P = ?$

Yechish: Real gazning holat tenglamasi –Van-der-Vaals tenglamasini yozamiz:

$$\left(\text{P} + \frac{\nu^2 a}{V^2}\right)(V - \nu b) = \nu RT, \qquad (1)$$

bu yerda: $a = 0{,}133\frac{\text{N}\cdot\text{m}^4}{\text{mol}^2}$, $b = 3{,}17\cdot10^{-5}\frac{\text{m}^3}{\text{mol}}$ – kislorod uchun Van-der-Vaals doimiylari. $R = 8{,}31\frac{\text{J}}{\text{K}\cdot\text{mol}}$ gaz molyar doimiysi.

(1) ifodadan *P* ni topamiz:

$$P = \frac{\nu RT}{(V - \nu b)} - \frac{\nu^2 a}{V^2}. \qquad (2)$$

Endi ideal gaz holat tenglamasi – Mendeleyev–Klapeyron tenglamasini yozamiz:

$$PV = \nu RT. \qquad (3)$$

(3) dan ham *P* ni topamiz: $P = \frac{\nu RT}{V}$.

(2) va(4) larga kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$P = \left(\frac{1 \cdot 8{,}31 \cdot 300}{5 \cdot 10^{4} - 1 \cdot 3{,}17 \cdot 10^{-5}} - \frac{(1)^2 \cdot 0{,}133}{(5 \cdot 10^{-4})^2} \right) \text{Pa} = \left(\frac{24{,}93 \cdot 10^{6}}{5{,}317} - \frac{0{,}133}{25} \cdot 10^{8} \right) \text{Pa} =$$

$$= (5{,}32 - 0{,}53) 10^6 \,\text{Pa} = 4{,}79 \cdot 10^6 \,\text{Pa} = 4{,}79 \text{MPa}.$$

$$P = \frac{1 \cdot 8{,}31 \cdot 300}{5 \cdot 10^{-4}} \text{Pa} = \frac{24{,}93}{5} \cdot 10^6 \,\text{Pa} = 4{,}99 \cdot 10^6 \,\text{Pa} = 4{,}99 \text{MPa}.$$

Javob: Van-der-Vaals tenglamasi bo'yicha *P*=4,79 MPa;
Mendeleev-Klapeyron tenglamasi bo'yicha *P*=4,99 MPa.

3-misol. Agar kiritik harorat 126 K va bosim 3,39 MPa lar ma'lum bo'lsa, azot uchun Van-der-Vaals tenglamasidagi *a* va *b* doimiylari hisoblansin.

Berilgan:
T_{kp}=126 K;
P_{kp}=3,39 MPa=3,39 · 10^6Pa

a=?
b=?

Yechish: Gazning kritik parametrlari T_{kp} va P_{kp} hamda Van-der-Vaals doimiylari *a* va *b* lar orasida quyidagi munosabatlar mavjud:

$$P_{kr} = \frac{a}{27b^2} \qquad (1)$$

va

$$T_{kr} = \frac{8a}{27Rb}. \qquad (2)$$

Bu yerda $R = 8{,}31 \dfrac{\text{J}}{\text{K} \cdot \text{mol}}$ – gaz molyar doimiysi.

(1) tenglamadan a ni topamiz:

$$a = 27 \cdot b^2 \cdot \text{P}_{kr} \tag{3}$$

va uni (2) ga qo'yib, natijadan b ni aniqlaymiz:

$$T_{kr} = \frac{8 \cdot 27 \cdot b^2 \cdot \text{P}_{kr}}{27R \cdot b} = \frac{8b\text{P}_{kr}}{R},$$

$$b = \frac{R \cdot T_{kr}}{8 \cdot \text{P}_{kr}}. \tag{4}$$

b ning bu ifodasini (3) ga qo'yamiz:

$$a = 27 \cdot P_{kr} \left(\frac{R \cdot T_{kr}}{8 \cdot P_{kr}} \right) = \frac{27R^2 \cdot \text{T}^2{}_{kr}}{64P_{kr}}.$$

Kattaliklarning son qiymatlarini qo'yib topamiz:

$$b = \frac{8{,}31 \cdot 126}{8 \cdot 3{,}39 \cdot 10^6} \frac{\text{m}^3}{\text{mol}} = 38{,}6 \cdot 10^{-6} \frac{\text{m}^3}{\text{mol}} = 3{,}86 \cdot 10^{-6} \frac{\text{m}^3}{\text{mol}};$$

$$a = \frac{27(8{,}31)^2(126)^2}{64 \cdot 3{,}39 \cdot 10^6} \frac{\text{N} \cdot \text{m}^4}{\text{mol}^2} = 136435{,}44 \cdot 10^{-6} \frac{\text{N} \cdot \text{m}^4}{\text{mol}^2} = 0{,}136 \frac{\text{N} \cdot \text{m}^4}{\text{mol}^2}.$$

Javob: $a = 0{,}136 \dfrac{\text{N} \cdot \text{m}^4}{\text{mol}^2}$; $b = 3{,}86 \cdot 10^{-6} \dfrac{\text{m}^3}{\text{mol}}$.

4-misol. Agar hajmi V va bosimi P, mos kritik V_{kr} va P_{kr} qiymatlardan k=3 marta ortiq bo'lsa, azot oksidi qanday haroratda turibdi? Azot oksidining kritik harorati T_{kr}=180 K.

Berilgan:

$$\frac{V}{V_{kr}} = \frac{P}{P_{kr}} = k;$$

$k = 3;$

$T_{kr} = 180K.$

$T = ?$

Yechish: Ixtiyoriy miqdordagi real gaz uchun Van-der-Vaals tenglamasini yozamiz

$$\left(P + \frac{\nu^2 a}{V^2}\right)(V - \nu b) = \nu RT \qquad (1)$$

va uni quyidagi parametlar yordamida o'zgartiramiz

$$\pi = \frac{P}{P_{kr}}; \quad \omega = \frac{V_m}{V_{kr}}; \quad \tau = \frac{T}{T_{kr}}. \qquad (2)$$

Unda kritik parametrlar va Van-der-Vaals doimiylari orasidagi munosabatlar quyidagi ko'rinishni oladi:

$$P = \frac{a}{27b^2}\pi; \quad V_m = 3b\omega; \qquad T = \frac{8a}{27Rb}\cdot\tau. \qquad (3)$$

Agar $V = \nu V_m$ ni nazarda tutib, (3) ni (1) ga qo'ysak,

$$\left(\frac{a}{27b^2}\pi + \frac{a\nu^2}{(3\nu b\omega)^2}\right)(3\nu b\omega - \nu b) = \nu R\frac{8a}{27Rb}\tau.$$

Uncha murakkab bo'lmagan soddalashtirishlardan keyin quyidagi Van-der-Vaals tenglamasining keltirilgan shakli hosil bo'ladi:

$$\left(\pi + \frac{3}{\omega^2}\right)(3\omega - 1) = 8\tau. \qquad (4)$$

$\tau = \frac{T}{T_{kr}}$ ligidan $T = \frac{T_{kr}}{8}\left(\pi + \frac{3}{\omega^2}\right)(3\omega - 1)$. (5)

Berilgan masala holida $\pi = k = 3, \omega = k = 3$.

Kattaliklarning son qiymatlarini hisoblaymiz:

$$T = \frac{180}{8}\left(3 + \frac{3}{3^2}\right)(3\cdot 3 - 1)K = \frac{180}{8}(3 + 0{,}33)(9 - 1)K = 599{,}4K.$$

Javob: T=599,4 K.

5-misol. 0,1 kg massali karbonat angidrid gazining hajmi 10^3 *l* dan to 10^4 *l* gacha ortdi. Gazning bu kengayishida molekulalar ichki o'zaro ta'sir kuchlarining ishi topilsin.

Berilgan:
m=0,1kg

$V_1 = 10^3 l = 1\text{m}^3$;

$V_2 = 10^4 l = 10\text{m}^3$.

A=?

Yechish: A ish dastlabki va keyingi holatdagi ichki energiyalarning farqiga teng ya'ni

$$A = U_2 - U_1. \quad (1)$$

Ichki energiya uchun quyidagi ifodalarni yozamiz:

$$U_1 = \frac{m}{M}\left(\frac{5}{2}RT - \frac{a \cdot m}{V_1 M}\right), \quad (2)$$

$$U_2 = \frac{m}{M}\left(\frac{5}{2}RT - \frac{a \cdot m}{V_2 M}\right). \quad (3)$$

(2) va (3) larni (1) ga qo'yamiz:

$$A = U_2 - U_1 = \frac{m}{M}\left(\frac{5}{2}RT - \frac{a \cdot m}{V_2 M}\right) - \frac{m}{M}\left(\frac{5}{2}RT - \frac{a \cdot m}{V_1 M}\right) = \frac{a \cdot m^2}{M^2}\left(\frac{1}{V_1} - \frac{1}{V_2}\right). (4)$$

Bu yerda: $a = 0{,}361\dfrac{\text{N} \cdot \text{m}^4}{\text{mol}^2}$ – karbonat angidrid uchun Van-der-Vaals doimiysi, $M = 44 \cdot 10^{-3}\dfrac{\text{kg}}{\text{mol}}$ – molyar massasi.

O'lchamliklar yordamida (4) ni tekshirib ko'ramiz:

$$[\text{A}] = \frac{[a][m^2]}{[M^2]}\frac{[1]}{[V]} = \frac{1\dfrac{\text{N} \cdot \text{m}^4}{\text{mol}^2} \cdot 1\text{kg}^2}{1\dfrac{\text{kg}^2}{\text{mol}^2} \cdot 1\text{m}^3} = 1\text{N} \cdot m = 1\text{J}.$$

Berilganlarni (4) ga qo'yib hisoblaymiz:

$$A = \frac{(0{,}631 \cdot (0{,}1)^2)}{(44 \cdot 10^{-3})^2}\left(\frac{1}{1} - \frac{1}{10}\right)\text{J} = \frac{0{,}361 \cdot 0{,}9}{1936} \cdot 10^4\,\text{J} = 1{,}68\text{J}.$$

Javob: A=1,68J.

6-misol. Kapillyar naydan oqib chiqayotgan 100 tomchi spirtning massasi 0,1 g. Agar uzilish paytida tomchi bo'yining diametri 1 mm bo'lsa, spirtning sirt tarangligi aniqlansin.

Berilgan:

$n = 100;$

$m = 0{,}71\text{g} = 7{,}1\cdot 10^{-4}\,\text{kg};$

$d = 1\text{mm} = 10^{-3}\,\text{m}.$

$\sigma = ?$

Yechish: Sirt tarangligi

$$\sigma = \frac{F}{l} \qquad (1)$$

ifoda yordamida aniqlanadi.
Bu yerda

$$F = P = m_{\text{T}}g = \left(\frac{m}{n}\right)g \qquad (2)$$

suyuqlik sirti o'rab turgan konturga ta'sir etuvchi kuch. Mazkur holda u bitta tomchining og'irlik kuchiga teng. l – konturning uzunligi

$$l = 2\pi r = \pi d \cdot \qquad (3)$$

(2) va (3) larni (1) ga qo'ysak,

$$\sigma = \frac{\left(\frac{m}{n}\right)g}{\pi d} = \frac{mg}{\pi n d} \qquad (4)$$

hosil bo'ladi. $$[\sigma] = \frac{[m][g]}{[d]} = \frac{1\text{kg}\cdot 1\frac{\text{m}}{\text{s}^2}}{1\text{m}} = 1\frac{\text{N}}{\text{m}}.$$

Kattaliklarning qiymatlarini (4) ga qo'yamiz:

$$\sigma = \frac{7{,}1\cdot 10^{-4}\cdot 9{,}8}{3{,}14\cdot 100\cdot 10^{-3}}\frac{\text{N}}{\text{m}} = \frac{7{,}1\cdot 9{,}8}{3{,}14}\cdot 10^{-3}\frac{\text{N}}{\text{m}} = 22{,}16\cdot 10^{-3}\frac{\text{N}}{\text{m}} = 22{,}16\frac{\text{mN}}{\text{m}}.$$

Javob: $\sigma = 22{,}16\dfrac{\text{mN}}{\text{m}}.$

7-misol. Sovun pufagining diametri 1 sm dan 11 sm gacha ortishi uchun qanday A ish bajarishi kerak? Jarayon izotermik deb hisoblansin.

Berilgan:
d_1=1sm=10^{-2}m;
d_2=11sm=$11 \cdot 10^{-2}$m.

A=?

Yechish: Pufakning diametrini orttirishda bajarilgan ish pufakning dastlabki va keyingi erkin energiyalarining farqiga teng:

$$A = E_2 - E_1. \qquad (1)$$

Ma'lumki, suyuqlik sirtining erkin energiyasi $E = \sigma \cdot S$ ko'paytma yordamida aniqlanadi.

Lekin sovun pufagining ikkita: tashqi va ichki sirtlari borligi va pufakning qalinligi kichik bo'lganligi tufayli bu sirtning erkin energiyasi $E = 2\sigma S$ ifoda yordamida aniqlanadi, ya'ni

$$A = 2\sigma S_2 - 2\sigma S_1, \qquad (2)$$

bu yerda σ – sirt taranglik, sovunli suv uchun $\sigma = 4 \cdot 10^{-2} \frac{\text{N}}{\text{m}}$.

Agar $S = 4\pi r^2 = \pi d^2$ ni nazarda tutsak,

$$A = 2\delta\pi d_2^2 - 2\delta\pi d_1^2 = 2\delta\pi\left(d_2^2 - d_1^2\right). \qquad (3)$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$A = 2 \cdot 4 \cdot 10^{-2} \cdot 3{,}14(121 \cdot 10^{-4} - 10^{-4})\text{J} = 8 \cdot 3{,}14 \cdot 120 \cdot 10^{-6} \cdot \text{J} = 3 \cdot 10^{-3}\,\text{J} = 3\text{mJ}.$$

Javob: A=3mJ.

8-misol. Har birining radiusi 1mm dan bo'lgan ikkita simob tomchisi, bitta katta tomchiga birlashishadi. Bu qo'shilishda qanday energiya ajraladi? Jarayon izotermik hisoblansin.

Berilgan:
n=2;
$r = 1\text{mm} = 10^{-3}\text{m}$.

ΔE=?

Yechish: Parda sirti ichki energiyasining o'zgarishi quyidagi ifoda yordamida aniqlanadi:

$$\Delta E = \sigma \cdot ES, \qquad (1)$$

bunda $\sigma = 0{,}5\frac{M}{m}$ simobning sirt tarangligi.

Jarayon izotermik bo'lgani uchun u o'zgarmay qoladi. ΔS – simob pardasi yuzasining o'zgarishi

$$\Delta S = nS_0 - S_1; \qquad (2)$$

S_0 – dastlabki tomchilarning har birining sirtining yuzasi, S_1 – katta tomchi sirtining yuzasi.

$$S_0 = 4\pi r_0^2, \qquad (3)$$

$$S_1 = 4\pi r_1^2. \qquad (4)$$

Katta tomchining radiusini topish uchun dastlabki va keyingi massalarning tengligidan foydalanamiz *nm=M*.

Agar tomchini shar shakliga ega deb olsak unda:

$$n\frac{4}{3}\pi r_0^3 \cdot \rho = \frac{4}{3}\pi_1^3 \cdot \rho, \qquad (5)$$

bu yerda: ρ – simobning zichligi va $m = \rho \cdot V$ dan foydalandik. (5) dan $r_1 = \sqrt[3]{n} \cdot r_0$. (6)

(6) ni (4) ga qo'ysak, $S_1 = 4\pi (n)^{2/3} r_0^2$ (7)

ni hosil qilamiz. Unda ΔS uchun quyidagini olamiz:

$$\Delta S = n \cdot 4\pi r_0^2 - 4\pi (n)^{2/3} r_0^2 = 4\pi r_0^2 (n - (n)^{2/3}) = 4\pi n r_0^2 (1 - (n)^{-\frac{1}{3}}). \qquad (8)$$

(8) yordamida (1) ni qayta yozsak $\Delta E = 4\pi n r_0^2 \sigma \left(1 - \frac{1}{\sqrt[3]{n}}\right)$. (9)

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$\Delta E = 4 \cdot 3{,}14 \cdot 2 \cdot (10^{-3})^2 \cdot 0{,}5\left(1 - \frac{1}{\sqrt[3]{2}}\right)\text{J} = 12{,}56(1 - 0{,}79) \cdot 10^{-6}\,\text{J} =$$

$$= 2{,}64 \cdot 10^{-6}\,\text{J} = 2{,}64\text{mkJ}.$$

Javob: $\Delta E = 2{,}64$mkJ.

9-misol. 10^{25} ta klassik, uch o'lchamli, erkli garmonik ossilyatorlardan tashkil topgan sistemaning energiyasi *U* va issiqlik sig'imi *C* aniqlansin. Harorat 300 K.

Berilgan:

$N = 10^{25}$;

$T = 300K$.

$U = ?$

$C = ?$

Yechish: Energiyaning erkinlik darajalari bo'yicha tekis taqsimlanish qonuniga muvofiq, bir o'lchamli garmonik ossilyator kT energiyaga ega (kinetik va potensial energiyalarning yig'indisi). Uch o'lchamli erkli garmonik ossilyatorlarning o'rtacha energiyasi:

$$< \varepsilon >= 3kT \cdot \qquad (1)$$

N ta ossilyatordan tashkil topgan sistemaning energiyasi esa

$$U = N \cdot < \varepsilon >= 3kT \cdot N \cdot \qquad (2)$$

Issiqlik sig'imi esa

$$C = \frac{dU}{dT} = 3kT, \qquad (3)$$

bu yerda k=1,38 · $10^{-23} J/K$ — Bolsman doimiysi.

Kattaliklarning son qiymatlarini (2) va (3) larga qo'yib hisoblaymiz:

$$U = 3 \cdot 1{,}38 \cdot 10^{-23} \cdot 300 \cdot 10^{25} \text{J} = 9 \cdot 1{,}38 \cdot 10^{4} \text{J} = 12{,}42 \cdot 10^{4} \text{J} = 124{,}2\text{kJ}.$$

$$C = 3 \cdot 1{,}38 \cdot 10^{-23} \cdot 10^{25} \frac{\text{J}}{\text{K}} = 3 \cdot 1{,}38 \cdot 10^{2} \frac{\text{J}}{\text{K}} = 4{,}14 \cdot 10^{2} \frac{\text{J}}{\text{K}} = 414 \frac{\text{J}}{\text{K}} \cdot$$

Javob: $U = 124{,}2\text{kJ}$; $C = 414 \frac{\text{J}}{\text{K}}$.

Mustaqil yechish uchun masalalar

1. 4 l hajmli idishda 220 g karbonat angidrid saqlanadi. 1) gazning ichki bosimi, 2) molekulalarning xususiy hajmi aniqlansin. [564,1 kPa, 2) 53,5 sm^3.]

2. 290 K haroratli, 2,1 kg massali azot hajmi 20 l bo'lgan idishda turibdi. Gazning bosimi Van-der-Vaals va Mendeleyev — Klapeyron formulalari bo'yicha hisoblansin. [8,67 MPa va 9,04 MPa.]

3. Modda miqdori 150 mol bo'lgan karbonat angidrid gazi 1000 kPa bosim ostida, 3,75 m^3 hajmli idishda saqlanmoqda. Gazning harorati aniqlansin. [302K.]

4. Hajmi V: 1) 2 *l*; 2) 20 *l*; 3) 200 *l* bo'lgan idishdagi m=1,6 kg massali, harorati t=350 K bo'lgan kislorodning bosimi aniqlansin. [10253,2 MPa; 2) 7,1 MPa; 3) 0,65 MPa.]

5. 1) azot uchun; 2) xlor uchun; 3) kislorod uchun; 4) suv uchun kritik bosim P_{kr}, kritik hajm V_{mkr} va kritik harorat T_{kr} lar aniqlansin.

$\left[1)3{,}39\,\text{MPa};\ 11{,}58\cdot10^{-5}\dfrac{\text{m}^3}{\text{mol}};\ 126\,\text{K};\quad 2)7{,}71\,\text{MPa};\ 16{,}86\cdot10^{-5}\dfrac{\text{m}^3}{\text{mol}};\ 417\,\text{K};\right.$

$\left.3)5{,}08\,\text{MPa};\ 9{,}51\cdot10^{-5}\dfrac{\text{m}^3}{\text{mol}};\ 155\,\text{K};\ 4)22{,}1\,\text{MPa};\ 9{,}12\cdot10^{-5}\dfrac{\text{m}^3}{\text{mol}};\ 647\,\text{K}\right]$

6. 1) 2 kg massali suvning, 2) 1kg massali azotning kritik hajmlari topilsin. [1)10 sm^3; 2)9,1 sm^3].

7. Massasi 28 g, kritik harorati 126 K bo‘lgan azotning 1) 20 l: 2) 2l; 3) 0,2 *l* hajmlar holi uchun ichki energiyasi *U* aniqlansin. [1) 2,61 kJ: 2) 2,56 kJ: 3) 1,94 kJ.]

8. 200 g suv bug‘ining hajmi 10 l dan 7 *l* gacha kamaydi. Hajmning bu kamayishida molekulalarning o‘zaro tortishish kuchlariga qarshi qancha ish bajarilganligi aniqlansin. [2,88 kJ.]

9. 84 g azot vakuumda kengayib hajmini 1*l* dan 5*l* gacha orttirdi. Azotning harorati oldingidek qolaverishi uchun unga qancha issiqlik miqdori berilishi kerak? [972 J.]

10. Azotning vakuumda adiabatik ravishda kengayishi haroratini $\Delta T = 1\,\text{K}$ ga pasaytirdi. Molekulalararo tortishish kuchlariga qarshi bajarilgan ish aniqlansin. Azotning massasi 56 g. [83,1 J.]

11. Ingichka nayning quyi uchida osilib turgan, uzilish oldidan shar shakliga ega bo‘lgan suv tomchisining diametri 4,4 mm bo‘lsa, nayning ichki radiusi topilsin. [1 mm.]

12. Diametri 6 mm bo‘lgan sovun pufagini izotermik ravishda kattalashtirish uchun 0,9 m J ish bajarilgan. Kattalashtirilgan pufakning diametri aniqlansin. [6 sm.]

13. Har birining radiuslari 1 mm dan bo‘lgan, 2 tomchi birlashishi natijasida 2,64 mkJ energiya ajralib chiqadi. Suyuqlikning turi aniqlansin. Jarayon izotermik deb hisoblansin. [Simob.]

14. Sovun pufagining ichidagi bosim atmosfera bosimidan 200 Pa ga katta. Pufakning diametri aniqlansin. [1,6 mm.].

15. Ichki diametrlari mos ravishda 0,5 mm va 1 mm bo‘lgan ikkita kapilyar nay tik ravishda spirtga tushirilgan. Naylardagi suyuqlik sathlarining farqi topilsin. [11,6 mm.]

16. Sovunli suvga tik tushirilgan kapilyarga 25,6 mg suyuqlik kirgan. Kapilyarning ichki radiusi qancha? [1 mm.]

17. Dyulong-Pti qonunidan foydalanib: 1) temirning; 2) oltinning; 3)alyuminiyning solishtirma issiqlik sig‘imlari aniqlansin. [445 J/(kg · K); 2) 126,5J/(kg · K); 3) 924J/(kg · K).]

18. Massasi 0,2 kg, harorati 290 K bo‘lgan mis parchasini qizitish uchun 4,7 kJ issiqlik miqdori sarflangan. Dyulong-Pti qonunidan foydalanib mis parchasining keyingi harorati aniqlansin. [350 K.]

13-§. Suyuqliklar va gazlar mexanikasi

Asosiy formulalar

h chuqurlikda suyuqlik sathi ko'rsatadigan gidrostatik bosim:

$$P = \rho_s gh,$$

bunda: ρ_s – suyuqlikning zichligi, g – erkin tushish tezlanishi.

Arximed kuchi, suyuqlikka botirilgan jismni itarib chiqaruvchi kuch:

$$F_A = \rho_S gV,$$

bunda V – siqib chiqarilgan suyuqlikning hajmi.

Naydagi suyuqlik oqimi uchun uzluksizlik tenglamasi:

$$\vartheta_1 S_1 = \vartheta_2 S_2, \text{ yoki } S\vartheta = \text{const},$$

bunda: S_1 va S_2 – oqim nayidagi ikki joyning ko'ndalang kesim yuzalari, ϑ_1 va ϑ_2 – mos oqim tezliklari.

Ideal siqilmaydigan suyuqlikning statsionar oqimi uchun Bernulli tenglamasining umumiy ko'rinishi:

$$\frac{\rho\vartheta^2}{2} + \rho gh + P = \text{const},$$

bunda: P – suyuqlik oqimining nayning ma'lum ko'ndalang kesimiga ko'rsatadigan statik bosimi, $\rho\vartheta^2/2$ – suyuqlikning shu kesimga dinamik bosimi, h – shu kesim joylashgan balandlik, ϑ – suyuqlikning shu kesimdagi tezligi.

Gorizontal joylashgan oqim nayi holida:

$$\frac{\rho\vartheta^2}{2} + P = \text{const}.$$

Suyuqlikning, ochiq keng idishning kichkina teshigidan oqish tezligi, Torichelli formulasi

$$\vartheta = \sqrt{2gh},$$

bu yerda h – idishdagi suyuqlik sathiga nisbatan teshikning joylashish chuqurligi.

Uzun naydan t vaqtda oqadigan suyuqlikning (gazning) hajmi – Puazeyl formulası:

$$V = \frac{\pi r^4 \cdot t\Delta P}{8l\eta},$$

bunda: r – nay radiusi, l – uning uzunligi, ΔP – nay uchlaridagi bosimlar farqi; η – suyuqlikning dinamik qovushqoqligi (ichki ishqalanish koeffitsienti).

Oqayotgan suyuqlik qatlamlari orasidagi ichki ishqalanish kuchi:

$$F = \eta\left|\frac{\Delta\vartheta}{\Delta x}\right|S,$$

bunda: $\frac{\Delta\vartheta}{\Delta x}$ – tezlik gradienti, S – tegib turuvchi qatlamlarning yuzasi.

Suyuqlikning harakat xarakterini aniqlovchi Reynolds soni:

$$\mathrm{Re} = \rho < \vartheta > \frac{d}{\eta},$$

bunda: ρ – suyuqlikning zichligi, $< \vartheta >$ – suyuqlikning nay kesimi bo'yicha o'rtacha oqim tezligi; d – nayning diametri.

Sharchaning suyuqlikdagi harakati uchun

$$\mathrm{Re} = \rho\frac{\vartheta d}{\eta},$$

bunda: ϑ – sharchaning tezligi; d – uning diametri.

Umumiy holda

$$\mathrm{Re} = f(\rho,\eta,l,v).$$

Re<Re_{kr} da oqim laminar, Re<Re_{kr} da suyuqlikning oqimi turbulentga o'tadi.

Sharning suyuqlikdagi harakati uchun Re_{kr}=0,5; uzun naydagi suyuqlik oqimi uchun Re_{kr}=2300.

Yopishqoq muhitda sekin harakatlanayotgan sharchaga ta'sir etuvchi qarshilik kuchini aniqlovchi formula – Stoks formulasi:

$$F = 6\pi\eta r\vartheta,$$

bunda: r – sharcha radiusi, ϑ – uning tezligi.

Peshona qarshilik:

$$R_p = S_p \frac{\rho \vartheta^2}{2} S,$$

bu yerda: S_p — qarshilikning o'lchamsiz koeffitsienti, ρ — muhitning zichligi, ϑ — jismning harakat tezligi, S — jismning eng katta ko'ndalang kesimining yuzasi.

Ko'tarish kuchi:

$$R_k = S_p \frac{\rho \vartheta^2}{2} S_k,$$

bunda: S_k — ko'tarish kuchining o'lchamsiz koeffitsienti.

Masala yechishga misollar

1-misol. Kovak mis sharning havodagi og'irligi 3 N, suvda esa 2 N. Havoning itarish kuchini e'tiborga olmay, shar ichki bo'shlig'ining hajmi aniqlansin.

Berilgan:	**Yechish:** Shar ichki bo'shlig'ining hajmi V_i
$P = 3N$;	sharning hajmi V_{sh} dan mis materialining
$P' = 2N$.	hajmi V_m ni olinganiga teng:
$V_u = ?$	$V_i = V_{sh} - V_m$. (1)

Sharning havodagi og'irlik kuchini quyidagidek aniqlash mumkin. (30-rasm)

$$P = mg = \rho_m \cdot V_m \cdot g.$$

Bunda

$$V_m = \frac{P}{\rho_m \cdot g}, \qquad (2)$$

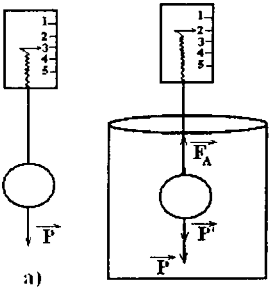
bu yerda: $\rho_m = 8930 \, kg/m^3$ — misning zichligi, $g = 9{,}8 \, m/s^2$ — erkin tushish tezlanishi.

Sharning suvdagi og'irlik kuchi esa

$$P' = P - F_A \qquad (3)$$

kabi aniqlanadi (30-rasm). Bu yerda $F_A = \rho_s V_{sh} \cdot g$ — Arximed kuchi, $\rho_s = 1000 \, {kg}/{m^3}$ — suvning zichligi. (3) dan

$$F_A = P - P', \quad \text{yoki} \quad \rho_s V_{sh} g = P - P'$$



30-rasm

Bundan esa

$$V_{sh} = \frac{P - P'}{\rho_s \cdot g}. \qquad (4)$$

(2) va (4) ni (1) ga qo'ysak,

$$V_t = \frac{P - P'}{\rho_s \cdot g} - \frac{P}{\rho_m g} \qquad (5)$$

hosil bo'lgan ifodani o'lchamliklar yordamida tekshirib ko'ramiz:

$$[V] = \frac{[p]}{[\rho][g]} = \frac{1\text{N}}{1 \, {kg}/{m^3} \cdot 1 \, {m}/{s^2}} = 1 \frac{kg \frac{m}{s^2}}{kg \frac{m}{s^2}} \cdot m^3 = 1 m^3$$

va kattaliklarning son qiymatlarini qo'yamiz:

$$V_t = \left(\frac{3-2}{1000 \cdot 9{,}8} - \frac{3}{8930 \cdot 9{,}8} \right) m^3 \approx \left(10^{-4} - 0{,}34 \cdot 10^{-4}\right) m^3 = 0{,}66 \cdot 10^{-4} m^3 = 66 sm^3.$$

Javob: $V_t = 66 sm^3$.

3-misol. O'zgaruvchan kesimli, gorizontal joylashgan quvurdan suv oqadi. Quvurning keng qismida suvning tezligi $20 \, {sm}/{s}$. Quvurning keng qismining diametri d_1 tor qismining diametri d_2 dan 1,5 marta katta bo'lsa, suvning, quvurning tor qismidagi tezligi ϑ_2 aniqlansin.

Berilgan:

$\vartheta_1 = 20\frac{sm}{s} = 0{,}2\,{}^{m}\!/\!{}_{s}$;

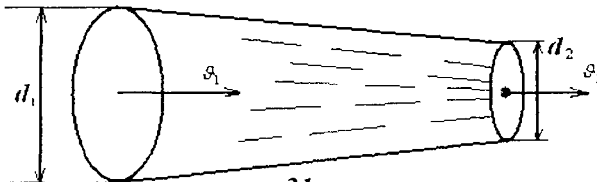
$\frac{d_1}{d_2} = 1{,}5.$

$\vartheta_2 = ?$

Yechish: Quvurdagi suyuqlik oqimi uchun uzluksiz tenglamasini yozamiz:

$$\vartheta_1 S_1 = \vartheta_2 S_2 . \qquad (1)$$

Bunda: S_1 va S_2 – oqim nayidagi ikki joyning ko'ndalang kesim yuzalari, ϑ_1 va ϑ_2 – mos oqim tezliklari. (1) dan



31-rasm

$$\vartheta_2 = \frac{S_1}{S_2}\vartheta_1 . \qquad (2)$$

Oqim nayining ko'ndalang kesim yuzalarini quyidagicha aniqlaymiz:

$$S_1 = \pi\left(\frac{d_1}{2}\right)^2, \qquad S_2 = \pi\left(\frac{d_2}{2}\right)^2 . \qquad (3)$$

(3) ni (2) ga qo'yamiz:

$$\vartheta_2 = \frac{\pi\left(\frac{d_1}{2}\right)^2}{\pi\left(\frac{d_2}{2}\right)^2}\vartheta_1 = \left(\frac{d_1}{d_2}\right)^2\vartheta_1 . \qquad (4)$$

Kattaliklarning son qiymatlarini qo'ysak,

$$\vartheta_2 = (1{,}5)^2 \cdot 0{,}2\text{m/s} = 2{,}25 \cdot 0{,}2\text{m/s} = 0{,}45\text{m/s}.$$

Javob: $\vartheta_2 = 0{,}45\text{m/s}$.

4-misol. Gorizontal joylashgan quvurning keng qismida neftning oqish tezligi 8ms. Agar quvurning keng va tor qismlaridagi statik bosimlar farqi 6,65 kPa bo'lsa, quvurning tor qismida neftning oqish tezligi ϑ_2 aniqlansin.

Berilgan:

$\vartheta_1 = 2\text{m/s}$;

$\Delta p = 6{,}65\text{kPa} = 6{,}65 \cdot 10^3\,\text{Pa}$.

$\vartheta_2 = ?$

Yechish: Gorizontal joylashgan quvurdagi neft oqimi uchun Bernulli tenglamasini yozamiz:

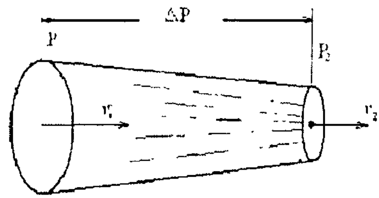
$$p_1 + \frac{\rho\vartheta_1^2}{2} = p_2 + \frac{\rho\vartheta_2^2}{2}. \quad (1)$$

Bu yerda $\rho = 900\text{kg/m}^3$ – neftning zichligi. Bundan

$$P_1 - P_2 + \frac{\rho\vartheta_1^2}{2} = \frac{\rho\vartheta_2^2}{2} \text{ yoki } \frac{\rho\vartheta_2^2}{2} = \Delta P + \frac{\rho\vartheta_1^2}{2}. \quad (2)$$

ϑ_2 ni aniqlasak,

$$\vartheta_2 = \sqrt{\frac{2 \cdot \Delta P}{\rho} + \vartheta_1^2}. \quad (3)$$



32-rasm

Kattaliklarning son qiymatlarini qo'ysak,

$$\vartheta_2 = \sqrt{\frac{2 \cdot 6{,}65 \cdot 10^3}{900} + 4}\,\frac{\text{m}}{\text{s}} = \sqrt{14{,}78 + 4}\,\frac{\text{m}}{\text{s}} = 4{,}3\,\frac{\text{m}}{\text{s}}.$$

Javob: $\vartheta_2 = 4{,}3\,\frac{\text{m}}{\text{s}}$.

5-misol. Balandligi 1,5 m bo'lgan bak suv bilan limmo-lim qilib to'ldirilgan. Bakning yuqori chegarasidan 1 m masofada kichik diametrli teshik hosil bo'ladi. Teshikdan chiqadigan suv oqimi polga bakdan qanday l masofada tushadi?

Berilgan:
H=1,5 *m*;
h=1 *m*.

---

l=?

Yechish: Suvning oqimi uchun Bernulli tenglamasini yozamiz:

$$\frac{\rho\vartheta_1^2}{2} + \rho g H + p_0 = \frac{\rho\vartheta_2^2}{2} + \rho g(H-h) + p_0, \qquad (1)$$

bu yerda: ρ – suvning zichligi, g – erkin tushish tezlanishi. Boshqa kattaliklar 33-rasmda ko'rsatilgan. Statik bosim p_0 bir xilligidan olamiz.

$$\frac{\rho}{2}\left(\vartheta_2^2 - \vartheta_1^2\right) = \rho g(H - H + h) = \rho g h.$$

Agar teshikning kichikligini nazarda tutsak, $\vartheta_2 >> \vartheta_1$ ni olamiz.
Demak,

$$\vartheta_2 = \sqrt{2gh}. \qquad (2)$$

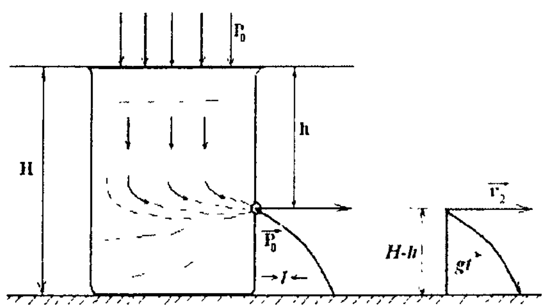
Torrichelli formulasi hosil bo'ladi.
33-rasmdan ko'rinib turibdiki,

$$H - h = \frac{gt^2}{2}. \qquad (3)$$

Bu ifodadan suvning tushish vaqti t ni topsak,

$$t = \sqrt{\frac{2(H-h)}{g}}. \qquad (4)$$

(2) va (4) lar yordamida masofani quyidagicha aniqlaymiz:



33-rasm

$$l = \vartheta_2 \cdot t = \sqrt{2gh} \cdot \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}.$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$l = 2\sqrt{1 \cdot (1{,}5 - 1)}\,\text{m} = 2 \cdot \sqrt{0{,}5}\,\text{m} = 1{,}41\,\text{m}.$$

Javob: $l = 1{,}41\text{m}$.

6-misol. Idishning yon sirtiga ichki diametri 2 mm va uzunligi 1,2 sm bo'lgan gorizontal kapillyar qo'yilgan. Kapillyardan kanakunjut moyi oqib chiqmoqda. Moyning sathi kapillyardan 30 sm balandlikda o'zgarmas qilib saqlanadi. Kapillyardan 10 sm^3 moy oqib chiqishi uchun qancha vaqt ketishi aniqlansin.

Berilgan:

$d = 2mm = 2 \cdot 10^{-3} m;$

$l = 1{,}2sm = 1{,}2 \cdot 10^{-2} m;$

$h = 30sm = 0{,}3m;$

$V = 10sm^3 = 10^{-5} m^3.$

$t = ?$

Yechish: Kapillyardan oqib chiqadigan moyning hajmi Puazeyl formulasi yordamida aniqlanadi:

$$V = \frac{\pi r^4 t \cdot \Delta P}{8l\eta}. \qquad (1)$$

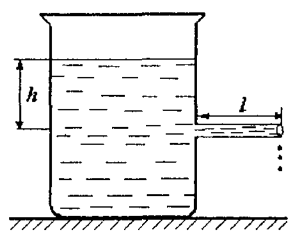
Bu formuladan suyuqlikning oqib chiqish vaqti t ni aniqlasak

$$t = \frac{8l\eta V}{\pi r^4 \cdot \Delta P}, \qquad (2)$$

bunda: $\eta = 0{,}99 Pa \cdot s$ – kanakunjut moyining dinamik qovushqoqligi, ΔP – kapillar nay uchlaridagi bosimlar farqi. Bu bosimlar farqi h chuqurlikdagi suyuqlik sathiga ko'rsatiladigan gidrostatik bosimga teng, ya'ni

$$\Delta P = \rho g h, \qquad (3)$$

bu yerda: $\rho = 960 \, kg/m^3$ – kanakunjut moyining zichligi, $g = 9{,}8 \, m/s^2$ – erkin tushish tezlanishi.



34-rasm

(3) ni (2) ga qo'ysak,

$$t = \frac{8l\eta V}{\pi r^4 \cdot \rho g h} \qquad (4)$$

ni hosil qilamiz. Kattaliklarning o'lchamliklari yordamida (4) ni tekshirib ko'ramiz:

$$[t]=\frac{[l][\eta][V]}{[r^4][\rho][g][h]}=\frac{1\text{m}\cdot 1\text{Pa}\cdot\text{s}\cdot 1\text{m}^3}{1\text{m}^4\cdot 1\frac{\text{kg}}{\text{m}^3}\cdot 1\frac{\text{m}}{\text{s}^2}\cdot 1\text{m}}=1\frac{\text{m}\cdot\text{s}^3}{\text{kg}}\cdot\frac{\text{N}}{\text{m}^2}=1\text{c}\frac{\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}=1\text{s}.$$

va uning to‘g‘riligiga ishonch hosil qilgandan keyin son qiymatlarini qo‘yamiz

$$t=\frac{8\cdot 1{,}2\cdot 10^{-2}\cdot 0{,}99\cdot 10^{-5}}{3{,}14\cdot(1\cdot 10^{-3})^4\cdot 960\cdot 9{,}8\cdot 0{,}3}\text{s}=\frac{9{,}5\cdot 10^{-7}}{8{,}86\cdot 10^{-9}}\text{s}=1{,}07\cdot 10^2\text{s}=107\text{s}\ .$$

Javob: t=107 s.

9-misol. *12 g* massali, havoda erkin tushayotgan qo‘rg‘oshin sharcha olishi mumkin bo‘lgan eng katta tezlik aniqlansin. S_k koeffitsienti 0,5 ga teng deb olinsin.

Berilgan:

$m=12\text{g}=12\cdot 10^{-3}\text{kg};$

$S_k=0{,}5.$

$\vartheta=?$

Yechish: Havoda erkin tushayotgan sharchaga ikkita: og‘irlik kuchi:

$$P=mg, \qquad (1)$$

va qarshilik kuchi

$$R_k=S_k\frac{\rho_h\cdot\vartheta^2}{2}S \qquad (2)$$

ta’sir qiladi. Bu yerda: $g=9{,}8\,\text{m/s}^2$ – erkin tushish tezlanishi, $\rho_h=1{,}29\,\text{kg/m}^3$ – havoning zichligi, $S=\pi r^2$ – sharcha ko‘ndalang kesimining yuzasi, r – sharcha radiusi.

Sharcha eng katta tezligiga

$$P=R_k \qquad (3)$$

shart bajarilganda erishadi va so‘ngra o‘zgarmas tezlik bilan harakatlana boshlaydi. Demak,

$$mg=S_k\frac{\rho_x\vartheta^2}{2}\pi r^2.$$

Bundan, $\vartheta^2 = \dfrac{2m \cdot g}{\pi r^2 \rho_h S_k}$. (4)

Sharchaning radiusini quyidagidek topamiz:

$$m = \rho_q \cdot V = \frac{4}{3}\pi r^3 \cdot \rho_q,$$

bu yerda: $\rho_q = 11300\,\text{kg/m}^3$ – qurg'oshinning zichligi.

Bundan $r = \left(\dfrac{3m}{4\pi\rho_q}\right)^{1/3}$ (5)

Radius uchun topilgan bu ifodani (4) ga qo'yib ϑ ni aniqlaymiz.

$$\vartheta = \left(\frac{2mg}{\pi\rho_h S_k}\right)^{1/2}\left(\frac{4\cdot\pi\rho_q}{3m}\right)^{1/3} \quad (6)$$

Ifodani kattaliklarning o'lchamliklari yordamida tekshirib ko'ramiz:

$$[\vartheta] = \left(\frac{[m][g]}{[\rho]}\right)^{1/2}\left(\frac{[\rho]}{[m]}\right)^{1/3} = \left(\frac{1\text{kg}\cdot 1\text{m/s}^2}{1\text{kg/m}^3}\right)^{1/2}\left(\frac{1\text{kg/m}^3}{1\text{kg}}\right)^{1/3} =$$

$$= \left(1\frac{\text{m}^4}{\text{s}^2}\right)^{1/2}\left(1\frac{1}{\text{m}^3}\right)^{1/3} = 1\frac{\text{m}^2}{\text{s}}\frac{1}{\text{m}} = 1\frac{\text{m}}{\text{s}};$$

$$\vartheta = \left(\frac{2\cdot 12\cdot 10^{-3}\cdot 9{,}8}{3{,}14\cdot 1{,}29\cdot 0{,}5}\right)^{1/2}\left(\frac{4\cdot 3{,}14\cdot 11300}{3\cdot 12\cdot 10^{-3}}\right)^{1/3}\frac{\text{m}}{\text{s}} =$$

$$= (0{,}12)^{1/2}\cdot\left(3{,}94\cdot 10^6\right)^{1/3}\frac{\text{m}}{\text{s}} = 0{,}34\cdot 158\frac{\text{m}}{\text{s}} = 53{,}72\frac{\text{m}}{\text{s}}$$

Javob: $\vartheta = 53{,}72\dfrac{\text{m}}{\text{s}}$.

Mustaqil yechish uchun masalalar

1. Ichki bo'shlig'ining hajmi 150 sm^3 bo'lgan kovak alyuminiy sharning havodagi og'irligi 4 N. Sharning suvdagi og'irligi aniqlansin. [1N.]

2. Radiusi 2 sm bo'lgan quvurdan 20sm/s tezlik bilan glitserin oqib chiqadi. 200 kg glitserin sig'adigan idishni to'ldirish uchun qancha vaqt kutish kerak? [10,5 min.]

3. Gorizontal joylashgan, kesim yuzasi o'zgaruvchan quvurning tor qismidagi suvning oqim tezligi J_1, keng qismidagi oqim tezligi J_2 dan n=3 marta katta bo'lsa, quvurning keng qismining diametri d_2 tor qismining diametri d_1 dan necha marta katta bo'ladi? [1,73 marta.]

4. Agar oldingi masalada J_2=50 sm/s bo'lsa, quvurning tor va keng qismlarida bosimlar farqi ΔP aniqlansin. [1000 Pa.]

5. Bakda to'la bo'lmagan holda suv saqlanadi. Ma'lum vaqtdan so'ng idish devori poldan 1m balandlikdan teshilib suv otilib ketdi va polga 1,8m masofada tushdi. Idishdagi suv sathining balandligi topilsin. Bak devorlarining qalinligi hisobga olinmasin. [1,81 m.]

6. Idishning devorida hosil bo'lgan teshikdan suv 4 m/s tezlik bilan otilib chiqadi. Teshik suv sathidan qancha pastda hosil bo'lgan? [82 sm.]

7. Quvurdan mashina yog'i oqmoqda. Yog'ning bu quvurdagi harakati laminar bo'lib qoladigan maksimal tezlik 3,2 sm/s. Shu quvurning o'zida, qanday V tezlikda glitslrinning harakati laminarlikdan turbulentlikka o'tadi? [1,94 sm/s.]

8. Glitserin bilan to'ldirilgan keng idishda 1 sm/s o'zgarmas tezlik bilan 1 mm radiusli po'lat sharcha tushmoqda. Glitserinning dinamik qovushqoqligi topilsin. [1,48 $Pa \cdot s$.]

9. Diametri 0,8sm bo'lgan sharcha kanakunjut moyida, o'zgarmas tezlik bilan tushmoqda. Sharchaning suyuqlikdagi harakatini xarakterlovchi Reynolds soni 1,9. Sharchaning zichligi va qanday materialdan yasalganligi aniqlansin. [7900 kg/m^3; temir.]

10. Havoda (10sm/s tezlik bilan harakatlanayotgan, diametri 50 sm bo'lgan sharchaga qanday qarshilik kuchi ta'sir qiladi? Qarshilikning o'lchamsiz koeffitsienti S_k=0,25 deb olinsin. [4,9 N.]

11. Havoda erkin tushayotgan po'lat sharcha erishadigan eng katta tezlik 55 m/s ga teng. S_k =0,5 deb hisoblab sharchaning massasi aniqlansin. [33 g.]

III BOB. ELEKTROSTATIKA

14-§. Elektrostatika elementlari

Asosiy formulalar

Kulon qonuni: $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 \cdot Q_2}{r^2}$, bunda F – ikkita nuqtaviy zaryad

Q_1 va Q_2 larning ta'sir kuchi; r – ular orasidagi masofa,

$$\varepsilon_0 = \frac{1}{4\pi \cdot 9 \cdot 10^9} \mathrm{F/m} = 8{,}85 \cdot 10^{-12}\,\mathrm{F/m}.$$

– elektr doimiysi.

Elektr zaryadining saqlanish qonuni:

$$\sum_{i=1}^{n} Q_i = \mathrm{const},$$

n – zaryadlar soni.

Elektr maydoni kuchlanganligi va potensiali:

$$\vec{E} = \vec{F}/Q, \quad \text{va} \quad \varphi = \frac{\Pi}{Q}, \quad \text{yoki} \quad \varphi = \frac{A_\infty}{Q}$$

bunda: $\vec{F}$ – maydonda joylashtirilgan musbat Q zaryadga ta'sir etayotgan kuch, $P - Q$ zaryadning potensial energiyasi, A_∞ – zaryadni maydonning shu nuqtasidan cheksizlikka ko'chirishda bajarilgan ish.

Elektr maydon kuchlanganligi vektori $\vec{E}$ ning oqimi:

$$\Phi_E = \int_S E \cdot \cos\alpha ds = \int_S E_n ds.$$

Ostrogradskiy – Gauss teoremasi:

$$\Phi_E = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} Q_i = \frac{1}{\varepsilon_0}\int_V \rho dV,$$

bu yerda: $\rho = \frac{dQ}{dV}$ – zaryadning hajmiy zichligi.

Nuqtaviy zaryadning r masofada hosil qilgan maydon kuchlanganligi va potensiali:

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad \text{va} \quad \varphi=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}.$$

Q zaryadi bir tekis taqsimlangan R radiusli sharning shar markazidan r masofadagi nuqtada hosil qiladigan elektr maydonning kuchlanganligi va potensiali:

$r < R$ da (nuqta shar ichida)

$$E=O \quad \text{va} \quad \varphi=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r};$$

$r = R$ da (nuqta shar sirtida)

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \quad \text{va} \quad \varphi=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R};$$

$r > R$ da (nuqta shardan tashqarisida)

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad \text{va} \quad \varphi=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}.$$

Bir tekis zaryadlangan R radiusli cheksiz silindrning, silindr o'qidan r masofadagi nuqtada hosil qiladigan elektr maydonining kuchlanganligi:

$r < R$ da (nuqta silindr ichida)

$$E=O,$$

$r \geq R$ da (nuqta silindrdan tashqarida)

$$E=\frac{1}{2\pi\varepsilon_0}\frac{\tau}{r},$$

bu yerda $\tau=\frac{dQ}{dl}$ – zaryadning chiziqli zichligi.

Bir tekis zaryadlangan cheksiz tekislik maydonining kuchlanganligi:

$$E=\frac{\sigma}{2\varepsilon_0},$$

bu yerda: $\sigma = \dfrac{dQ}{dS}$ – zaryadning sirtiy zichligi.

Ikkita parallel, cheksiz va turli xil zaryadlangan tekisliklar hosil qiladigan maydon kuchlanganligi:

$$E = \frac{\sigma}{\varepsilon_0}.$$

Elektr maydon kuchlanganligining va potensialining superpozitsiya prinsipi:

$$\vec{E} = \sum_{i=1}^{n} \vec{E_i} \quad \text{va} \quad \varphi = \sum_{i=1}^{n} \varphi_i .$$

$n = 2$ bo'lganda hosil bo'lgan kuchlanganlik vektorining moduli:

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1 \cdot E_2 \cdot \cos\alpha}.$$

bunda α – $\vec{E_1}$ va $\vec{E_2}$ vektorlar orasidagi burchak.

Nuqtaviy zaryadlar sistemasining o'zaro ta'sir energiyasi:

$$W = \frac{1}{2}\sum_{i=1}^{n} Q_i \varphi_i ,$$

bu yerda φ_i – $(n-1)$ ta zaryadning Q_i zaryad turgan nuqtada hosil qiladigan maydon potensiali. Q zaryadning maydonida undan r masofada turgan Q_0 zaryadning potensial energiyasi:

$$\Pi = \frac{1}{4\pi\varepsilon_0}\frac{QQ_0}{r}.$$

Elektrostatik maydon potensiali va kuchlanganligi orasidagi munosabat

$$\vec{E} = -grad\varphi .$$

Sferosimmetriklik xususiyatiga ega maydon uchun:

$$E = -\frac{d\varphi}{dr}.$$

Bir jinsli maydon holida:

$$E = \frac{\varphi_1 - \varphi_2}{d},$$

bu yerda: φ_1 va φ_2 – sirt nuqtalaridagi potensiallari, d – ular orasidagi masofa.

Nuqtaviy zaryadni φ_1 potensialli nuqtadan φ_2 potensialli nuqtaga ko'chirishda maydon kuchlarining ishi:

$$A = Q(\varphi_1 - \varphi_2) \quad \text{yoki} \quad A = Q\int_e \mathrm{E}_e dl.$$

Bir jinsli maydon holida:

$$A = Q \cdot E \cdot l \cdot \cos\alpha,$$

bu yerda: l – ko'chish, α – $\vec{E}$ va ko'chish yo'nalishi orasidagi burchak.

Masala yechishga misollar

1-misol. Har birining massasi $m = m_1 = m_2 = 1\text{g}$ dan bo'lgan ikkita sharcha berilgan. Ularning elektrostatik ta'sir kuchi, sharchalarning Nyuton qonuni bo'yicha tortishish kuchiga teng bo'lishi uchun har bir sharchaga qanday Q zaryad berilishi kerak? Sharchalar moddiy nuqtalar sifatida qaralsin.

Berilgan:

$m = m_1 = m_2 = 1\text{g} = 10^{-3}\text{kg};$

$F_k = F_N$

$Q = ?$

Yechish: Ikkita bir ismli zaryadlangan nuqtaviy zaryadlar orasidagi itarish kuchi Kulon qonuniga muvofiq aniqlanadi:

$$F_k = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 \cdot Q_2}{r^2}, \qquad (1)$$

bu yerda: $k = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9\ \text{F}/\text{m}$ – doimiylik.

Nuqtaviy jismlarning Nyuton qonuni bo'yicha o'zaro tortishish kuchi:

$$F_N = G\frac{m_1 m_2}{r^2}, \tag{2}$$

bu yerda: $G = 6{,}67\cdot 10^{-11}\,\dfrac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$ – gravitatsion doimiylik.

Masalaning shartiga muvofiq

$$F_K = F_N,$$

yoki (1) va (2) ni hamda $Q = Q_1 = Q_2$ va $m = m_1 = m_2$ ligini e'tiborga olsak,

$$k\frac{Q^2}{r^2} = G\frac{m^2}{r^2}.$$

Bundan

$$Q = m\sqrt{\frac{G}{k}} \tag{3}$$

ni olamiz.

$$[Q] = [m]\frac{[G]^{1/2}}{[k]^{1/2}} = 1\,\mathrm{kg}\frac{\sqrt{1\dfrac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{kg}^2}}}{\sqrt{1\dfrac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}^2}}} = 1\,\mathrm{kg}\sqrt{1\frac{\mathrm{C}^2}{\mathrm{kg}^2}} = 1\,\mathrm{C}.$$

Berilganlarni (3) ga qo'yib topamiz:

$$Q = 10^{-3}\sqrt{\frac{6{,}67\cdot 10^{-11}}{9\cdot 10^9}}\,\mathrm{C} = 86{,}7\cdot 10^{-15}\,\mathrm{C}.$$

Javob: $Q = 86{,}7\cdot 10^{-15}\,\mathrm{C}$.

2-misol. Bir xil zaryadlangan ikkita sharchalar uzunligi bir xil iplar bilan bir nuqtaga osilgan. Bunda ular α burchakka ajralishdi. Sharchalar yog'li idishga tushirilganda ham α burchak o'zgarmay qoldi. Sharcha materialining zichligi $1{,}5\cdot10^3\,\frac{\text{kg}}{\text{m}^3}$, yog'ning dielektrik kirituvchanligi 2,2 bo'lsa, yog'ning zichligi ρ_0 nimaga teng?

Berilgan:

$q_1 = q_2 = q$;

$\rho_m = 1{,}5\cdot10^3\,\frac{\text{kg}}{\text{m}^3}$;

$\varepsilon = 2{,}2$

$\rho_0 = ?$

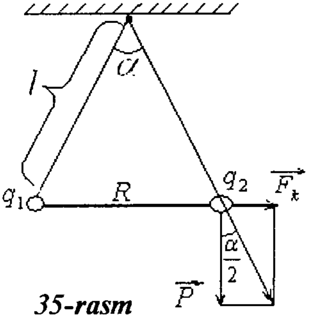
Yechish: Bo'shliqdagi bu ikki zaryadlangan sharcha orasidagi o'zaro ta'sir kuchi Kulon qonuni bilan aniqlanadi.:

$$F_k = \frac{1}{4\pi\varepsilon_0}\frac{q_1\cdot q_2}{R^2}. \qquad (1)$$

Shuningdek, 35-rasmdan

$$tg\frac{\alpha}{2} = \frac{F_k}{\text{P}} \qquad (2)$$

ekani ko'rinib turibdi.



35-rasm

Sharchalar yog'li idishga tushirilganda esa ular orasidagi elektrostatik ta'sir kuchi

$$F_k' = \frac{1}{4\pi\varepsilon_0\varepsilon}\frac{q_1\cdot q_2}{R^2} \qquad (3)$$

bo'ladi. Bu yerda ε – yog'ning dielektrik kirituvchanligi. Bundan tashqari ularga og'irlik kuchiga qarama-qarshi yo'nalgan Arximed kuchi ham ta'sir qiladi (36-rasm).

Demak,

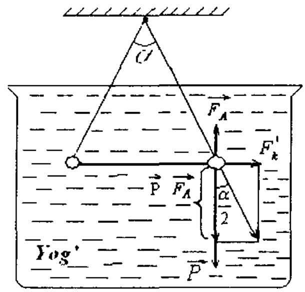
$$\text{tg}\frac{\alpha}{2} = \frac{F_k'}{P - F_A} \qquad (4)$$

Agar (2) va (4) larni tenglashtirsak, quyidagini olamiz:

$$\varepsilon = \frac{\text{P}}{P - F_A}. \qquad (5)$$

Sharning og'irligi $P = mg = \rho_{sh} \cdot V \cdot g$; Arximed kuchi esa $F_A = \rho_0 Vg$ kabi aniqlanadi. Bu yerda V – sharchaning hajmi. Unda (5) dan:

$$\varepsilon = \frac{\rho_{sh}}{\rho_{sh} - \rho_0},$$



36-rasm

yoki ρ_0 uchun

$$\rho_0 = \rho_{sh}\left(\frac{\varepsilon - 1}{\varepsilon}\right). \qquad (6)$$

Berilganlarni (6) ga qo'ysak,

$$\rho_0 = 1{,}5 \cdot 10^3 \left(\frac{2{,}2 - 1}{2{,}2}\right)\frac{\text{kg}}{\text{m}^3} = 0{,}82 \cdot 10^3 \frac{\text{kg}}{\text{m}^3} = 820 \frac{\text{kg}}{\text{m}^3}.$$

Javob: $\rho_0 = 820 \frac{\text{kg}}{\text{m}^3}$.

3-misol. 2 mkC va –3 mkC bo'lgan ikkita nuqtaviy zaryadlar 5 sm masofada joylashgan. Musbat zaryaddan 3 sm va manfiy zaryaddan 4 sm uzoqlikda joylashgan nuqtadagi maydon kuchlanganligi aniqlansin.

Berilgan:

$q_1 = 2\text{mkC} = 2 \cdot 10^{-6}\text{C};$

$q_2 = -3\text{mkC} = -3 \cdot 10^{-6}\text{C};$

$R = 5\text{sm} = 5 \cdot 10^{-2}\text{m};$

$R_1 = 3\text{sm} = 3 \cdot 10^{-2}\text{m}$

$R_2 = 4\text{sm} = 4 \cdot 10^{-2}\text{m}$

$E = ?$

Yechish: Kuchlanganlik vektorining musbat zaryaddan chiqishi va manfiy zaryadda tugashini e'tiborga olib, ko'rilayotgan nuqtadagi maydon kuchlanganligining yo'nalishini 37-rasmdagidek aniqlaymiz, ya'ni elektr maydon kuchlanganligining superpozitsiya prinsipiga muvofiq:

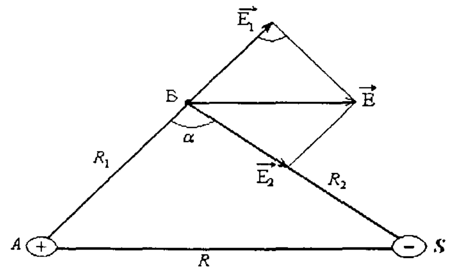
$$\vec{E} = \vec{E_1} + \vec{E_2}.$$

Shu bilan birga ΔABC to'g'ri burchakli uchburchakdir $\left(\alpha = \frac{\pi}{2}\right)$, chunki $R^2 = R_1^2 + R_2^2$. Demak,

$$E^2 = E_1^2 + E_2^2, \qquad (1)$$

yoki (1) dan

$$E = \sqrt{E_1^2 + E_2^2}. \qquad (2)$$



37-rasm

O'z navbatida nuqtaviy zaryad maydon kuchlanganligini aniqlash formulasidan:

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1^2}{R_1^2}, \qquad \text{va}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2^2}{R_2^2}. \qquad (3)$$

Bularni (2) ga qo'ysak,

$$E = \frac{1}{4\pi\varepsilon_0}\sqrt{\frac{q_1^2}{R_1^4} + \frac{q_2^2}{R_2^4}}. \qquad (4)$$

Berilganlarni o'rniga qo'ysak, $\left(\frac{1}{4\pi\varepsilon_0} = 9\cdot10^9\,\frac{m}{F}\right.$ dan foydalanamiz):

$$E = 9\cdot10^9\sqrt{\frac{4}{81} - \frac{9}{256}}\cdot10^{-2}\,\frac{\text{N}}{\text{C}} = 9\cdot10^7\sqrt{0{,}049 - 0{,}035}\,\frac{\text{N}}{\text{C}} = 9\cdot10^6\cdot1{,}1\,\frac{\text{N}}{\text{C}} = 9{,}9\cdot10^6\,\frac{\text{N}}{\text{C}}.$$

Javob: $E = 9{,}9\cdot10^6\,\frac{\text{N}}{\text{C}}$.

4-misol. Tomonining uzunligi 10 sm bo'lgan kvadratning uchlarida joylashgan to'rtta bir xil 10 nC nuqtaviy zaryadlar sistemasining potensial energiyasi P qanday bo'ladi?

Berilgan:

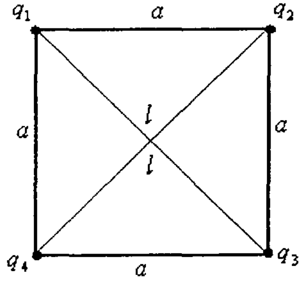
$q_1 = q_2 = q_3 = q_4 = q = 10\text{C};$

$a = 10\text{sm} = 0{,}1\text{m}.$

$P = ?$

Yechish: Ikki nuqtaviy zaryadlarning o‘zaro ta’sir potensial energiyasi quyidagi ifoda yordamida aniqlanadi:

$$\Pi = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{q_1 \cdot q_2}{r} \qquad (1)$$



38-rasm

bu yerda r – zaryadlar orasidagi masofa. Agar maydonni n ta nuqtaviy zaryad hosil qilayotgan bo‘lsa, unda superpozitsiya prinsipiga muvofiq umumiy potensial energiya har bir ikki zaryad o‘zaro ta’sir potensial energiyalarining yig‘indisi sifatida aniqlanadi. Ya’ni

$$\Pi = \Pi_{12} + \Pi_{13} + \Pi_{14} + \Pi_{23} + \Pi_{24} + \Pi_{34}. \qquad (2)$$

(1) ga asosan va 38-rasmdan har bir ikki nuqtaviy zaryad orasidagi potensial energiyalar:

$$\Pi_{12} = \Pi_{14} = \Pi_{23} = \Pi_{34} = \frac{1}{4\pi\varepsilon_0\varepsilon}\frac{q^2}{a}; \qquad (3)$$

$$\Pi_{13} = \Pi_{24} = \frac{1}{4\pi\varepsilon_0\varepsilon}\frac{q^2}{l}. \qquad (4)$$

Pifagor teoremasiga ko‘ra:

$$l^2 = a^2 + a^2 = 2a^2 \qquad \text{yoki} \qquad l = \sqrt{2}a. \qquad (5)$$

(5) ni (4) ga qo‘ysak,

$$\Pi_{13} = \Pi_{24} = \frac{1}{4\pi\varepsilon_0\varepsilon}\frac{q^2}{\sqrt{2}a}. \qquad (6)$$

(3) va (6) ga asosan (2)ni qayta yozsak $(\varepsilon = 1)$:

$$\Pi = \frac{q_2}{4\pi\varepsilon_0 a}\left[4 + \sqrt{2}\right] \qquad (7)$$

$$[\Pi] = \frac{[q]^2}{[\varepsilon][a]} = \frac{1\text{C}^2}{1\frac{\text{F}}{\text{m}} \cdot 1\text{m}} = 1\frac{\text{C}^2 \cdot \text{V}}{\text{C}} = 1\text{C} \cdot \frac{\text{J}}{\text{C}} = 1\text{J}.$$

Kattaliklarning son qiymatlarini qo'yamiz $\left(\varepsilon_0 = 8{,}85 \cdot 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}\right)$:

$$\Pi = \frac{(10^{-8})^2}{4 \cdot 3{,}14 \cdot 8{,}85 \cdot 10^{-12} \cdot 0{,}1}\left[4 + \sqrt{2}\right]\mathrm{J} \approx 0{,}05 \cdot 10^{-3}\,\mathrm{J} = 50 \cdot 10^{-6}\,\mathrm{J} = 50\,\mathrm{mkJ}.$$

Javob: $\Pi \approx 50\,\mathrm{mkJ}$.

5-misol. Uzunligi 10 sm bo'lgan ingichka tayoqchada tekis taqsimlangan 1 nC zaryad bor. Tayoqcha o'qida yotuvchi va uning yaqin uchidan 20 sm masofada joylashgan nuqtadagi elektr maydon potensiali φ aniqlansin.

Berilgan:

$l = 10\,\mathrm{sm} = 0{,}1\,\mathrm{m}$;

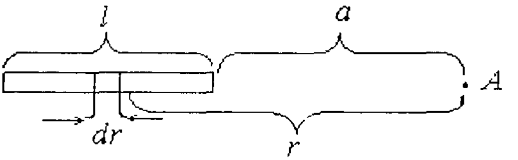
$q = 1\,\mathrm{nC} = 10^{-9}\,\mathrm{C}$;

$a = 20\,\mathrm{sm} = 0{,}2\,\mathrm{m}$

$\varphi = ?$

Yechish: Tayoqcha elementi dr ning, tayoqcha o'qida yotuvchi r masofadagi A nuqtada hosil qilgan maydon potensiali $d\varphi$ quyidagicha aniqlanadi:

$$d\varphi = \frac{\tau dr}{4\pi\varepsilon_0 \varepsilon r}. \qquad (1)$$



39-rasm

Bu yerda $\tau = \frac{q}{l}$ – zaryadning chiziqli zichligi. (1) ifodani tayoqchaning butun uzunligi bo'yicha integrallab, l uzunlikli tayoqcha hosil qilgan maydon potensialini topamiz:

$$\varphi = \int_a^{l+a} \frac{\tau dr}{4\pi\varepsilon_0 \varepsilon r} = \frac{\tau}{4\pi\varepsilon_0 \varepsilon}\int_a^{l+a}\frac{dr}{r} = \frac{\tau}{4\pi\varepsilon_0\varepsilon}\ln|r|\Big|_a^{l+a} = \frac{\tau}{4\pi\varepsilon_0\varepsilon}\ln\left|\frac{l+a}{a}\right|.$$

Demak, A nuqtada l tayoqcha hosil qiladigan maydon potensiali:

$$\varphi = \frac{\tau}{4\pi\varepsilon_0\varepsilon}\ln\left|\frac{l+a}{a}\right| = \frac{q}{4\pi\varepsilon_0\varepsilon l}\ln\left|\frac{l+a}{a}\right| \qquad (2)$$

$$[\varphi]=\frac{[\tau]}{[\varepsilon_0]}=\frac{1\frac{\text{C}}{\text{m}}}{1\frac{\text{F}}{\text{m}}}=1\frac{\text{C}\cdot\text{V}}{\text{C}}=1\text{V}.$$

Kattaliklarning qiymatlarini (2) ga qo'yib, hisoblaymiz:

$$\left(\varepsilon=1;\varepsilon_0=8{,}85\cdot10^{-12}\,\text{F}/\text{m}\right).$$

$$\varphi=\frac{10^{-9}}{4\cdot3{,}14\cdot8{,}85\cdot10^{-12}\cdot0{,}1}\ln\left|\frac{0{,}1+0{,}2}{0{,}2}\right|\text{V}=36{,}5\text{V}.$$

Javob: $\varphi=36{,}5\text{V}$.

7-misol. Gorizontal yo'nalishda $1{,}6\frac{\text{Mm}}{\text{s}}$ tezlik bilan harakatlanayotgan elektron tik yuqoriga yo'nalgan 90V/sm kuchlanganlikli bir jinsli elektr maydoniga uchib kirdi. 1ns dan keyin elektron tezligining yo'nalishi va moduli qanday bo'ladi?

Berilgan:

$\vartheta_0=1{,}6\cdot10^6\frac{\text{m}}{\text{s}}$;

$\text{E}=90\frac{\text{V}}{\text{sm}}=9000\frac{\text{V}}{\text{m}}$;

$t=1\text{ns}=10^{-9}\text{s}$.

$\vartheta=?$

$\varphi=?$

Yechish: Elektronga yuqoriga yo'nalgan

$$F_e=e\text{E} \qquad (1)$$

kuch ta'sir qiladi. Bu kuch ta'sirida elektron

$$a=\frac{F_e}{m}=\frac{e\text{E}}{m} \qquad (2)$$

tezlanish oladi. 40-rasmda ko'rsatilgandek qilib sanoq sistemasi tanlanganda

$$\vartheta_y=a\cdot t=\frac{eE}{m}\cdot t. \qquad (3)$$

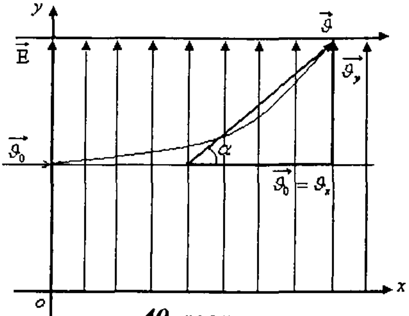
Undan ko'rinib turibdiki, $\vartheta=\sqrt{\vartheta_x^2+\vartheta_y^2}=\sqrt{\vartheta_0^2+\left(\frac{eEt}{m}\right)^2}$. (4)

Endi φ burchakning tangensini aniqlaylik:

$$\mathrm{tg}\varphi=\frac{\vartheta_y}{\vartheta_x}=\frac{e\,\mathrm{E}t}{\vartheta_0\cdot m}$$

Bundan $\varphi=\mathrm{arctg}\left(\frac{e\mathrm{E}t}{\vartheta_0\cdot m}\right)$.

(5)



40-rasm

Berilganlarni (4) va (5) larga qo'yib topamiz:

$$\vartheta=\sqrt{(1{,}6\cdot10^6)^2+\left(\frac{1{,}6\cdot10^{-19}\cdot9000\cdot10^{-9}}{9{,}1\cdot10^{-31}}\right)^2}=2{,}24\cdot10^6\frac{\mathrm{m}}{\mathrm{s}}=2{,}24\frac{\mathrm{Mm}}{\mathrm{s}};$$

$$\varphi=\mathrm{arctg}\left(\frac{1{,}6\cdot10^{-19}\cdot9000\cdot10^{-9}}{1{,}6\cdot10^6\cdot9{,}1\cdot10^{-31}}\right)\approx\mathrm{arctg}(1)=45^0$$

Bu yerda $e=1{,}6\cdot10^{-19}\mathrm{C}$ va $m=9{,}1\cdot10^{-31}\mathrm{kg}$ dan foydalanildi.

Javob: $\vartheta=2{,}24\frac{\mathrm{Mm}}{\mathrm{s}}$; $\varphi=45^0$.

Mustaqil yechish uchun masalalar

1. Ikkita elektronning gravitatsion o'zaro ta'sir kuchi ularning elektrostatik o'zaro ta'sir kuchidan necha marta kichik? $[2{,}4\cdot10^{-45}.]$

2. Tomonlarining uzunligi 10sm bo'lgan teng tomonli uchburchakning uchlariga $q_1=q_2=q_3=2nC$ zaryadlar joylashtirilgan. Zaryadlarning birortasiga qolgan ikkitasi ko'rsatadigan itarish kuchining kattaligi aniqlansin. [11,5 mkN.]

3. Ikkita q va 9 q zaryadlar bir-biridan 10sm masofada joylashtirilgan. Muvozanat holatida bo'lishi uchun shu zaryadlardan o'tuvchi o'qda joylashtiriladigan uchinchi zaryadning ishorasi qanday va u zaryadlardan

qanday masofada bo‘lishi kerak? Zaryadlar o‘q bo‘ylab harakat qila oladi. [2,5sm; 7,5 sm.]

4. Tomonining uzunligi 10 sm bo‘lgan kvadratning uchlarida to‘rtta bir xil 10 nC nuqtaviy zaryadlar joylashgan. Shu zaryadlarning birortasiga qolgan uchtasi tomonidan ta’sir etadigan Kulon kuchi topilsin. [90 mkN].

5. Zaryadning sirtiy zichligi 4 mkC/m^2 bo‘lgan cheksiz tekislikka, zaryadning chiziqli zichligi 100 nC/m bo‘lgan cheksiz uzun sim parallel joylashgan. Simning 1 m uzunligiga tekislik tomonidan ta’sir etadigan kuch topilsin. [22 mN.]

6. Ikkita 8 nC va −5,3 nC nuqtaviy zaryadlar orasidagi masofa 40 sm. Zaryadlarning o‘rtasida yotgan nuqtadagi maydon kuchlanganligi $\vec{E}$ hisoblansin. Agar ikkinchi zaryad musbat bo‘lsa, kuchlanganlik qanday bo‘ladi? [2,99 kV/m; 607 V/m.]

7. Ikkita $2q$ va $-q$ nuqtaviy zaryadlar bir-biridan d masofada joylashgan. Shu zaryadlardan o‘tuvchi to‘g‘ri chiziqda yotuvchi va maydon kuchlanganligi nolga teng bo‘lgan nuqtaning o‘rni topilsin. $[(\sqrt{2}+1)d.]$

8. Diametri 20 sm bo‘lgan yupqa sirtli cheksiz silindrda zaryad 4mC/m^2 sirtiy zichlik bilan tekis taqsimlangan. Silindr sirtidan 15 sm masofada joylashgan nuqtadagi elektr maydon kuchlanganligi topilsin.[0,45 MV/m.]

9. Bir tekis zaryadlangan cheksiz tekislikka shunday ismli 0,6 nC zaryadli 50 mg massali sharcha ip bilan osilgan. Sharcha osilgan ipning tarangligi 0,7 mN bo‘lsa, tekislikdagi zaryadning sirtiy zichligi topilsin. [14,5 mkC/m^2.]

10. 20 sm radiusli shar markazida 10 nC nuqtaviy zaryad turibdi. Shar sirtining yuzasi 20 sm_2 bo‘lgan qismi orqali kuchlanganlik vektorining oqimi aniqlansin. [4,5V · m.]

11. 20 nC zaryadni maydonning ikki nuqtasi orasiga ko‘chirishda tashqi kuchlar tomonidan 4mkJ ish bajarilgan. Maydon kuchlarining ishi va maydonning shu nuqtalari orasidagi potensiallar farqi aniqlansin. [−4 mkJ; 200V.]

12. Maydonni 1nC nuqtaviy zaryad hosil qilgan. Zaryaddan 20sm masofadagi nuqtada maydon potensiali nimaga teng? [45V.]

13. Ingichka tayoqchalar tomonining uzunligi a bo‘lgan kvadrat hosil qiladi. Tayoqchalar 1,33 nC/m chiziqli zichlik bilan zaryadlangan. Kvadrat markazidagi potensial topilsin. [33,6V.]

14. Yupqa aylana plastina tekislik bo‘ylab bir tekis taqsimlangan 1nC zaryadga ega. Plastina radiusi 5 sm. Quyidagi ikki nuqtada elektr maydonining potensiali aniqlansin: 1) plastina markazida; 2) plastina tekisligiga tik o‘qda yotuvchi va markazidan 5 sm uzoqlikdagi nuqtada. [1) 360V; 2) 149V.]

15. Diametri 2sm bo'lgan metall shar 15V potensialgacha manfiy zaryadlangan. Shar sirtida nechta elektron bor? [$1{,}04 \cdot 10^9$ ta.]

16. Ingichka tayoqcha yarim xalqadek qayrilgan. Tayoqcha 133nC/m chiziqli zichlik bilan zaryadlangan. 6,7 nC zaryadni yarim halqa markazidan cheksizlikka ko'chirish uchun qanday ish bajariladi. [25,2 mkJ.]

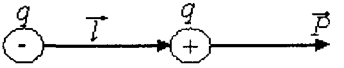
17. Beshta elektroni bor 1pg massali zarra bo'shliqda 3MV tezlashtiruvchi potensiallar farqidan o'tdi. Zarraning kinetik energiyasini toping. U qanday tezlikka erishadi? [15 MeV; $2{,}19 \cdot 10^9$ m/s.]

15-§. Dielektriklar. Elektr sig'imi. Kondensatorlar

Asosiy formulalar

Dipolning elektr momenti:

$$\vec{p} = |q|\vec{l},$$



41-rasm

bu yerda: $|q|$ – dipol zaryadi, $\vec{l}$ – dipol yelkasi deyilib, dipolning manfiy zaryadidan musbat zaryadiga o'tkazilgan vektor.

Dipol o'qida yotuvchi nuqtada dipol maydonining kuchlanganligi

$$E = \frac{P}{2\pi\varepsilon_0\varepsilon r^3},$$

r – dipol markazidan ko'rilayotgan nuqtagacha bo'lgan masofa.

Dipol o'qida yotuvchi nuqtada dipol maydonining potensiali.

$$\varphi = \frac{P}{4\pi\varepsilon_0\varepsilon r^2}.$$

Dielektrikdagi o'rtacha maydon kuchlanganligi E tashqi maydon kuchlanganligi E_0 bilan quyidagicha bog'langan:

$$E = \frac{E_0}{\varepsilon}.$$

ε – dielektrik kirituvchanlik.

Yakkalangan o'tkazgichning elektr sig'imi:

$$C = \frac{Q}{\varphi},$$

Q – o'tkazgichga berilgan zaryad, φ – shu zaryad vujudga keltirgan potensiallar farqi.

Yassi kondensatorning elektr sig'imi:

$$C = \frac{\varepsilon\varepsilon_0 S}{d},$$

bu yerda: ε – dielektrik kirituvchanlik, ε_0 – elektrostatik doimiy, S – kondensator qoplamasining yuzasi, d – ular orasidagi masofa.

R radiusli sharning elektr sig'imi:

$$C=\pi \varepsilon_0 \varepsilon R .$$

Qalinliklari d_i, dielektrik kirituvchanliklari ε_i bo'lgan n ta dielektrik qatlami bilan to'ldirilgan yassi kondensatorning elektr sig'imi:

$$C=\frac{\varepsilon_0 S}{\frac{d_1}{\varepsilon_1}+\frac{d_2}{\varepsilon_2}+---+\frac{d_n}{\varepsilon_n}}.$$

R_1 va R_2 radiusli konsentrik silindrlarning orasiga ε – singdiruvchanlikli dielektrik to'ldirilgan sistemaning (silindrik kondensatorning) elektr sig'imi

$$C=\frac{2\pi\varepsilon\varepsilon_0 l}{\ln(\frac{R_1}{R_2})}.$$

l – silindr sirtining uzunligi.

Ketma-ket ulangan kondensatorlar batareyasining elektr sig'imi:

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+...+\frac{1}{C_n}.$$

Ikkita kondensator holida:

$$C=\frac{C_1 \cdot C_2}{C_1+C_2}.$$

Parallel ulangan kondensatorlar batareyasining elektr sig'imi:

$$C=C_1+C_2+...+C_n.$$

Masala yechishga misollar

1-misol. 100nC·m elektr momentli dipol kuchlanganligi 10 kV/m bo'lgan bir jinsli eletkr maydonida erkin joylashgan. Dipolni 60^0 burganda uning potensial energiyasining o'zgarishi $\Delta\Pi$ aniqlansin.

Berilgan:

$P = 100\text{nC}\cdot\text{m} = 10^{-10}\text{C}\cdot\text{m};$

$E = 10\text{kV/m} = 10^4\, \text{V}/\text{m};$

$\alpha = 60^0 = \dfrac{\pi}{3}.$

$\Delta\Pi = ?$

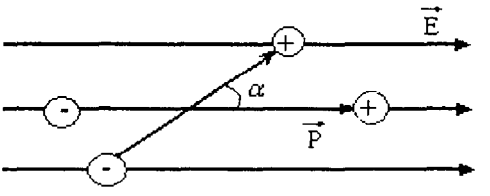
Yechish: Dipolni burish natijasida uning potensial energiyasining o'zgarishi $\Delta\Pi$ son jihatdan uni burish uchun bajariladigan A ishga teng bo'ladi:

$$\Delta\Pi = A. \qquad (1)$$

Dipolni α burchakka burishda bajariladigan ish esa

$$dA = Md\alpha.$$

Bu yer $M = p\cdot E\sin\alpha$ – kuchlanganligi $\vec{E}$ bo'lgan elektr maydonda joylashtirilgan $\vec{p}$ momentli dipolga ta'sir etadigan mexanik moment.



42-rasm

Demak,

$$dA = p\cdot E\sin\alpha d\alpha, \qquad (2)$$

Topilgan ifodani integrall ko'rinishda yozamiz (α burchak$[0:\dfrac{\pi}{3}]$ da o'zgaradi):

$$A = \int_0^{\pi/3} p\cdot E\sin\alpha d\alpha = p\cdot E\int_0^{\pi/3}\sin\alpha d\alpha = p\cdot E(-\cos\alpha)\Big|_0^{\pi/3} =$$

$$= pE(\cos O - \cos\frac{\pi}{3}) = \frac{1}{2}p\text{E}$$

Demak, $$A = \frac{1}{2}p\text{E},$$

$$[A] = [p]\cdot[E] = 1\text{C}\cdot\text{m}\cdot 1\frac{\text{N}}{\text{C}} = 1\text{N}\cdot\text{m} = 1\text{J}. \qquad (3)$$

Berilganlarni (3) ga qo'yib va (1) ni nazarga olib, quyidagini topamiz:

$$\Delta\Pi = A = \frac{1}{2}10^{-10}\text{C}\cdot\text{m}\cdot 10^4\, \text{V}/\text{m} = 0{,}5\cdot 10^{-6}\text{J} = 0{,}5\text{mkJ}.$$

Javob: $\Delta\Pi = 0{,}5$ mkJ.

2-misol. 10 V potensialgacha zaryadlangan to‘rtta bir xil simob tomchisi qo‘shilib bitta tomchi hosil qiladi. Hosil bo‘lgan katta tomchining potensiali φ topilsin.

Berilgan:
$\varphi_0 = 10V;$
$V = 4V_0.$

$\varphi = ?$

Yechish: Katta tomchining potensiali:

$$\varphi = \frac{Q}{S} \qquad (1)$$

ifoda yordamida aniqlanadi. Bu yerda:

$$S = 4\pi\varepsilon_0\varepsilon R. \qquad (2)$$

Katta tomchining elektr sig‘imi (uni sharsimon kondensator sifatida qarash mumkin) va zaryadning saqlanish qonuniga muvofiq:

$$Q = 4q, \qquad (3)$$

bunda: Q – katta tomchidagi zaryad miqdori, q – har bir kichik tomchidagi zaryad miqdori. Demak,

$$\varphi = \frac{4q}{4\pi\varepsilon_0\varepsilon R}. \qquad (4)$$

Agar kichik tomchidagi zaryad miqdori

$$q = C_0 \cdot \varphi_0 \qquad (5)$$

kabi aniqlanishini nazarda tutsak, bu yerda $C_0 = 4\pi\varepsilon_0\varepsilon r$ (r– kichik tomchining radiusi) olamiz:

$$\varphi = \frac{4C_0\varphi_0}{4\pi\varepsilon_0\varepsilon R} = \frac{4\varphi_0 \cdot 4\pi\varepsilon_0\varepsilon r}{4\pi\varepsilon_0\varepsilon R} = \frac{4\varphi_0 r}{R}. \qquad (6)$$

Shu bilan birga katta tomchi va kichik tomchilar hajmlari orasida quyidagi munosabat o‘rinli:

$$V=4V_0$$

yoki $V = \frac{4}{3}\pi R^3$ va $V_0 = \frac{4}{3}\pi r^3$ ligidan

$$R^3 = 4r^3, \qquad R = \sqrt[3]{4}r \cdot \tag{7}$$

(7) ni (6) ga qo'ysak:

$$\varphi = \frac{4\varphi_0}{\sqrt[3]{4}} . \tag{8}$$

Shunday qilib, $\varphi = \frac{4 \cdot 10}{\sqrt[3]{4}} \text{V} \approx 25\text{V}$.

Javob: $\varphi \approx 25\text{V}$.

3-misol. Yassi kondensatorning elektr sig'imi *1,5* mkF. Qoplamalar orasidagi masofa 5 mm. Agar pastdagi qoplamaga 3 mm qalinlikli ebonit taxtachasi qo'yilsa, kondensatorning elektr sig'imi qanday bo'ladi?

Berilgan:

C_0=1,5mkF=$1{,}5 \cdot 10^{-6}$ F;

d =5 mm=$5 \cdot 10^{-3}$m;

d_1=3 mm=$3 \cdot 10^{-3}$ m;

$\varepsilon_1 = 3{,}0$;

$\varepsilon = 1$.

C=?

Yechish: Pastdagi qoplamaga ebonit taxtacha qo'yish natijasida hosil bo'lgan qurilmani ikkita ketma-ket ulangan yassi kondensatorlar sistemasi sifatida qarash mumkin. Ya'ni uning umumiy sig'imi:

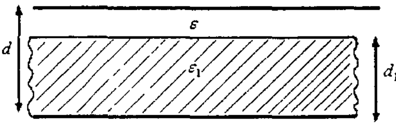
$$C = \frac{C_1 \cdot C_2}{C_1 + C_2} \tag{1}$$

Bu yerda: C_1 qalinligi d_1 ga teng bo'lgan qoplamalari orasiga ebonit qo'yilgan yassi kondensatorning sig'imi:

$$C_1 = \frac{\varepsilon_1 \varepsilon_0 S}{d_1}, \tag{2}$$

C_2– qalinligi (d-d_1) va qoplamalari oralig'i bo'sh ($\varepsilon = 1$) bo'lgan yassi kondensatorlarning sig'imi:

$$C_2 = \frac{\varepsilon\varepsilon_0 S}{d - d_1}. \qquad (3)$$



43-rasm

(2) va (3) larni (1) ga qo‘ysak,

$$C = \frac{\varepsilon_0\varepsilon\varepsilon_1 S}{\varepsilon_1(d - d_1) + \varepsilon d_1}. \qquad (4)$$

Dastlabki (ebonit taxtachasi qo‘yilmasdan oldin) yassi kondensatorning

sig‘imi $\quad C_0 = \frac{\varepsilon\varepsilon_0 S}{d}.$

Bundan S ni topsak, $S = \frac{d \cdot C_0}{\varepsilon\varepsilon_0}.$ (5)

(5) ni (4) ga qo‘ysak, quyidagini olamiz:

$$C = \frac{\varepsilon_1 d C_0}{\varepsilon_1(d - d_1) + \varepsilon d_1}. \qquad (6)$$

Kattaliklarning qiymatlarini (6) ga qo‘yamiz:

$$C = \frac{3 \cdot 5 \cdot 10^{-3} \cdot 1{,}5 \cdot 10^{-6}}{3(5 \cdot 10^{-3} - 3 \cdot 10^{-3}) + 1 \cdot 3 \cdot 10^{-3}}\,\mathrm{F} = 2{,}5 \cdot 10^{-6}\,\mathrm{F} = 2{,}5\,\mathrm{mkF}$$

Javob: S=*2,5* mkF.

4-misol: Yassi kondensator qoplamalari orasida zich yopishib turgan shisha taxtacha bor. Kondensator 100V potensiallar farqigacha zaryadlangan. Agar shisha taxtacha kondensatordan chiqarib olinsa, potensiallar farqi qanday bo‘ladi?

Berilgan:
U_1=100 V
$\varepsilon_1 = 7$.

U_2=?

Yechish: Kondensator sig‘imi va potensiallar farqi orasida quyidagi munosabat mavjud:

$$C = \frac{Q}{U}. \qquad (1)$$

Bundan $$U = \frac{Q}{C}. \qquad (2)$$

Yassi kondensatorning sig'imi formulasi $C = \dfrac{\varepsilon\varepsilon_0 S}{d}$ ga muvofiq shisha

plastinkali kondensator uchun $$U_1 = \frac{Qd}{\varepsilon_1\varepsilon_0 S}, \qquad (3)$$

va shisha plastinkasiz kondensator uchun $$U_2 = \frac{Qd}{\varepsilon\varepsilon_0 S}. \qquad (4)$$

(3) ni (4) ga hadma-had bo'lib, quyidagini olamiz $\dfrac{U_1}{U_2} = \dfrac{\varepsilon}{\varepsilon_1}$.

Bundan $$U_2 = \frac{\varepsilon_1}{\varepsilon} U_1 \qquad (5)$$

ni topamiz. Kattaliklarning qiymatlarini ($\varepsilon = 1_1$) (5) ga qo'ysak ,

$$U_2 = \frac{7}{1} 100V = 700V.$$

Javob: U_2 =*700* V.

5-misol: Har birining sig'imi 10 pF dan 450 pF gacha o'zgaradigan o'zgaruvchan sig'imli ikkita kondensatordan tuzilgan sistemalarning sig'imi qanday chegarada o'zgaradi?

Berilgan:
C_1=C_2=(10÷450) pF.

C_{kk}=?

C_p=?

Yechish: 1. Kondensatorning ketma-ket ulangan hol ifodasi:

$$C_{kk} = \frac{C_1 \cdot C_2}{C_1 + C_2} \qquad (1)$$

ga dastlab eng kichik qiymatlarni,

$$C_{kk} = \frac{10 \cdot 10}{10 + 10} \text{pF} = 5\text{pF},$$

so'ngra esa eng katta qiymatlarni qo'yamiz:

$$C_{kk}=\frac{450\cdot 450}{450+450}\text{pF}=225\text{pF}.$$

2. Kondensatorlar parallel ulanganda

$$C_p = C_1 + C_2. \qquad (2)$$

ga dastlab eng kichik qiymatlarni,

$$C_p = 10\text{pF}+10\text{pF}=20\text{pF},$$

so'ngra esa eng katta qiymatlarni qo'yamiz:

$$C_p = 450\text{pF}+450\text{pF}=900\text{pF}.$$

Javob: $C_{kk}=(5\div 25)\text{pF}; \quad C_p=(20\div 900)\text{pF}.$

Mustaqil yechish uchun masalalar

1. Radiusi 4 sm bo'lgan sharcha 1000 V potensialgacha manfiy zaryadlanadi. Bunda sharchaga berilgan elektronlarning umumiy massasi topilsin. [2,53 $\cdot$ 10^{-20}kg.]

2. Qoplamalarining yuzasi 100 sm^2, orasidagi masofa esa 0,1mm bo'lgan yassi kondensatorning elektr sig'imi aniqlansin. [8,85 mkF.]

3. Yassi kondensatorga uning qoplamalariga jipslashib turadigan qilib qalinligi 1sm bo'lgan parafin taxtachani surib kiritdilar. Dastlabki sig'imni hosil qilish uchun qoplamalar orasidagi masofani qancha o'zgartirish kerak? [0,5 sm.]

4. Radiuslari 2 sm va 2,1 sm bo'lgan ikkita konsentrik metall sharlar, sharsimon kondensatorlarni hosil qiladi. Agar sharlar orasidagi bo'shliq parafin bilan to'ldirilgan bo'lsa, kondensatorning sig'imi aniqlansin. [93,3 pF.]

5. Sig'imlari 100 pF dan bo'lgan ikkita bir xil havo yassi kondensatorlari ketma-ket ulangan. Agar kondensatorlardan birining qoplamalari orasi parafin bilan to'ldirilsa, kondensatorlar batareyasining sig'imi qanday o'zgaradi? [16,7 pF.]

6. 0,6 mkF sig'imli kondensator 300V potensiallar farqigacha zaryadlangan va sig'imi 0,4 mkF bo'lgan, 150 V potensiallar farqigacha zaryadlangan ikkinchi kondensatorga ketma-ket ulangan. Bunda birinchi kondensatordan ikkinchisiga qancha zaryad oqib o'tadi? [36 mC.]

7. Uchta bir xil yassi kondensatorlar ketma-ket ulangan. Bunda kondensatorlar batareyasining umumiy sig'imi 89 pF. Kondensator qoplamasining yuzasi 100 sm^2 dielektrik shisha. Shishaning qalinligi topilsin. [2,32 mm.]

16-§. Zaryadlangan o'tkazgich energiyasi. Elektr maydon energiyasi

Asosiy formulalar

Zaryadlangan o'tkazgich energiyasi:

$$W=\frac{1}{2}C\varphi^2=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}Q\varphi.$$

Bu yerda: Q – zaryad miqdori, φ – o'tkazgichning potensiali, C – elektr sig'imi.

Zaryadlangan kondensatorning energiyasi:

$$W=\frac{1}{2}CU^2=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}QU,$$

bunda: C – kondensatorning sig'imi, Q – zaryadi, U – qoplamalari orasidagi potensiallar farqi.

Elektr maydon energiyasining hajmiy zichligi:

$$\omega=\frac{1}{2}\varepsilon_0\varepsilon E^2=\frac{1}{2}ED,$$

E – elektr maydon kuchlanganligi, ε – muhitning dielektrik singdiruvchanligi, D – elektr siljishi.

Masala yechishga misollar

1-misol. Agar diametri 20 sm bo'lsa, 100 nC zaryad berilgan metal sharning elektrostatik maydon energiyasi nimaga teng bo'ladi?

Berilgan:	Yechish: Zaryadlangan metall sharning energiyasi
d =20 sm=0,2m;	
Q=100nC=10^{-7}C.	$$W=\frac{Q^2}{2C},\qquad (1)$$
W=?	

ifoda yordamida aniqlanadi. Bu yerda C– $R=\frac{d}{2}$ radiusli sharning elektr sig'imi,

$$C=4\pi\varepsilon_0\varepsilon R=2\pi\varepsilon_0\varepsilon d.\qquad (2)$$

(2) ni (1) ga qo'yib sharning energiyasi uchun olamiz:

$$W = \frac{Q^2}{4\pi\varepsilon_0\varepsilon d}. \qquad (3)$$

$$[W] = \frac{[Q]^2}{[\varepsilon_0][d]} = \frac{1\text{C}^2}{1\frac{\text{F}}{\text{m}}\cdot 1\text{m}} = \frac{1\text{C}^2\cdot 1\text{V}}{1\text{C}} = 1\text{C}\frac{\text{J}}{\text{C}} = 1\text{J}.$$

Berilganlarni (3) ga qo'yib olamiz ($\varepsilon = 1$, $\varepsilon_0 = 8{,}85\cdot 10^{-12}\ \text{F}/\text{m}$)

$$W = \frac{(10^{-7})^2}{4\cdot 3{,}14\cdot 8{,}85\cdot 10^{-12}\cdot 0{,}2}\text{J} \approx 4{,}50\cdot 10^{-4}\text{J} = 450\text{mkJ}.$$

Javob: $W = 450\text{mkJ}$.

2-misol. Radiuslari 5 sm va 10 sm bo'lgan ikkita metall sharlar mos ravishda 40 nC va 20 nC zaryadlarga ega. Agar sharlar o'tkazgich yordamida ulansa, razryadlanish natijasida ajralib chiqadigan energiya topilsin.

Berilgan:

R_1=5 sm = $5\cdot 10^{-2}$ m;

R_2= 10 sm = 10^{-1} m;

Q_1=40 nC = $4\cdot 10^{-8}$ C;

Q_2=−20 nC=$2\cdot 10^{-8}$ C.

$\Delta W = ?$

Yechish: Energiyaning saqlanish qonuniga muvofiq razryadlanish natijasida ajralib chiqadigan energiya quyidagicha aniqlanadi

$$\Delta W = (W_1 + W_2) - (W_1' + W_2'). \qquad (1)$$

Bu yerda

$$W_1 = \frac{Q_1^2}{2C_1} \quad \text{va} \quad W_2 = \frac{Q_2^2}{2C_2} \qquad (2)$$

sharlarning dastlabki energiyalari.

$$W_1' = \frac{(Q_1')^2}{2C_1} \quad \text{va} \quad W_2' = \frac{(Q_2^1)^2}{2C_2} \qquad (3)$$

sharlarning razryadlanish ro'y bergandan keyingi energiyalari. Shu bilan birga sharlar ulangandan keyin ularning potensiallari tenglashadi, ya'ni

$$\varphi_1 = \varphi_2, \quad \text{yoki} \quad \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \tag{4}$$

Shuningdek, zaryadning saqlanish qonuniga muvofiq:

$$Q_1 + Q_2 = Q_1' + Q_2', \quad \text{yoki} \quad Q_2' = (Q_1 + Q_2) - Q_1'. \tag{5}$$

(5) ni (4) ga qo'ysak, Q_1' uchun quyidagini olamiz:

$$Q_1' = \frac{(Q_1 + Q_2)C_1}{C_1 + C_2}. \tag{6}$$

Shu usul bilan Q_2' ni ham topish mumkin

$$Q_2' = \frac{(Q_1 + Q_2)S_2}{S_1 + S_2}. \tag{7}$$

Q_1' va Q_2' lar uchun topilgan ifodalarni (3) ga qo'yib sharlarning razryadlanish ro'y bergandan keyingi energiyalari uchun quyidagini olamiz:

$$W_1' = \frac{(Q_1 + Q_2)^2 C_1}{2(C_1 + C_2)^2} \quad \text{va} \quad W_2' = \frac{(Q_1 + Q_2)^2 C_2}{2(C_1 + C_2)}. \tag{8}$$

Sharning elektr sig'imi $S = 4\pi\varepsilon_0\varepsilon R$ ekanligini nazarda tutib va (2), (8) larni (1) ga qo'yib, quyidagini olamiz:

$$\Delta W = \frac{1}{8\pi\varepsilon_0\varepsilon}\left[\left(\frac{Q_1^2}{R_1} + \frac{Q_2^2}{R_2}\right) - \frac{(Q_1 + Q_2)^2}{R_1 + R_2}\right]. \tag{9}$$

Sharlar bo'shliqda turganligi uchun $\varepsilon = 1$; $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ \frac{\text{F}}{\text{m}}$ va boshqa berilganlarni (9) ga qo'yamiz:

$$\Delta W = \frac{1}{8 \cdot 3{,}14 \cdot 8{,}85 \cdot 10^{-12}}\left[\left(\frac{(4 \cdot 10^{-8})^2}{5 \cdot 10^{-2}} + \frac{(-2 \cdot 10^{-8})^2}{10^{-1}}\right) - \frac{(4 \cdot 10^{-8} - 2 \cdot 10^{-8})^2}{5 \cdot 10^{-2} + 10^{-1}}\right] J =$$

$$= \frac{10^{-2}}{8 \cdot 3{,}14 \cdot 8{,}85}\left[\left(\frac{16}{5} + \frac{4}{10}\right) - \frac{(4-2)^2}{5+10}\right] J = 1{,}5 \cdot 10^{-4}\, J.$$

Javob: $\Delta W = 1{,}5 \cdot 10^{-4}\,\text{J} = 150\text{mkJ}$.

3-misol: Elektr sig'imlari 1mkF, 2mkF, 3mkF bo'lgan kondensatorlar 1,1kV kuchlanishli zanjirga ulangan. 1) ular ketma-ket ulangan; 2) parallel ulangan hollarda har bir kondensatorning energiyalari aniqlansin.

Berilgan:

C_1=1 mkF=10^{-6}F;

C_2=2 mkF=$2\cdot10^{-6}$F;

C_3=3 mkF= $3\cdot10^{-6}$F;

U=1,1 kV=$1,1\cdot10^3$V.

1) $W_1^{kk}=?$, $W_2^{kk}=?$, $W_3^{kk}=?$

2) $W_1^{P}=?$, $W_2^{P}=?$, $W_3^{P}=?$

Yechish:

1. Ketma-ket ulangan kondensatorlarning energiyalarini

$$W=\frac{Q^2}{2C} \qquad (1)$$

ifodadan topamiz.

Bunday ulanishda

$$Q=Q_1=Q_2=Q_3 \qquad (2)$$

tenglik o'rinli bo'ladi. Zaryad miqdorini esa

$$Q=C_{kk}\cdot U \qquad (3)$$

ifodadan aniqlaymiz.

Ketma-ket ulangan kondensatorning umumiy sig'imi

$$\frac{1}{C_{kk}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

dek aniqlanadi. Yoki

$$C_{kk}=\frac{C_1\cdot C_2\cdot C_3}{C_2\cdot C_3+C_1C_3+C_1C_2}=\frac{6}{11}\cdot10^{-6}\text{F}. \qquad (4)$$

Demak, (2), (3) va (4) larni hamda berilganlarni e'tiborga olib har bir kondensatorning energiyasi uchun quyidagini olamiz:

$$W_1^{kk}=\frac{Q^2}{2C_1}=\frac{(C_{kk}U)^2}{2C_1}=\frac{(\frac{6}{11}\cdot10^{-6}\cdot1,1\cdot10^3)^2}{2\cdot10^{-6}}\text{J}=0,18\text{J};$$

$$W_2^{kk} = \frac{Q^2}{2C_2} = \frac{(C_{kk}\cdot U)^2}{2C_2} = \frac{(\frac{6}{11}\cdot 10^{-6}\cdot 1{,}1\cdot 10^3)^2\,\text{J}}{2\cdot 2\cdot 10^{-6}} = 0{,}09\text{J};$$

$$W_3^{kk} = \frac{Q_2}{2C_3} = \frac{(C_{kk}U)^2}{2C_3} = \frac{(\frac{6}{11}\cdot 10^{-6}\cdot 1{,}1\cdot 10^3)^2}{2\cdot 3\cdot 10^{-6}}\text{J} = 0{,}06\text{J}.$$

2) Parallel ulangan kondensatorlarning energiyalarini

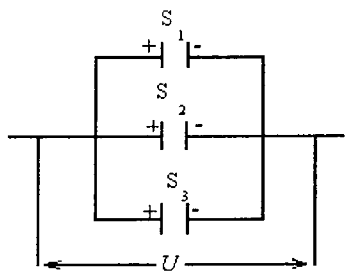
$$W = \frac{CU^2}{2} \qquad (5)$$

ifodadan topamiz.

Ma'lumki, parallel ulanganda

$$U = U_1 = U_2 = U_3 \qquad (6)$$

shart bajariladi. Endi (5) va (6) lar yordamida berilganlarni qo'yib topamiz:



44-rasm

$$W_1^P = \frac{C_1U^2}{2} = \frac{10^{-6}\cdot(1{,}1\cdot 10^3)^2\,\text{J}}{2} = 0{,}605\text{J};$$

$$W_2^P = \frac{C_2U_2}{2} = \frac{2\cdot 10^{-6}(1{,}1\cdot 10^3)^2}{2}\text{J} = 1{,}21\text{J};$$

$$W_3^P = \frac{C_3U^2}{2} = \frac{3\cdot 10^{-6}(1{,}1\cdot 10^3)^2}{2}\text{J} = 1{,}815\text{J}.$$

Javob: 1) $W_1^{kk} = 0{,}18\text{J}$; $W_2^{kk} = 0{,}09\text{J}$; $W_3^{kk} = 0{,}06\text{J}$.

2) $W_1^P = 0{,}605\text{J}$; $W_2^P = 1{,}21\text{J}$; $W_3^P = 1{,}815\text{J}$.

4-misol. Kondensator qoplamalari orasidagi tortishish kuchi 50 mN. Har bir qoplamaning yuzasi 200 sm². Kondensator maydoni energiyasining zichligi ω topilsin.

Berilgan:

$F = 50\text{mN} = 5\cdot10^{-2}\,\text{N};$

$S = 200\text{sm}^2 = 2\cdot10^{-2}\,\text{m}^2.$

$\omega = ?$

Yechish: Kondensator elektrostatik maydonining energiyasi quyidagicha aniqlanadi:

$$W = \frac{\varepsilon_0 \varepsilon E^2}{2} V. \qquad (1)$$

Energiya zichligi esa

$$\omega = \frac{W}{V} = \frac{\varepsilon_0 \varepsilon E^2}{2}, \qquad (2)$$

bu yerda: V – kondensatorning hajmi.

Ikkinchi tomondan kondensator qoplamalari orasidagi tortishish kuchi

$$F = \frac{\varepsilon_0 \varepsilon E^2}{2} S, \qquad (3)$$

dek aniqlanadi. (3) ning ko'rinishini o'zgartirib yozamiz:

$$\frac{F}{S} = \frac{\varepsilon_0 \varepsilon E^2}{2} \ . \qquad (4)$$

(2) va (4) larni solishtirib quyidagini olamiz:

$$\omega = \frac{F}{S}. \qquad (5)$$

$$[\omega] = \frac{[F]}{[S]} = \frac{1\text{N}}{1\text{m}^2} = 1\frac{\text{N}\cdot\text{m}}{\text{m}^3} = 1\frac{\text{J}}{\text{m}^3}$$

Berilganlarni o'rniga qo'yib topamiz:

$$\omega = \frac{5\cdot10^{-2}\,\text{J}}{2\cdot10^{-2}\,\text{m}^3} = 2{,}5\frac{\text{J}}{\text{m}^3}.$$

Javob: $\omega = 2{,}5\frac{\text{J}}{\text{m}^3}.$

Mustaqil yechish uchun misollar

1. 4sm radiusli, 500V potensialgacha zaryadlangan yakkalangan sharning energiyasi topilsin.[0,55 mkJ.]

2. 5sm radiusli yaxlit ebonit shar 10nC/m^3 hajmiy zichlik bilan bir tekis zaryadlangan. Shar hajmida mujassamlashgan elektrostatik maydon energiyasi topilsin.[0,164 pJ.]

3. 5sm radiusli dielektrikdan yasalgan yaxlit shar 10nC/m^3 hajmiy zichlik bilan bir tekis zaryadlangan. Shar o'rab turgan bo'shliqdagi elektrostatik maydon energiyasi topilsin. [2,46 pJ.]

4. Sirtiy zaryad zichligi 10 mkC/m^3 va potensiali 500V bo'lgan shar dielektrik kirituvchanligi 2 bo'lgan yog'ga botirilgan. 1) sharning radiusi; 2) sharning zaryadi; 3) sharning hajmi; 4)sharning energiyasi aniqlansin [1)9,74 mm; 2) 1,19 pC; 3)2,38 pF; 4) 0,3 mkJ.]

5. Elektr sig'imi 10pF bo'lgan kondensatorga 1pC zaryad berilgan. Kondensatorning energiyasi aniqlansin.[0,05 mkJ.]

6. Yassi kondensator qoplamalari orasidagi masofa 2sm, potensiallar farqi 6kV, har bir qoplamadagi zaryad 10 nC bo'lsa, kondensator maydonining energiyasi va qoplamalari orasidagi o'zaro tortishish kuchi hisoblansin. [30 mk J, 15 mH.]

7. Agar qoplamalari orasidagi potensiallar farqi 16kV, masofasi 1mm, dielektrik – slyuda va qoplamasining yuzasi 300sm^2 bo'lsa, yassi kondensatorning razryadlanishida qancha issiqlik miqdori ajraladi? [0,21 J.]

8. Yassi kondensator qoplamalari orasidagi bo'shliq 100 sm^3 bo'lib, u chinni bilan to'ldirilgan. Qoplamadagi zaryadning sirt zichligi $8{,}85\,{}^{nC}/_{m^2}$. Dielektrikni kondensatordan chiqarib olish uchun qanday ish bajarish kerak? Dielektrik va qoplamalar orasidagi ishqalanish hisobga olinmasin. [63,5 nJ.]

9. Yassi kondensator qoplamalari orasidagi potensiallar farqi 100V. Qoplamalarning har birining yuzasi 200 sm^2, orasidagi masofa 0,5mm va parafin bilan to'ldirilgan. Qoplamalar orasidagi tortishi kuchi topilsin. [7,08 mN.]

10. Har bir qoplamasining yuzasi 200 sm^2 bo'lgan kondensator 2kV potensiallar farqigacha zaryadlangan. Qoplamalar orasidagi masofa 2 sm va shisha bilan to'ldirilgan. Kondensator maydonining energiyasi va maydon energiyasining hajmiy zichligi topilsin. [124 mkJ; $\omega = 0{,}31\frac{J}{m^3}$.]

17-§. O'zgarmas elektr toki

Asosiy formulalar

O'zgarmas tokning kuchi:

$$I = \frac{Q}{t},$$

bunda: Q – o'tkazgichning ko'ndalang kesimidan t vaqtda o'tadigan elektr zaryadi.

Elektr tokining zichligi:

$$\vec{j} = \frac{I}{S}\vec{k},$$

bu yerda: S – ko'ndalang kesim yuzasi, $\vec{k}$ – yo'nalishi musbat zaryad tashuvchilar yo'nalishi bilan mos keluvchi birlik vektor.

Bir jinsli o'tkazgichning qarshiligi:

$$R = \rho\frac{l}{S} = \frac{1}{\gamma}\frac{l}{S},$$

bunda: ρ – o'tkazgichning solishtirma qarshiligi, l – uzunligi, S – ko'ndalang kesim yuzasi, γ – solishtirma o'tkazuvchanlik.

Solishtirma qarshilikning tempraturaga bog'liqligi:

$$\rho = \rho_0(1 + \alpha t),$$

ρ va ρ_0 lar mos ravishda t va 0^0C lardagi solishtirma qarshiliklar, α – qarshilikning termik koeffitsienti.

O'tkazgichlar ketma-ket ulanganda umumiy qarshilik:

$$R_{kk} = \sum_{i=1}^{n} R_i.$$

O'zkazgichlar parallel ulanganda umumiy qarshilik:

$$\frac{1}{R_p} = \sum_{i=1}^{n}\frac{1}{R_i}.$$

Ikkita o'tkazgich parallel ulangan holida:

$$R_p = \frac{R_1 \cdot R_2}{R_1 + R_2}.$$

Om qonuni: zanjirning bir jinslimas qismi uchun

$$I = \frac{(\varphi_1 - \varphi_2) \pm \varepsilon_{12}}{R} = \frac{U}{R};$$

zanjirning bir jinsli qismi uchun

$$I = \frac{\varphi_1 - \varphi_2}{R} = \frac{U}{R};$$

yopiq zanjir uchun $(\varphi_1 = \varphi_2)$

$$I = \frac{\varepsilon}{R}.$$

Bunda: $(\varphi_1 - \varphi_2)$ – zanjirning qismi uchlaridagi potensiallar farqi, ε_{12} – shu qismga kiruvchi manbaning EYuK, U – kuchlanish, R – qarshilik, ε – zanjirdagi barcha manbalarning EYuK.

Kirxgofning birinchi qoidasi: tugunda qo'shiluvchi tok kuchlarining algebraik yig'indisi nolga teng:

$$\sum_{i=1}^{n} I_i = 0.$$

Kirxgofning ikkinchi qoidasi: Yopiq konturda, konturning barcha qismlaridagi kuchlanishlarning algebraik yig'indisi EYuK larning algebraik yig'indisiga teng:

$$\sum_{i=1}^{n} I_i R_i = \sum_{i=1}^{k} \varepsilon_i,$$

n – aktiv qarshiligi bo'lgan qismlar soni, k – tok manbai soni.

O'zgarmas tok zanjirida t vaqtdagi ish:

$$A = IUt.$$

Tokning quvvati:

$$P = IU.$$

Joul–Lens qonuni:

$$Q = I^2 Rt,$$

$Q - t$ vaqtda zanjir qismida ajraladigan issiqlik miqdori.

Masala yechishga misollar

2-misol. Shuntlangan ampermetr 10 A gacha tok kuchini o'lchaydi. Agar ampermetrning qarshiligi 0,02 Ω va shunt qarshiligi 5mΩ bo'lsa, ampermetr shunsiz qanday eng katta tok kuchini o'lchashi mumkin?

Berilgan:
I_0=10A;
R_A=0,02 Ω =20.10^{-3} Ω;
$R_{sh} = 5\text{m}\Omega = 5\cdot 10^{-3}\Omega.$

I=?;

Yechish: Ampermetr o'lchash mumkin bo'lgan eng katta tok kuchi Om qonuni yordamida aniqlanadi (45-rasm):

$$I = \frac{U}{R_A} \quad (1)$$

har ikkala holda ham kuchlanish bir xillligini nazarda tutib, shunt ulangan hol uchun (45-rasm).

Om qonunini yozamiz (shunt ampermetrga parallel ulanadi):

$$I_0 = \frac{U}{R'}, \quad \text{yoki} \quad U = I_0 R'. \quad (2)$$

Bu yerda: R' umumiy qarshilik bo'lib, parallel ulangan ampermetr va shunt uchun quyidagicha topiladi:

$$R' = \frac{R_A \cdot R_{sh}}{R_A + R_{sh}}. \quad (3)$$

(2)va (3) larni (1) ga qo'ysak, quyidagini olamiz:

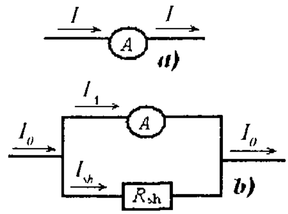
$$I = \frac{I_0 R'}{R_A} = \frac{I_0 \cdot R_A \cdot R_{sh}}{R_A(R_A + R_{sh})} = \frac{I_0 \cdot R_{sh}}{R_A + R_{sh}}.$$

Demak, $$I = \frac{I_0 \cdot R_{sh}}{R_A + R_{sh}}. \quad (4)$$

Berilganlarni (4) ga qo'yib hisoblaymiz:

$$I=\frac{10\cdot5\cdot10^{3}}{20\cdot10^{-3}+5\cdot10^{-3}}\mathrm{A}=\frac{50}{25}\mathrm{A}=2\mathrm{A}.$$

Javob: $I=2\mathrm{A}$.



45-rasm

3-misol. Ketma-ket ulangan g‘altak va ampermetr tok manbaiga ulangan. G‘altakka ichki qarshiligi 4kΩ bo‘lgan voltmetr ulangan. Ampermetr 0,3 A tok kuchini, voltmetr esa 120 V kuchlanishni ko‘rsatadi. G‘altakning qarshiligi topilsin. Agar g‘altakning qarshiligini hisoblashda voltmetrning qarshiligi hisobga olinmasa, xatolik necha foizni tashkil qiladi?

Berilgan:

$R_V=4k\Omega=4\cdot10^{3}\Omega;$

$I_A=0{,}3A;$

$U_V=120V.$

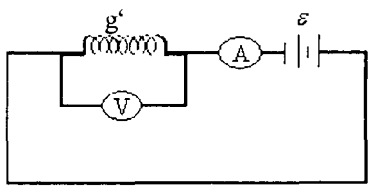
$R_{g'}=?$

$\frac{\Delta R}{R_{g'}}\cdot100\%=?$

Yechish: G‘altakning qarshiligini zanjirning bir qismi uchun Om qonunidan foydalanib topish mumkin:

$$R_{g'}=\frac{U_{g'}}{I_{g'}}=\frac{U_V}{I_{g'}}. \qquad (1)$$

Shu bilan birga



46-rasm

$$I_{g'}=I_A-I_V=I_A-\frac{U_V}{R_V}=\frac{I_A\cdot R_V-U_V}{R_V}. \qquad (2)$$

(2) ni (1) ga qo‘yib olamiz:

$$R_{g'}=\frac{U_V\cdot R_V}{I_A\cdot R_V-U_V}. \qquad (3)$$

Agar voltmetrning qarshiligi hisobga olinmasa, $I_A = I_{g'}$ bo'ladi va

$$R'_{g'} = \frac{U_V}{I_A}. \qquad (4)$$

Unda

$$\Delta R = R_{g'} - R'_{g'} = R_{g'} - \frac{U_V}{I_A}. \qquad (5)$$

Endi so'ralgan nisbatni tuzsak

$$\frac{\Delta R}{R_{g'}} = 1 - \frac{R'_{g'}}{R_{g'}} = 1 - \frac{U_V}{R_{g'} \cdot I_A} \qquad (6)$$

ni hosil qilamiz. Berilganlarni (3) va (6) larga qo'yib olamiz

$$R_{g'} = \frac{120 \cdot 4 \cdot 10^3}{0{,}3 \cdot 4 \cdot 10^3 - 120} \Omega = 444{,}4\Omega,$$

$$\frac{\Delta R}{R_{g'}} \cdot 100\% = \left(1 - \frac{120}{444{,}4 \cdot 0{,}3}\right) \cdot 100\% = 0{,}1 \cdot 100\% = 10\%.$$

Javob: $R_{g'} = 444{,}4\Omega$; $\frac{\Delta R}{R_{g'}} \cdot 100\% = 10\%$.

4-misol. Tashqi qarshilik 80 Ω bo'lganda zanjirdagi tok kuchi 0,8 A, tashqi qarshilik 15 Ω bo'lganda tok kuchi 0,5 A ga teng bo'ladi. EYuK manbayining qisqa tutashuvdagi tok kuchi I_{qt} aniqlansin.

Berilgan:	Yechish:
$R_1 = 80\Omega$;	**Yechish:** Qisqa tutashuv holida $R=0$ bo'ladi
$I_1 = 0{,}8A$;	va to'la zanjir uchun Om qonuni
$R_2 = 15\Omega$;	$I_{qt} = \frac{\varepsilon}{r}$ (1)
$I_2 = 0{,}5A$	ko'rinishni oladi. Bu yerda ε – EYuK, r – manbaning
$I_{qt} = ?$	ichki qarshiligi.

Dastlabki ikki hol uchun Om qonunini yozamiz:

$$I_1 = \frac{\varepsilon}{R_1 + r}, \quad \text{va} \quad I_2 = \frac{\varepsilon}{R_2 + r},$$

yoki bulardan

$$\varepsilon = I_1(R_1 + r), \quad \text{va} \quad \varepsilon = I_2(R_2 + r) \qquad (2)$$

Bu tengliklarni tenglashtirib,

$$I_1(R_1 + r) = I_2(R_2 + r),$$

va undan r ni topsak,

$$r = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}. \qquad (3)$$

(3) ni (2) ga qo‘ysak ε uchun topamiz:

$$\varepsilon = \frac{I_1 \cdot I_2 (R_2 - R_1)}{I_1 - I_2}. \qquad (4)$$

(3) va (4) larni (1) ga qo‘yib quyidagini olamiz:

$$I_{qt} = \frac{I_1 I_2 (R_2 - R_1)}{I_2 R_2 - I_1 R_1}. \qquad (5)$$

Kattaliklarning son qiymatlarini qo‘yib hisoblaymiz:

$$I_{qt} = \frac{0{,}8 \cdot 0{,}5(15 - 80)}{0{,}5 \cdot 15 - 0{,}8 \cdot 80}\text{A} = \frac{0{,}4(15 - 80)}{7{,}5 - 64}\text{A} = 0{,}46\text{A}.$$

Javob: $I_{qt} = 0{,}46\text{A}$.

5-misol. EYuK lari 12 V; 5 V; 10 V va ichki qarshiliklari teng 1 Ω bo‘lgan uchta batareya bir xil ismli qutblari bilan o‘zaro ulangan. Tutashtiruvchi simlarning qarshiliklari juda kichik. Har bir batareyadan oqayotgan tok kuchi aniqlansin.

Berilgan:

$\varepsilon_1 = 12V;$

$\varepsilon_2 = 5V;$

$\varepsilon_3 = 10V;$

$r = r_1 = r_2 = r_3 = 1\Omega.$

$I_1 = ?$

$I_2 = ?$

$I_3 = ?$

Yechish: Batareyalarning ulanishi va toklarning yo'nalishi 47-rasmda ko'rsatilgan. D nuqta uchun Kirxgofning birinchi qoidasini yozamiz:

$$I_2 - I_1 - I_3 = 0. \quad (1)$$

Zanjirning *ABDS* qismi

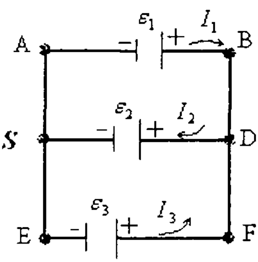
$$I_2 r_2 + I_1 r_1 = \varepsilon_1 - \varepsilon_2 \quad (2)$$

va *SDFE* qismlari uchun Kirxgofning ikkinchi qoidasi:

$$I_3 r_3 + I_2 r_2 = \varepsilon_3 - \varepsilon_2. \quad (3)$$

Bu uchta, uch noma'lumli (I_1, I_2, I_3) tenglamalardan sistema tuzsak,

$$\begin{cases} -I_1 + I_2 - I_3 = 0, \\ I_1 r_1 + I_2 r_2 = \varepsilon_1 - \varepsilon_2, \\ I_2 r_2 + I_3 r_3 = \varepsilon_3 - \varepsilon_2. \end{cases} \quad (4)$$



47-rasm

Bu tenglamalar sistemasini Kramer usulida yechsak:

$$\Delta = \begin{vmatrix} -1 & 1 & -1 \\ r_1 & r_2 & 0 \\ 0 & r_2 & r_3 \end{vmatrix} = -3; \qquad \Delta I_1 = \begin{vmatrix} 0 & 1 & -1 \\ \varepsilon_1 - \varepsilon_2 & r_2 & 0 \\ \varepsilon_3 - \varepsilon_2 & r_2 & r_3 \end{vmatrix} = -9;$$

$$\Delta I_2 = \begin{vmatrix} -1 & 0 & -1 \\ r_1 & \varepsilon_1 - \varepsilon_2 & 0 \\ 0 & \varepsilon_3 - \varepsilon_2 & r_3 \end{vmatrix} = -12; \quad \Delta I_3 = \begin{vmatrix} -1 & +1 & 0 \\ r_1 & r_2 & \varepsilon_1 - \varepsilon_2 \\ 0 & r_2 & \varepsilon_3 - \varepsilon_2 \end{vmatrix} = -3.$$

Bu yerda $r_1 = r_2 = r_3 = 1\ \Omega$ va $\varepsilon_1 - \varepsilon_2 = 7V;\ \ \varepsilon_3 - \varepsilon_2 = 5V$ ekanligi hisobga olingan.

Kramer formulalari yordamida quyidagi topamiz:

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{-9}{-3}\text{A} = 3\text{A}; \quad I_2 = \frac{\Delta I_2}{\Delta} = \frac{-12}{-3}\text{A} = 4\text{A};$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{-3}{-3}\text{A} = 1\text{A}.$$

Javob: $I_1 = 3\text{A}; \; I_2 = 4\text{A}; \; I_3 = 1\text{A}.$

6-misol. Elektr qaynatgichning chulg‘ami ikki qismdan iborat. Faqat birinchi qismi ulanganda suv 15 min da, faqat ikkinchi qismi ulanganda esa, shuncha suv 30 min da qaynaydi. Agar ikkala qism ham: 1) ketma-ket; 2) parallel ulansa shu suv necha minutda qaynaydi?

Berilgan:

$t_1 = 15\,\text{min} = 900\text{s};$

$t_2 = 30\,\text{min} = 1800\text{s}.$

$1)\, t_{kk} = ?$

$2)\, t_p = ?$

Yechish: Barcha hollarda qaynaydigan suvning miqdori bir xil bo‘lganidan, qaynatish uchun zarur bo‘lgan issiqlik miqdorlari ham teng bo‘ladi, ya’ni

$$Q = Q_1 = Q_2 = Q_{kk} = Q_p. \qquad (1)$$

Shuningdek, 48-rasmdan ko‘rinib turibdiki,

$$I = I_1 = I_2 = I_3 = I_4. \qquad (2)$$

Joul-Lens qonuniga muvofiq birinchi qismdan ajraladigan issiqlik miqdori

$$Q = I^2 R_1 t_1, \qquad (3)$$

ikkinchi qismdan

$$Q = I^2 R_2 t_1. \qquad (4)$$

Ketma-ket ulangan

$$Q = I^2 \cdot R_{kk} t_{kk} = I^2 (R_1 + R_2) t_{kk}, \qquad (5)$$

va parallel ulangan hollar uchun

$$Q = I^2 R_P t_P = I^2 \left(\frac{R_1 \cdot R_2}{R_1 + R_2} \right) t_P. \qquad (6)$$

(3)va (4) larni tenglashtirib olamiz

$$I^2R_1t_1 = I^2R_2t_2, \quad \text{yoki} \quad R_1t_1 = R_2t_2.$$

Bundan

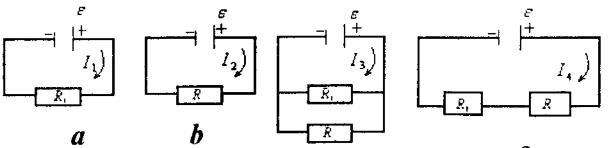
$$R_1 = \frac{R_2t_2}{t_1}. \tag{7}$$

(4) va (5) larni tenglashtirib olamiz:

$$I^2R_2t_2 = I^2(R_1 + R_2)t_{kk}, \text{ yoki} \quad R_2t_2 = (R_1 + R_2)t_{kk}.$$

Bundan

$$t_{kk} = \frac{R_2t_2}{R_1 + R_2}$$



a *b* *d* *e*

48-rasm

(7) ni e'tiborga olsak t_{kk} uchun topamiz

$$t_{kk} = \frac{t_1 \cdot t_2}{t_1 + t_2}. \tag{8}$$

Shuningdek (4) va (6) larni tenglashtirib

$$t_p = \frac{(R_1 + R_2)\cdot t_2}{R_1}$$

va (7) ni e'tiborga olib t_p uchun topamiz:

$$t_p = t_1 + t_2. \tag{9}$$

Berilganlarni (8) va (9) larga qo'ysak.

$$t_{kk} = \frac{900\cdot 1800}{900+1800}\text{s} = 600\text{s} = 10\text{min};$$

$$t_p = 900\text{s} + 1800\text{s} = 2700\text{s} = 45\text{min}.$$

Javob: $t_{kk} = 10\,\text{min}$; $t_p = 45\,\text{min}$.

7-misol. Qarshiligi 12 Ω bo'lgan o'tkazgichdagi tok kuchi 10 s davomida 5 A dan 0 gacha bir tekisda kamayadi. Shu vaqtda o'tkazgichdan qancha issiqlik miqdori ajraladi?

Berilgan:	**Yechish:** O‘zgaruvchan tok uchun Joul-Lens qonuniga muvofiq cheksiz kichik
$R=12\ \Omega$;	
$I_0=5$A;	*dt* vaqtda o‘tkazgichda ajraladigan issiqlik miqdori:
$I=0$	$dQ = I^2(t)\cdot Rdt$. (1)
$\Delta t = 10$s	Agar tok o‘zgaruvchan ekanligini nazarda tutsak,
$Q=?$	uni chiziqli funksiya sifatida tasvirlashimiz mumkin:

$$I(t) = k\cdot t. \qquad (2)$$

k koeffitsiyenti quyidagicha aniqlanadi:

$$k = \frac{\Delta I}{\Delta t} = \frac{I - I_0}{\Delta t}. \qquad (3)$$

(2) ni (1) ga qo‘yib topamiz:

$$dQ = k^2 Rt^2 dt. \qquad (4)$$

$\Delta t = t - t_0$ ekanidan $t_0 = 0$ dan $t = 10$s oraliq uchun integrallab so‘ralgan issiqlik miqdorini topamiz:

$$Q = \int_0^t k^2 Rt^2 dt = k^2 R\int_0^t t^2 dt = \frac{1}{3}k^2 Rt^3. \qquad \textbf{(5)}$$

(3) ni (5)ga qo‘ysak,

$$Q = \frac{1}{3}\left(\frac{I - I_0}{\Delta t}\right)^2\cdot Rt^3. \qquad (6)$$

$$[Q] = \frac{[I]^2}{[t]^2}[R][t]^3 = \frac{1\text{A}^2}{1\text{s}^2}\cdot 1\,\Omega\cdot 1\text{s}^3 = 1\text{A}^2\cdot\Omega\cdot\text{s} = 1\text{J}.$$

Berilganlarni hamda $\Delta t = t - t_0 = t$ ekanligini hisobga olib, quyidagini topamiz:

$$Q = \frac{1}{3}(0-5)^2\cdot 12\cdot 10\text{J} = 1000\text{J} = 1\text{kJ}.$$

Javob: $Q=1$ kJ.

Mustaqil yechish uchun misollar

1. O'tkazgichdagi tok kuchi 5 s davomida 0 dan 2 A gacha tekis o'sadi. O'tkazgichdan o'tgan zaryad miqdori aniqlansin. [5C.]

2. Ko'ndalang kesim yuzasi 1,6 mm^2 bo'lgan o'tkazgichdan 2s davomida $2\cdot10^{19}$ ta elektron o'tdi. Tok zichligi aniqlansin. [1A/mm^2.]

3. Uzunligi 20 m bo'lgan temir simning uchlariga 16 V kuchlanish qo'yilgan. Uning ko'ndalang kesim yuzasidan bir sekundda o'tadigan elektronlar soni aniqlansin. [$5\cdot10^{25}$.]

4. Uchta R_1, R_2, R_3 qarshiliklar o'zaro parallel ulangan. Ampermetr esa qarshiliklar batareyasiga ketma-ket ulangan. Agar R_2=2 Ω, R_3=6 Ω bo'lsa, unda R_1 dan oqadigan tok I_1=0,5 A bo'ladi va ampermetr I=1,5 A ni ko'rsatadi. R_1 qarshilik va R_2, R_3 qarshiliklardan oqadigan tok kuchlari I_1, I_2 lar aniqlansin. [3 Ω; 0,75 A; 0,25 A.]

5. Kuchlanishi 100 V bo'lgan manbaga qarshiligi 2 k Ω bo'lgan g'altak va voltmetr ketma-ket ulangan. Voltmetr 80 V ni ko'rsatdi. G'altakni boshqasi bilan almashtirishganda voltmetr 60 V ni ko'rsatadi. Ikkinchi g'altakning qarshiligi topilsin. [750 Ω.]

6. Batareyaning EYuK 12 V. Tok kuchi 4A bo'lganda uning FIK 0,6 bo'lsa, batareyaning ichki qarshiligi aniqlansin. [1,2 Ω.]

7. 3 1.3 tok kuchida akkumulyatorlar batareyasining tashqi zanjirida 18W quvvat ajraladi, 1A tok kuchida esa, mos ravishda 10W. Batareyaning EYuK va ichki qarshiligi aniqlansin. [12 V, 2 Ω.]

8. Batareyaning EYuK 20V. Tashqi zanjirning qarshiligi 2 Ω, tok kuchi 4 A. Batareyaning FIK topilsin. Tashqi qarshilik R ning qanday qiymatida FIK 99% ga teng bo'ladi? [0,4; 297 Ω.]

9. 15 Ω qarshilikli o'tkazgichdagi tok kuchi 5 s davomida 0 dan biror maksimal qiymatgacha bir tekisda o'sadi. Bu vaqtda o'tkazgichdan 10 kJ issiqlik miqdori ajraladi. Shu vaqt oralig'i uchun o'tkizgichdagi tok kuchining o'rtacha qiymati $\langle I\rangle$ topilsin. [10 A.]

10. O'tkazgichdagi tok kuchi 10 s vaqt davomida 0 dan biror maksimal qiymatgacha bir tekisda ortadi. Bu vaqt ichida o'tkazgichda 1kJ issiqlik miqdori ajraldi. Agar o'tkazgichning qarshiligi 3 Ω bo'lsa, undagi tokning o'sish tezligi aniqlansin. [1A/s.]

18-§. Metallarda, suyuqliklarda va gazlarda elektr toki

Asosiy formulalar

Zaryad tashuvchilar batartib harakatining o'rtacha tezligi $\langle\vec{V}\rangle$, konsentratsiyasi n va tok zichligi $\vec{j}$ orasidagi munosabat:

$$\vec{j} = e \cdot n\langle\vec{V}\rangle,$$

bu yerda: $e = 1{,}6 \cdot 10^{-19}$ C – elementar zaryad.

Om qonunining differensial ko'rinishi:

$$\vec{j} = \gamma \vec{E},$$

γ – solishitirma o'tkazuvchanlik, $\vec{E}$ – elektr maydon kuchlanganligi.

Joul–Lens qonunining differinsial ko'rinishi:

$$\omega = \gamma E^2,$$

ω – energiyaning hajmiy zichligi.

Solishtirma elektr o'tkazuvchanlik

$$\gamma = \frac{1}{2}\frac{e^2\langle l\rangle n}{m \cdot u}$$

ifoda bilan aniqlanadi. Bu yerda m – elektronning massasi, $< l >$– o'rtacha erkin yugurish uzunligi, u – issiqlik betartib harakatining o'rtacha tezligi.

Videman-Frans qonuni:

$$\frac{\lambda}{\gamma} = 3\frac{k^2}{e^2}T.$$

Bunda: λ – solishtirma issiqlik o'tkazish koeffitsienti, k – Bolsman doimiysi, T – absolyut harorat.

Termoparada vujudga keladigan issiqlik EYuK

$$\varepsilon = \alpha \cdot \Delta T = \alpha(T_1 - T_2),$$

bunda: α – solishtirma issiqlik EYuK yoki termopara doimiysi ΔT – uloqlardagi haroratlar farqi.

Elektroliz uchun Faradeyning birinchi qonuni: $m = ka$;

ikkinchi qonuni: $k = M/(Fz)$

va umumlashgan qonunlari:

$$m = \frac{1}{F}\frac{M}{z}Q = \frac{1}{F}\frac{M}{z}I \cdot t\,.$$

Ionlarning harakatchanligi:

$$b = \frac{\langle \vartheta \rangle}{\mathrm{E}}\,.$$

Bunda: $\langle \vartheta \rangle$ – ionlar betartib harakatining o‘rtacha tezligi, E – elektr maydon kuchlanganligi.

Gazlardagi to‘yinishdan uzoq bo‘lgan mustaqil razryadlar sohasi va letrolitlar uchun Om qonuning differensial ko‘rinishi

$$\vec{j} = Q \cdot n \cdot (b_+ + b_-)\vec{\mathrm{E}},$$

Bunda: Q – ionning zaryadi, n – ularning konsentratsiyasi, b_+ va b_- – mos ravishda musbat va manfiy ionlarning harakatchanligi.

To‘yinish tokning zichligi:

$$j_{to'y} = Q \cdot n_0 d\,,$$

bunda: d – elektrodlar orasidagi masofa, $n_0 = \frac{N}{V \cdot t}$ – vaqt birligida, birlik hajmda hosil bo‘ladigan juft ionlar soni, V – elektrodlar orasidagi fazoning hajmi.

Masala yechishga misollar

1-misol. Alyuminiy simdagi tok zichligi $1 A/mm^2$. Alyuminiy hajm birligidagi erkin elektronlar soni atomlari soniga teng deb faraz qilib, elektronlar betartib harakatining o'rtacha tezligi aniqlansin.

Berilgan:

$$j = 1\frac{A}{mm^2} = 10^6\frac{A}{m^2};$$

$$n_e = n_A.$$

$$\langle\vartheta\rangle = ?$$

Yechish: Tok zichligi va elektronlar betartib harakatining o'rtacha tezligi orasidagi munosabatni yozamiz:

$$j = en\langle\vartheta\rangle. \qquad (1)$$

Bundan

$$\langle\vartheta\rangle = \frac{j}{en} \qquad (2)$$

bu yerda: $e = 1{,}6\cdot 10^{-19}C$ – elektronning zaryadi. $n = n_e$ – elektronlarning konsentratsiyasi:

$$n = n_e = n_A = \frac{N_A}{V_M} = \frac{N_A\cdot\rho}{M}, \qquad (3)$$

$N_A = 6{,}02\cdot 10^{23}mol^{-1}$ – Avogadro soni, $V_M = \frac{M}{\rho}$ – molyar hajm.

$M = 27\cdot 10^{-3}\,{kg}/{mol}$ – alyuminiy molyar massasi,

$\rho = 267\cdot 10^{3}\,{kg}/{m^3}$ – zichligi. (3) ni (2) ga qo'yib olamiz:

$$\langle\vartheta\rangle = \frac{j\cdot M}{N_A\cdot\rho\cdot e}; \qquad (4)$$

$$[\vartheta] = \frac{[j][M]}{[N_A][\rho][e]} = \frac{1\frac{A}{m^2}\cdot 1\frac{kg}{mol}}{1mol^{-1}\cdot 1\frac{kg}{m^3}\cdot 1C} = 1\frac{C\cdot m}{s\cdot C} = 1\frac{m}{s}.$$

Kattaliklarning qiymatini (4) ga qo'yib olamiz:

$$\langle \vartheta \rangle = \frac{10^6 \cdot 27 \cdot 10^{-3}}{6{,}02 \cdot 10^{23} \cdot 2{,}7 \cdot 10^3 \cdot 1{,}6 \cdot 10^{-19}} \frac{\text{m}}{\text{s}} \approx 10^{-4} \frac{\text{m}}{\text{s}} = 0{,}1 \frac{\text{mm}}{\text{s}}.$$

Javob: $\langle \vartheta \rangle = 0{,}1 \dfrac{\text{mm}}{\text{s}}$.

2-misol. 6 sm³ hajmli mis o‘tkazgichdan 1 min davomida tok o‘tganida 216 J issiqlik miqdori ajralib chiqdi. O‘tkazgichdagi elektr maydon kuchlanganligi *E* hisoblansin.

Berilgan:

$V = 6\text{sm}^3 = 6 \cdot 10^{-6}\text{m}^3$;

$t = 1\text{min} = 60\text{s}$;

$Q = 216\text{J}$;

$\rho = 1{,}7 \cdot 10^{-8}\,\Omega \cdot \text{m}$.

E=?

Yechish: Joul-Lens qonunining differensial ko‘rinishini yozamiz:

$$\omega = \gamma E^2. \qquad (1)$$

Ikkinchi tomondan issiqlik energiyasining hajmiy zichligi

$$\omega = \frac{Q}{V \cdot t} \qquad (2)$$

kabi aniqlash mumkin. (1) va (2) larni tenglashtirsak:

$\gamma E^2 = \dfrac{Q}{V \cdot t}$ ni olamiz. Bundan

$$E = \sqrt{\frac{Q}{\gamma \cdot V \cdot t}} \qquad (3)$$

Agar $\gamma = \dfrac{1}{\rho}$ – ekanligini e’tiborga olsak, (3) ni quyidagi ko‘rinishga keltiramiz:

$$E = \sqrt{\frac{\rho \cdot Q}{V \cdot t}}. \qquad (4)$$

(4) yordamida kuchlanganlik birligini hosil qilamiz:

$$[E]^2=\frac{[\rho][Q]}{[V]\cdot[t]}=\frac{1\,\Omega\cdot m\cdot 1J}{1m^3\cdot 1s}=1\frac{V\cdot J}{A\cdot m^2\cdot s}=1\frac{V\cdot J}{C\cdot m^2}=1\frac{V^2}{m^2}.$$

$$[E]=1\frac{V}{m}.$$

$$E=\sqrt{\frac{1{,}7\cdot 10^{-8}\cdot 216}{6\cdot 10^{-6}\cdot 60}}\cdot\frac{V}{m}=0{,}1\frac{V}{m}.$$

Javob: $E=0{,}1\frac{V}{m}$.

3-misol. Agar azotning ionlashuv potensiali 14,5 V ga teng bo'lsa, azot atomini ionlashtirish uchun elektron qanday eng kichik ϑ_{min} tezlikka ega bo'lishi kerak?

Berilgan:	Yechish:
$U_i=14{,}5I;$	Elektron azot atomini ionlashtira olishi uchun kamida
$\vartheta_{min}=?$	$E_i=eU_i$ (1)

ionlashuv energiyasiga teng energiyaga ega bo'lishi kerak, ya'ni:

$$E_e=E_i \qquad (2)$$

Agar elektronning energiyasi

$$E_e=\frac{m_e\vartheta_{min}^2}{2}$$

ekanligini nazarda tutsak, (1) va (2) lar yordamida quyidagini olamiz:

$$\frac{m_e\vartheta_{min}^2}{2}=eU_i$$

Bundan esa $$\vartheta_{min}=\sqrt{\frac{2eU_i}{m_e}} \qquad (3)$$

Bu yerda: $e=1,6\cdot10^{-19}$ C – elektronning zaryadi, $m_e=9,1\cdot10^{-31}$ kg – elektronning massasi

$$[\vartheta]^2=\frac{[e][U]}{[m]}=\frac{1\text{C}\cdot1\text{V}}{1\text{kg}}=1\frac{\text{C}}{\text{kg}}\frac{\text{J}}{\text{C}}=1\frac{\text{N}\cdot\text{m}}{\text{kg}}=1\frac{\text{kg}\cdot\text{m}^2}{\text{kg}\cdot\text{s}^2}=1\frac{\text{m}^2}{\text{s}^2};$$

$$[\vartheta]=1\frac{\text{m}}{\text{s}}.$$

Kattaliklarning son qiymatlarini qo'yamiz:

$$\vartheta_{\min}=\sqrt{\frac{2\cdot1.6\cdot10^{-19}\cdot14,5}{9,1\cdot10^{-31}}}\frac{\text{m}}{\text{s}}=2,3\cdot10^6\frac{\text{m}}{\text{s}}.$$

Javob: $\vartheta_{\min}=2,3\cdot10^6\frac{\text{m}}{\text{s}}$.

4-misol. Azot rentgen nurlanishi bilan ionlanadi. Agar muvozanat holatida gazning har bir kub santimetrida $10^7\frac{1}{\text{sm}^3}$ juft ion bo'lsa, azotning o'tkazuvchanligi γ aniqlansin. Musbat ionlarning harakatchanligi $b_+=1,28\,\text{sm}^2/(\text{V}\cdot\text{s})$, manfiylariniki $b_-=1,81\,\text{sm}^2/(\text{V}\cdot\text{s})$

Berilgan:

$n=10^7\frac{1}{\text{sm}^3}=10^{13}\frac{1}{\text{m}^3};$

$b_+=1,27\frac{\text{sm}^2}{\text{V}\cdot\text{s}}=1,27\cdot10^{-4}\frac{\text{m}^2}{\text{V}\cdot\text{s}};$

$b_-=1,81\frac{\text{sm}^2}{\text{V}\cdot\text{s}}=1,81\cdot10^{-4}\frac{\text{m}^2}{\text{V}\cdot\text{s}}.$

$\gamma=?$

Yechish: Muvozanat holatida gaz razryadi uchun Om qonunining differensial ko'rinishi quyidagicha:

$$\vec{j}=Q\cdot n\cdot(b_++b_-)\vec{E}\cdot \quad (1)$$

Bu yerda

$$\gamma=Q\cdot n(b_++b_-) \quad (2)$$

gazning o'tkazuvchanligi. Mazkur holda $Q=e=1,6\cdot10^{-19}$ C ligini nazarda tutib berilganlar yordamida topamiz:

$$\gamma = 1,6\cdot10^{-19}\cdot10^{13}\cdot(1,27\cdot10^{-4}+1,81\cdot10^{-4})\text{sm} = 4,9\cdot10^{-10}\text{sm} = 0,49\text{nsm}$$

Javob: $\gamma = 0,49\text{nsm}$.

5-misol: Ionizatsion kamera elektrodlari orasidagi gazning hajmi 0,5 *l*. Gaz rentgen nurlari bilan ionlashtiriladi. To'yinish tokining kuchi 4нA. Bir sekundda, bir kub santimetr hamda necha juft ionlar hosil bo'ladi? Har bir ionning zaryadi elementlar zaryadiga teng.

Berilgan:

$V = 0,5l = 5\cdot10^{-4}\text{m}^3;$

$I_{to'y} = 4\text{nA} = 4\cdot10^{-9}\text{A};$

$Q = l = 1,6\cdot10^{-19}\text{C}.$

$n_0 = ?$

Yechish: To'yinish tokining zichligi ifodasini yozib olamiz:

$$I_{to'y} = Q\cdot n_0 d\,. \qquad (1)$$

Bundan vaqt birligida birlik hajmda hosil bo'ladigan ionlar soni

$$n_0 = \frac{I_{to'y}}{Q\cdot d}. \qquad (2)$$

Bu yerda d – elektrodlar orasidagi masofa.

Agar tok kuchi va tok zichligi orasidagi $I = \frac{J}{S}$ munosabatdan foydalansak, (2) quyidagi ko'rinishni oladi:

$$n_0 = \frac{J_{to'y}}{Qd\cdot S}\,. \qquad (3)$$

Maxrajdagi $d\cdot S = V$ – elektrodlar orasidagi gazning hajmi ekanligidan

$$[n_0] = \frac{[I]}{[Q][V]} = \frac{1\text{A}}{1\text{C}\cdot1\text{m}^3} = 1\frac{\text{C}}{\text{s}\cdot\text{C}\cdot\text{m}^3} = 1\frac{1}{\text{m}^3\cdot\text{s}} = 10^6\frac{1}{\text{sm}^3\cdot\text{s}}$$

Kattaliklarning qiymatlari o'rniga qo'yib, hisoblaymiz:

$$n_0 = \frac{4\cdot10^{-9}}{1,6\cdot10^{-19}\cdot5\cdot10^{-4}}\frac{1}{\text{m}^3\cdot\text{s}} = 0,5\cdot10^{14}\frac{1}{\text{m}^3\cdot\text{s}} = 5\cdot10^7\frac{1}{\text{sm}^3\cdot\text{s}}.$$

Javob: $n_0 = 5\cdot10^7\frac{1}{\text{sm}^3\cdot\text{s}}$.

Mustaqil yechish uchun masalalar

1. Ko'ndalang kesimning yuzasi $1mm^2$ bo'lgan mis o'tkazgichdan 10 A tok oqmoqda. Har bir atomiga ikkitadan zaryad tashuvchi elektron to'g'ri keladi deb hisoblab, misdagi elektronlar batartib harakatining o'rtacha tezligi $\langle \vartheta \rangle$ aniqlansin. [37m/s.]

2. Uzunligi 2m, ko'ndalang kesimining yuzasi 0,4 mm^2 bo'lgan mis o'tkazgichdan tok o'tgan har bir sekundda 0,35 J issiqlik miqdori ajraladi. Bu o'tkazgichning ko'ndalang kesim yuzasidan bir sekundda nechta elektron o'tadi? [$1{,}27 \cdot 10^{19}$.]

3. Metall o'tkazgich $a=100m/s^2$ tezlanish bilan harakatlanmoqda. Erkin elektronlar modelidan foydalanib, o'tkazgichdagi elektr maydon kuchlanganligi aniqlansin.[568PV/m.]

4. Metalning solishtirma qarshiligi 10 msm/m. Agar erkin elektronlarning konsentratsiyasi $10^{28}m^{-3}$ bo'lsa, ularning erkin yugurish yo'li $\langle l \rangle$ topilsin. Elektronlar batartib harakatining o'rtacha tezligi $1\frac{mm}{s}$. [71nm.]

5. Kavsharlaridagi haroratlar farqi $50^0 K$ bo'lgan, 4Ω qarshilikli termopara va 80 Ω qarshilikli galvonometrdan tashkil topgan zanjirdagi tok kuchi 26 mkA. Termopara doimiysi aniqlansin. $\left[4{,}4 \cdot 10^{-5}\, V/K\right]$

6. Ikkita elektrolitik vannalar ketma-ket ulangan. Birinchi vannada 3,9 g rux, ikkinchisida esa shu vaqt davomida 2,24 g temir ajraldi. Rux ikki valentli. Temirning valentligi aniqlansin. [3.]

7. Vodorod atomining ionlashish energiyasi $2{,}18 \cdot 10^{-18} J$. Vodorodning ionlashish potensiali u_i aniqlansin. [13,6V.]

8. Atomlarning ilgarilanma harakat o'rtacha kinetik energiyasi to'qnashish yo'li bilan ionlashtirishga yetarli bo'lishi uchun atomar vodorod harorati qanday bo'lishi kerak? Atomar vodorodning ionlashish potensiali 13,6V. [210K.]

9. Kondensator qoplamalari orasidagi fazoning har bir kub santimetrida bir sekundda, zaryadi elementlar zaryadiga teng bo'lgan 10^8 ta ionlar jufti hosil bo'ladi. Kondensator qoplamalarining yuzasi $100sm^2$, oralaridagi masofa $d = 1$ sm bo'lsa to'yinish tokining kuchi topilsin. $\left[1{,}6 \cdot 10^{-9} A\right]$

10. Yassi elektrodlari orasidagi masofa 5 sm bo'lgan ionizatsion kameradan $j_{to'y} = 16\, mkA/m^2$ o'tadi. Kamera ichida bir sekundda har bir kub santimetrda hosil bo'ladigan juft ionlar soni aniqlansin. $\left[2 \cdot 10^9 sm^{-3} s^{-1}\right]$

IV BOB. ELEKTR VA MAGNETIZM

19-§. Magnit maydoni. Magnit maydonining harakatlanayotgan zaryadga va tokli o'tkazgichga ta'siri

Asosiy formulalar

Bio-Savar-Laplas qonuni: $d\vec{B} = \frac{\mu_0 \mu}{4\pi}\left[d\vec{l_1}\vec{r}\right]\frac{I}{r^3}$,

yoki $d\vec{B}$ ning moduli $d\vec{B} = \frac{\mu_0 \mu}{4\pi}\frac{I \cdot \sin\alpha}{r^2}dl$.

Bunda: $\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{H}/\mathrm{m} = 4\,\pi \cdot 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2}$ – magnit doimiysi, μ – magnit singdiruvchanlik (bo'shliq uchun $\mu = 1$), dl – o'tkazgich elementi, I – tok kuchi, r_0 – o'tkazgich elementi markazidan qaralayotgan nuqtagacha bo'lgan masofa, α – $d\vec{l}$ va $\vec{r}$ lar orasidagi burchak.

Magnit maydon induksiyasi $\vec{B}$ va kuchlanganligi $\vec{H}$ vektorlari orasidagi munosabat:

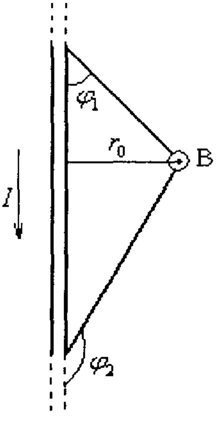
$$\vec{B} = \mu\mu_0\vec{H}.$$

Tokli aylana o'tkazgich markazidagi magnit induksiyasi:

$$B = \frac{\mu_0 \mu}{2}\frac{I}{R},$$

R – o'tkazgichning egrilik radiusi. Cheksiz uzun tokli o'tkazgichning o'z o'qida r masofadagi nuqtada hosil qiladigan magnit maydon induksiyasi

$$B = \frac{\mu_0 \mu}{2}\frac{I}{r}.$$



49-rasm

O'tkazgich bo'lagi hosil qiladigan magnit maydon induksiyasi (49-rasm):

$$B=\frac{\mu_0\mu}{4\pi}\frac{I}{r_0}(\cos\varphi_1-\cos\varphi_2).$$

O‘tkazgich uchlari magnit induksiyasi aniqlanayotgan nuqtaga nisbatan simmetrik joylashgan bo‘lsa, $\cos\varphi_2=\cos\varphi_1=\cos\varphi$ bo‘ladi.

Solenoid o‘zining o‘rta qismida (toroid o‘z o‘qida) hosil qiladigan magnit maydon induksyasi:

$$B=\mu_0\mu nI.$$

Magnit maydonining superpozisiya prinsipi $\vec{B}=\sum_{i=1}^{n}\vec{B_i}$.

Xususiy holda ikkita maydon uchun quyidagi o‘rinli:

$$B=\sqrt{B_1^2+B_2^2-2B_1B_2\cos\alpha}$$

$\alpha-\vec{B_1}$ va $\vec{B_2}$ lar orasidagi burchak.

Amper qonuni:

$$\vec{F}=I[\vec{l}\cdot\vec{B}],$$

yoki $\vec{F}$ vektorning moduli $F=IlB\sin\alpha$,

$\alpha-\vec{l}$ va $\vec{B}$ vektorlar orasidagi burchak.

Bir-biridan d masofada joylashgan, I_1 va I_2 tok oqayotgan parallel o‘tkazgichlarning l uzunlikli bo‘lagiga to‘g‘ri keluvchi ta’sir kuchi:

$$F=\frac{\mu_0\mu}{2\pi}\frac{I_1I_2}{d}l.$$

Tokli konturning magnit momenti: $\vec{P_m}=I\cdot\vec{S}$

va unga ta’sir etadigan mexanik moment: $\vec{M}=[\vec{P_m}\cdot\vec{B}]$,

yoki $M=P_m\cdot B\cdot\sin\alpha$.

Bu yerda: $\vec{S}$ – yuzasi S ga teng, yo‘nalishi normal bilan mos keluvchi vektor; $\alpha-\vec{P_m}$ va $\vec{B}$ vektorlar orasidagi burchak.

Magnit maydonidagi tokli konturning potensial energiyasi

$$P = \left(\overrightarrow{P_m} \cdot \overrightarrow{B}\right) = \mathrm{P}_m \cdot \mathrm{B} \cdot \cos \alpha,$$

va unga ta'sir etadigan kuch $F = \mathrm{P}_m \cdot \dfrac{\partial \mathrm{B}}{\partial x} \cdot \cos\alpha$.

Lorens kuchi $\vec{F} = q[\vec{V} \cdot \vec{B}]$ yoki $F = q\vartheta B \sin\alpha$,

$\vec{V} - q$ zaryadning harakat tezligi, $\alpha - \vec{V}$ va $\vec{\mathrm{B}}$ vektorlar orasidagi burchak.

Masala yechishga misollar

1-misol. 0,2 m radiusli ingichka o'tkazuvchi halqadan qanday I tok oqqanda, halqaning hamma nuqtalaridan bir xil 0,3 m uzoqlikda joylashgan nuqtada magnit maydon induksiyasi 2mkTl ga teng bo'ladi?

Berilgan:

$R = 0{,}2\text{m};$

$r = 0{,}3\text{m};$

$B_A = 20\text{mkTl} = 2 \cdot 10^{-5}\text{Tl}.$

$I = ?$

Yechish: A nuqtadagi magnit maydon induksiyasi halqaning har bir $I\overrightarrow{dl}$ tok elementi hosil qiladigan induksiyalarning yig'indisidan iborat (superpozitsiya prinsipi), ya'ni

$$\overrightarrow{\mathrm{B_A}} = \int_L d\vec{\mathrm{B}} \qquad (1)$$

Integrallash halqa uzunligi bo'ylab bajariladi. $d\vec{\mathrm{B}}$ ni $d\vec{\mathrm{B}}_\perp$ va $d\vec{\mathrm{B}}_{//}$ tashkil etuvchilarga ajratamiz. 50-rasmdan ko'rinib turibdiki, simmetriklik shartlaridan halqaning barcha elementlari uchun:

$$\int_L d\overrightarrow{\mathrm{B}_{//}} = 0 .$$

Demak,

$$\overrightarrow{\mathrm{B_A}} = \int_L (d\overrightarrow{\mathrm{B}_\perp} + d\overrightarrow{\mathrm{B}_{//}}) = \int_L d\vec{\mathrm{B}}_\perp . \qquad (2)$$

Shu bilan birga $dB_\perp = dB \cdot \cos\beta$. O'z navbatida

$$\cos\beta = \frac{R}{r} . \qquad (3)$$

Shunday qilib, (2) yordamida (1) ni yozsak,

$$B_A = \int_L dB \cdot \cos\beta \, . \qquad (4)$$

Idl — tok elementining *A* nuqtada hosil qiladigan magnit maydon induksiyasi dB ni Bio-Savar-Laplas formulasi

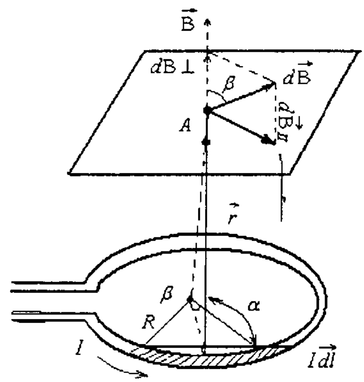
$$d\mathrm{B} = \frac{\mu_0 \mu}{4\pi} \frac{I \cdot \sin\alpha}{r^2} \cdot dl \qquad (5)$$

yordamida aniqlymiz. Mazkur masalada

$\alpha = \dfrac{\pi}{2}$ ligidan (50-rasmga qarang)

$\sin\alpha = 1$. Shuningdek, $\mu = 1$

Endi (5) ni (4) ga qo'yib va halqa uzunligi $L = 2\pi R$ ekanligini e'tiborga olsak,



50-rasm

$$B_A = \int_0^{2\pi R} \frac{\mu_0}{4\pi} \frac{I}{r^2} \cdot \cos\beta \cdot dl = \frac{\mu_0 I}{4\pi r^2} \cos\beta \int_0^{2\pi R} dl = \frac{\mu_0 I \cdot \cos\beta}{4\pi r^2} \cdot 2\pi R$$

(3) dan foydalanib yozamiz:

$$\mathrm{B}_A = \frac{\mu_0 I R^2}{2r^3} \, . \qquad (6)$$

(6) dan *I* ni topamiz:

$$I = \frac{2r^3 \mathrm{B}_A}{\mu_0 R^2} \, . \qquad (7)$$

I ning birligini tekshirib ko'ramiz:

$$[I] = \frac{[r]^3 \cdot [\mathrm{B}]}{[\mu_0] \cdot [R]^2} = \frac{1\mathrm{m}^3 \cdot 1\mathrm{Tl}}{1\dfrac{\mathrm{N}}{\mathrm{A}^2} \cdot 1\mathrm{m}^2} = 1\frac{\mathrm{A}^2 \cdot \mathrm{m}}{\mathrm{N}} \cdot \frac{\mathrm{N}}{\mathrm{A} \cdot \mathrm{m}} = 1\mathrm{A}.$$

Kattaliklarning qiymatlarini qo'yib hisoblaymiz:

$$I=\frac{2\cdot(0{,}3)^3\cdot 2\cdot 10^{-5}}{4\pi\cdot 10^{-7}\cdot(0{,}2)^2}\text{A}=21{,}5\text{A}.$$

Javob: $I=21{,}5\text{A}$.

2-misol. Ikkita cheksiz uzun, to'g'ri parallel o'tkazgichlardan qarama-qarshi yo'nalishlarda 50 A va 100 A toklar oqadi. Agar o'tkazgichlar orasidagi masofa 20 sm bo'lsa, birinchisidan 25 sm, ikkinchisidan 40 sm masofada joylashgan nuqtadagi magnit maydon induksiyasi B aniqlansin.

Berilgan:

$I_1=50A$;
$I_2=100A$;

d=20 sm=0,2 m;
r_1=25 sm=0,25 m;

r_2=40 sm=0,4 m.

$B=?$

Yechish: A nuqtadagi magnit maydon induksiyasi $\vec{B}$ superpozisiya prinsipiga muvofiq I_1 va I_2 toklar hosil qilayotgan $\vec{B}_1$ va $\vec{B}_2$ vektorlarning yig'indisiga teng (51-rasm):

$$\vec{B}=\vec{B_1}+\vec{B_2}$$

Yoki B ning moduli uchun:

$$B=\sqrt{B_1^2+B_2^2-2B_1\cdot B_2\cos\alpha}. \qquad (1)$$

B_1 va B_2 larni cheksiz uzun tokli o'tkazgichlar hosil qilishidan:

$$B_1=\frac{\mu_0\mu}{2\pi}\frac{I_1}{r_1} \quad \text{va} \quad B_2=\frac{\mu_0\mu}{2\pi}\frac{I_2}{r_2}. \qquad (2)$$

Shu bilan birga α burchak uchun kosinuslar teoremasidan:

$$d^2=r_1^2+r_2^2-2r_1r_2\cos\alpha$$

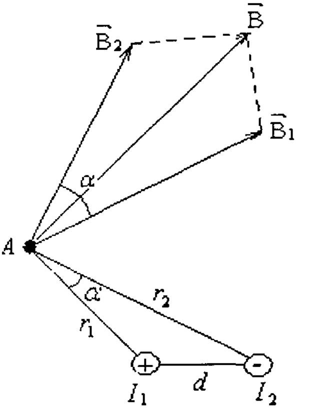
yoki $$\cos\alpha=\frac{1}{2r_1r_2}(r_1^2+r_2^2-d^2). \qquad (3)$$

(2), (3) larni (1) ga qo'yib va $\mu=1$ ligidan:

$$B=\frac{\mu_0}{2\pi}\sqrt{\frac{I_1^2}{r_1^2}+\frac{I_2^2}{r_2^2}-\frac{I_1I_2}{r_1^2r_2^2}(r_1^2+r_2^2-d^2)}. \qquad (4)$$

$$[B]=\frac{[\mu_0][I]}{[r]}=\frac{1\frac{N}{A^2}\cdot 1A}{1m}=1\frac{N}{A\cdot m}=1Tl.$$

Berilganlarning qiymatlarini qo'yib, quyidagini topamiz:



51-rasm

$$B=\frac{4\cdot 3{,}14\cdot 10^{-7}}{2\cdot 3{,}14}\sqrt{\left(\frac{50}{0{,}25}\right)^2+\left(\frac{100}{0{,}4}\right)^2-\frac{50\cdot 100}{(0{,}25)^2\cdot(0{,}4)^2}\left[(0{,}25)^2+\right.}$$

$$\left.+(0{,}4)^2-(0{,}2)^2\right]Tl=21{,}2\cdot 10^{-6}Tl=21{,}2mkTk.$$

Javob: B=21,2mkTl.

3-misol. Halqa shaklidagi o'tkazgichdan tok oqmoqda. Halqa markazidagi magnit maydon induksiyasi 60 mkTk. Tok kuchini o'zgartirmasdan o'tkazgich kvadrat shakliga keltirilgan bo'lsa, kvadratning diagonallari kesishadigan nuqtadagi magnit maydon induksiyasi nimaga teng bo'ladi?

Berilgan:
B_1=60 mkTl=6 · 10 $^{-5}$Tl;
$I=I_1=I_2$

B_2=?

Yechish: Magnit maydon induksiyasi vektorining superpozitsiya prinsipiga asosan kvadratning diagonallari kesishgan nuqtadagi induksiyani, kvadrat tomonlari hosil qiladigan induksiyalar yig'indisi sifatida topamiz, ya'ni

$$B_2=4B_0\,. \qquad (1)$$

B_0 o'tkazgich bo'lagi hosil qiladigan induksiya sifatida aniqlanadi. Shu bilan birga nuqta o'tkazgich uchlariga nisbatan simmetrik joylashgan:

$$B_0 = \frac{\mu_0 \mu}{2\pi} \frac{I}{r} \cos\varphi \,. \qquad (2)$$

Demak,

$$B_2 = \frac{2\mu_0 \mu}{\pi} \frac{I}{r} \cos\varphi \,. \qquad (3)$$

Halqa markazidagi magnit maydon induksiyasi:

$$B_1 = \frac{\mu_0 \mu}{2} \frac{I}{R} \,. \qquad (4)$$

Bu yerda: R – halqaning radiusi, halqa uzunligi (perimetri) esa

$$S = 2\pi R \,.$$

Agar halqa piremetri kvadratning piremetriga tengligini e'tiborga olsak,

$$2\pi R = 4l \,,$$

yoki

$$R = \frac{2l}{\pi} \,. \qquad (5)$$

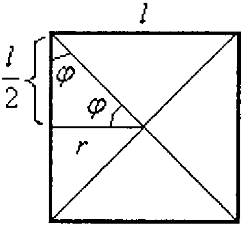
(4) ni (5) yordamida qayta yozsak,

$$B_1 = \frac{\pi \mu_0 \mu I}{4l} \,. \qquad (6)$$

(6) dan tok kuchini aniqlaymiz va (3) ga qo'yamiz:

$$B_2 = \frac{8l\mathrm{B}_1}{\pi^2 r} \cos\varphi \,.$$

Agar $r = \frac{l}{2}$ va $\varphi = \frac{\pi}{4}$, $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ ligini e'tiborga olsak (52-rasm):



52-rasm

$$B_2 = \frac{8\sqrt{2}}{\pi^2} B_1 \,. \qquad (7)$$

Kattaliklarning son qiymatlarini qo'yamiz:

$$B_2 = \frac{8\sqrt{2}}{(3{,}14)^2} \cdot 6 \cdot 10^{5}\,\mathrm{Tl} = 6{,}68 \cdot 10^{-5}\,\mathrm{Tl} = 68{,}6\,\mathrm{mkTl}$$

Javob: B_2=68,6 mkTl.

4-misol. Ikkita ingichka o'tkazgichlar 10 sm radiusli halqalardek qayirilgan va har biridan 10A dan tok oqadi. Agar halqalar yotgan tekisliklar parallel va ularning markazlari orasidagi masofa 1 mm bo'lsa, halqalarning o'zaro ta'sir kuchi topilsin.

Berilgan:

$R = 10\text{sm} = 0{,}1\text{m};$

$I_1 = I_2 = I = 10\text{A};$

$d = 1\text{mm} = 10^{-3}\text{m}$

$F = ?$

Yechish: Uzunliklari dl dan bo'lgan ikkita o'tkazgich elementlari orasidagi ta'sir kuchi quyidagi ifodadan aniqlanadi:

$$dF = \frac{\mu_0 \mu}{2\pi}\frac{I_1 I_2}{d} dl. \qquad (1)$$

Halqalar orasidagi ta'sir kuchini topish uchun (1) ni butun halqa uzunligi bo'ylab, ya'ni $l = 0$ dan $l = 2\pi R$ gacha integrallaymiz:

$$F = \int_0^{2\pi R} \frac{\mu_0 \mu}{2\pi}\frac{I_1 I_2}{d} dl = \frac{\mu_0 \mu}{2\pi}\frac{I_1 I_2}{d}\int_0^{2\pi R} dl = \frac{\mu_0 \mu}{2\pi}\frac{I_1 I_2}{d} 2\pi R = \frac{\mu_0 \mu I_1 I_2 R}{d} \qquad (2)$$

Agar $I_1 = I_2 = I$ va $\mu = 1$ ligini e'tiborga olsak, (2) quyidagi ko'rinishni oladi:

$$F = \frac{\mu_0 I^2 R}{d}, \qquad (3)$$

$$[F] = \frac{[\mu_0][I]^2[R]}{[d]} = \frac{1\frac{\text{N}}{\text{A}^2}\cdot 1\text{A}^2 \cdot 1\text{m}}{1\text{m}} = 1\text{N}.$$

Kattaliklrning qiymatlarini (3) ga qo'yib olamiz $\left(\mu_0 = 4\pi \cdot 10^{-7}\frac{\text{N}}{\text{A}^2}\right)$

$$F = \frac{4\pi \cdot 10^{-7} \cdot (10)^2 \cdot 0{,}1}{10^{-3}}\text{N} = 12{,}6 \cdot 10^{-3}\text{N} = 12{,}6\text{mN}.$$

Javob: F=12,6 mN.

5-misol. Halqa markazidagi magnit maydon induksiyasi 251,2mkTl. Halqaning magnit momenti 1A · m², halqaning radiusi va undagi tok kuchi topilsin.

Berilgan:

$B = 251{,}2\text{mkTl} = 251{,}2 \cdot 10^{-6}\text{Tl};$

$P_m = 1\text{Am}^2.$

$R = ?$

$I = ?$

Yechish: Halqa markazidagi induksiyani

$$B = \frac{\mu_0 \mu}{2} \frac{I}{R} \qquad (1)$$

ifodadan aniqlaymiz. Bundan

$$I = \frac{2RB}{\mu_0 \mu}. \qquad (2)$$

Shuningdek, tokli Halqaning magnit momenti

$$P_m = I \cdot S = I \cdot \pi R^2. \qquad (3)$$

Bu yerda halqa o'ragan yuza $S = \pi R^2$ ekanligi e'tiborga olingan. (3) dan tok kuchini aniqlasak,

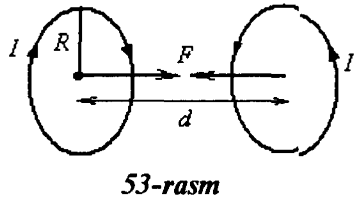
$$I = \frac{P_m}{\pi R^2}. \qquad (4)$$

(2) va (4) ni tenglashtirib R ni aniqlaymiz:

$$\frac{2RB}{\mu_0 \mu} = \frac{P_m}{\pi R^2} \qquad \text{yoki} \qquad R^3 = \frac{\mu_0 \mu P_m}{2\pi B},$$

$$R = \sqrt[3]{\frac{\mu_0 \mu P_m}{2\pi B}}. \qquad (5)$$

(5) ni (2) ga qo'yib I uchun quyidagini hosil qilamiz:



53-rasm

$$I = \frac{2B}{\mu_0 \mu} \sqrt[3]{\frac{\mu_0 \mu P_m}{2\pi B}} = \sqrt[3]{\frac{4B^2 P_m}{\pi \mu_0^2 \mu^2}} \,. \qquad (6)$$

Endi (5)-va (6) larni tekshirib ko'raylik

$$[R]^3 = \frac{[\mu_0][P_m]}{[B]} = \frac{1\frac{N}{A^2} \cdot 1A \cdot m^2}{1Tl} = 1\frac{N \cdot m^2 \cdot A \cdot m}{A \cdot N} = 1m^3; \quad [R] = 1m.$$

$$[I]^3 = \frac{[B]^2 [P_m]}{[\mu_0]^2} = \frac{1Tl^2 \cdot 1A \cdot m^2}{1\frac{N^2}{A^4}} = 1\frac{A^5 \cdot m^2 \cdot N^2}{N^2 \; A^2 \cdot m^2} = 1A^3; \quad [I] = 1A.$$

Kattaliklarning qiymatlarini (5) va (2) larga qo'yib olamiz $(\mu = 1)$:

$$R = \sqrt[3]{\frac{4\pi \cdot 10^{-7} \cdot 1}{2\pi \cdot 251{,}2 \cdot 10^{-6}}} m = 9{,}27 \cdot 10^{-2} m = 9{,}27 sm;$$

$$I = \frac{2 \cdot 9{,}27 \cdot 10^{-2} \cdot 251{,}2 \cdot 10^{-6}}{4\pi \cdot 10^{-7}} A = 37{,}08 A.$$

Javob: R=9,27 sm; I =37,08A.

6-misol. Induksiyasi 0,2Tl bo'lgan magnit maydonidagi elektronning aylanma orbita bo'ylab aylanish chastotasi aniqlansin.

Berilgan:	Yechish.
B =0,2Tl.	**Yechish.** Elektronning aylanma orbita bo'ylab aylanish chastotasi quyidagicha aniqlanadi.
$n = ?$	$n = \frac{\omega}{2\pi} = \frac{\vartheta}{2\pi R}$. (1)

Elektron aylanma orbita bo'ylab aylanganida unga ta'sir etuvchi kuchlar: Lorens kuchi:

$$F_l = q_e \vartheta B \sin\alpha \qquad (2)$$

va markazdan qochma kuch:

$$F_{m.k} = \frac{m_e \vartheta^2}{R} \qquad (3)$$

teng bo'lmog'i kerak. Ya'ni $F_l = F_{mk}$, yoki

$$q_e \cdot \vartheta \cdot \mathrm{B} \cdot \sin\alpha = \frac{m_e \vartheta^2}{R}. \qquad (4)$$

Mazkur holda $\alpha = \frac{\pi}{2}$ va $\sin\alpha = 1$. (4) dan ϑ ni topsak,

$$\vartheta = \frac{q_e \cdot \mathrm{B}R}{m_e}. \qquad (5)$$

(5) ni (1) ga qo'yib quyidagini topamiz:

$$n = \frac{q_e \cdot B}{2\pi m_e}; \qquad (6)$$

$$[n] = \frac{[q][\mathrm{B}]}{[m]} = \frac{1\mathrm{C} \cdot 1\mathrm{Tl}}{1\mathrm{kg}} = 1\frac{\mathrm{C} \cdot \mathrm{N}}{\mathrm{kg} \cdot \mathrm{A} \cdot \mathrm{m}} = 1\frac{\mathrm{C} \cdot \mathrm{s} \cdot \mathrm{kg} \cdot \mathrm{m}}{\mathrm{kg} \cdot \mathrm{C} \cdot \mathrm{m.s}^2} = 1\mathrm{s}^{-1} = 1\mathrm{Hz}.$$

Kattaliklarning qiymatlarini (6)ga qo'yib olamiz.

$$(q_e = 1{,}6 \cdot 10^{-19}\,C; \mathrm{m}_e = 9{,}1 \cdot 10^{-31}\,\mathrm{kg})$$

$$n = \frac{1{,}6 \cdot 10^{-19} \cdot 0{,}2}{2 \cdot 3{,}14 \cdot 9{,}1 \cdot 10^{-31}}\,\mathrm{Hz} = 5{,}6 \cdot 10^9\,\mathrm{Hz}$$

Javob: $n = 5{,}6 \cdot 10^9\,\mathrm{Hz}$.

Mustaqil yechish uchun masalalar

1. Maydonning vakuumdagi magnit induksiyasi 10 mTl bo'lsa, magnit maydon kuchlanganligi N aniqlansin. [7,96 kA/m.]

2. 10 A tok oqayotgan ingichka halqa markazidagi magnit induksiya topilsin. Halqaning radiusi 5sm. [126 mkTl.]

3. Uzunligi 20 sm bo'lgan g'altak 100 o'ramdan iborat. G'altak cho'lg'amidan 5 A tok oqmoqda. G'altakning diametri 20 sm. G'altak uchidan 10 sm masofada va uning o'qida yotuvchi nuqtadagi magnit induksiyasi aniqlansin. [606 mkTl.]

4. Ikkita uzun, parallel simlar bir-biridan 5sm masofada turibdi. Simlarning har biridan qarama-qarshi yo'nalishlarda bir xil 10 A tok

oqmoqda. Simlarning biridan 2 sm, ikkinchisidan 3sm masofada turgan nuqtadagi magnit maydon kuchlanganligi topilsin. [132 A/m.]

5. Teng tomonli uchburchak ko'rinishidagi konturdan 40 A tok oqmoqda. Uchburchak tomonining uzunligi 30 sm. Balandliklari kesishadigan nuqtadagi magnit maydon induksiyasi aniqlansin. [240 mkTl.]

6. Elektron, 0,02Tl induksiyali magnit maydonida 10 sm radiusli aylana bo'ylab harakatlanmoqda. Elektronning kinetik energiyasi aniqlansin. [0,35 MeV.]

7. Agar proton induksiyasi 15 mTl bo'lgan magnit maydonida 10^6m/s tezlik bilan harakatlanayotgan bo'lsa, proton traektoriyasining egrilik radiusi aniqlansin. [1,39 m.]

8. 1,5 MeV kinetik energiyali elektron bir jinsli magnit maydonida aylanma orbita bo'ylab harakatlanmoqda. Agar maydon induksiyasi 0,02Tl bo'lsa, elektronning aylanish davri aniqlansin. [7,02 ns.]

9. Qo'sh simli elektron uzatish sistemasi bir-biridan 4mm masofada turgan uzun, parallel to'g'ri simlardan iborat. Agar simlardan teng 50 A dan tok oqayotgan bo'lsa, ularning birlik uzunligiga to'g'ri keluvchi o'zaro ta'sir kuchi topilsin. [0,125 N/m.]

10. O'ramning magnit momenti 0,2J/Tl, diametri 10sm bo'lsa, undagi tok kuchi topilsin. [25,5 A.]

11. 10 sm radiusli ingichka halqada 10nKl zaryad bor. Halqa o'z markazidan o'tuvchi va halqa tekisligiga tik yo'nalgan o'qqa nisbatan $10s^{-1}$ chastota bilan bir tekis aylanadi. Halqa hosil qiladigan aylanma tokning magnit momenti topilsin. [$3{,}14 nA \cdot m^2$.]

12. Uzunligining har bir santimetrida 5 tadan o'ram bo'lgan uzun to'g'ri solenoid magnit meridiani tekisligiga tik holatda joylashgan. Solenoidning ichida, uning o'rta qismida Yerning magnit maydoni bo'ylab joylashgan magnit strelkasi turibdi. Solenoiddan tok o'tganida strelka 60^0ga burildi. Tok kuchi topilsin. [55 mA.]

20-§. To‘la tok qonuni. Magnit oqimi va oqim ilashuvi. Induktivlik

Asosiy formulalar

Bo‘shliqdagi magnit maydoni uchun to‘la tok qonuni:

$$\oint_L \vec{B}\vec{dl} = \oint_L B_e dl = \mu_0 \sum_{i=1}^{n} I_i .$$

Ixtiyoriy muhit uchun to‘la tok qonuni

$$\oint_L \vec{H} \cdot \vec{dl} = \oint_L H_e dl = \mu_0 \sum_{i=1}^{n} I_i .$$

Bu yerda: $\oint_L \vec{B} \cdot \vec{dl} = \oint_L B_e dl$ – magnit induksiya vektorining induksiyasi,

$\oint_L \vec{H} \cdot \vec{dl} = \oint_L H_e dl$ magnit maydon kuchlanganligining uyurmasi,

$\sum_{i=1}^{n} I_i$ – kontur o‘rab turgan toklarning algebraik yig‘indisi.

S yuzali yassi kontur orqali magnit oqimi:

a) bir jinsli maydon holida.

$$\Phi = B \cdot S \cdot \cos\alpha = B_n \cdot S$$

α – S ning normali bilan $\vec{B}$ orasidagi burchak;

b) ixtiyoriy maydon uchun $\Phi = \int_S B_n dS$.

Oqim ilashuvi, ya’ni solenoid yoki toroidning barcha chulg‘amlariga ilashgan to‘la magnit oqimi $\psi = N\Phi$,

N – o‘ramlar soni.

Konturning oqim ilashuvi $\psi = L \cdot I$,

L – konturning induktivligi.

Selenoidning (toroidning) induktivligi $L = \mu_0 \mu n^2 V$,

$n=N/l$ – uzunlik birligidagi o‘ramlar soni, V – solenoidning hajmi.

Masala yechishga misollar

1-misol. Bir tomonga oquvchi I_1=10A, I_2=15A qarama-qarshi tomonga oquvchi I_3=20 A toklarni qamrab olgan kontur bo'ylab induksiya vektorining uyurmasi hisoblansin.

Berilgan:	Yechish:
I_1=10 A, I_2=15 A, I_3=20 A.	**Yechish:** Magnit maydon uchun to'la tok qonunini yozamiz
$\oint\limits_L \vec{B}\cdot\vec{dl} = ?$	$\oint\limits_L \vec{B}\cdot\vec{dl} = \mu_0 \sum\limits_{i=1}^{n} I_i$. (1)

Mazkur masalada n=3. I_1 va I_2 larning yo'nalishlarini musbat deb qabul qilsak I_3 ning yo'nalishi manfiy bo'ladi. Demak, (1) quyidagi ko'rinishni oladi:

$$\oint\limits_L \vec{B}\cdot\vec{dl} = \mu_0(I_1 + I_2 - I_3). \quad (2)$$

Toklarning qiymatini va $\mu_0 = 4\pi\cdot10^{-7}\,\dfrac{\text{N}}{\text{A}^2}$ ni (2) ga qo'ysak olamiz.

$$\oint\limits_L \vec{B}\cdot\vec{dl} = 4\pi\cdot10^{-7}(10+15-20)\text{Tl}\cdot\text{m} = 62{,}8\cdot10^{-7}\text{Tl}\cdot\text{m}$$

Javob: $\oint\limits_L \vec{B}\cdot\vec{dl} = 6{,}8\cdot10^{-7}\text{Tl}\cdot\text{m}$.

2-misol. Ko'ndalang kesim yuzasi kvadratdek bo'lgan toroidda 100 ta o'ram bor. Toroidning tashqi diametri 40 sm, ichki diametri 20 sm. Agar o'ramlardan oqayotgan tok kuchi 10 A bo'lsa, toroidning magnit oqimi aniqlansin.

Berilgan:	Yechish:
	Yechish: Toroidning magnit oqimini
N=1000=10^3;	$\Phi = \int\limits_S BdS$ (1)
D =40 sm=0,4 m;	ifodadan aniqlaymiz. Bu yerda:
d=20 sm=0,2 m;	$B = \dfrac{\mu_0 NI}{2\pi x}$ (2)

I=10A.	toroidning magnit induksiyasi.
$\Phi=?$	x –toroidning o'rta radiusi.

$2\pi x$ – o'rta radius bo'yicha uzunligi. x – kichik radius $d/2$ dan katta radius $D/2$ gacha o'zgarishi mumkin.

Elementar yuzani esa quyidagicha topamiz:

$$dS=\left(\frac{D}{2}-\frac{d}{2}\right)dx=\frac{(D-d)}{2}dx. \qquad (3)$$

(2) va (3) larni (1) ga qo'yib va integrallab quyidagini olamiz:

$$\Phi=\int_{d/2}^{D/2}\frac{\mu_0 NI}{2\pi x}\cdot\frac{(D-d)}{2}dx=\frac{\mu_0 NI}{4\pi}(D-d)\int_{d/2}^{D/2}\frac{dx}{x}=\frac{\mu_0 NI}{4\pi}(D-d)\ln\frac{D}{d}.$$

Shunday qilib, $$\Phi=\frac{\mu_0 NI}{4\pi}(D-d)\ln\frac{D}{d} \qquad (4)$$

$$[F]=[\mu_0][I][n][S]=1\frac{\text{N}}{\text{A}^2}\cdot 1\text{A}\cdot\frac{1}{\text{m}}\cdot 1\text{m}^2=1\frac{\text{N}}{\text{A}\cdot\text{m}}\cdot\text{m}^2=1\text{Tl}\cdot\text{m}^2=1\text{Vb}$$

μ_0 ning va kattaliklarning qiymatlarini (4) ga qo'yib hisoblaymiz:

$$\Phi=\frac{4\pi\cdot 10^{-7}\cdot 10^3\cdot 10}{4\pi}(0,4-0,2)\ln\left(\frac{0,4}{0,2}\right)\text{Vb}=139\cdot 10^{-6}\text{Vb}=139\text{mkV}.$$

Javob: $\Phi=139$ mkVb.

3-misol. Induktivligi 1 mH ga teng bo'ladigan bir qatlamli g'altak hosil qilish uchun diametri 2sm bo'lgan karton silindrga juda yupqa 0,4mm izolyatsiyali simdan necha o'ram o'rash kerak? O'ramlar bir-biriga yopishib turadi.

Berilgan:	**Yechish**: G'altakning induktivligini
$L=1\text{mH}=10^{-3}\text{H}$; $D=2\text{sm}=2\cdot 10^{-2}\text{m}$; $d=0,4\text{mm}=4\cdot 10^{-4}\text{m}$.	$$L=\mu_0\mu\left(\frac{N}{l}\right)^2 V, \qquad (1)$$
$N=?$	ifodadan aniqlaymiz.

Bu yerda: $V = S \cdot l$ g'altakning hajmi. $S = \dfrac{\pi D^2}{4}$ – g'altakning ko'ndalang kesim yuzasi. l – g'altakning uzunligi bo'lib $l = N \cdot d$ kabi aniqlanadi. Shunday qilib,

$$V = \frac{\pi D^2}{4} \cdot Nd. \qquad (2)$$

Unda (1) quyidagi ko'rinishni oladi:

$$L = \mu_0 \mu \left(\frac{N}{Nd}\right)^2 \cdot \frac{\pi D^2}{4} \cdot Nd = \frac{\pi D^2}{4} \cdot \frac{\mu_0 \mu N}{d}. \qquad (3)$$

Bundan o'ramlar sonini aniqlasak,

$$N = \frac{4d \cdot L}{\mu_0 \mu \pi D^2}; \qquad (4)$$

$$[N] = \frac{[d][L]}{[\mu][D]^2} = \frac{1\text{m} \cdot 1\text{H}}{\dfrac{\text{H}}{\text{m}} \cdot 1\text{m}^2} = 1$$

Kattaliklarning qiymatlarini qo'yib topamiz ($\mu = 1$)

$$N = \frac{4 \cdot 4 \cdot 10^{-4} \cdot 10^{-3}}{4 \cdot 10^{-7} \cdot (3{,}14)^2 (2 \cdot 10^{-2})^2} = 1000 = 10^3 \text{ o'ram.}$$

Javob: N=1000 o'ram.

5-misol. Qo'sh simli o'tkazgichning 1 km li qismidagi induktivligi L topilsin. Simning radiusi 1 mm, o'q chiziqlari orasidagi masofa 0,4m.

Berilgan:

$l = 1\text{km} = 10^3\,\text{m};$

$R = 1\text{mm} = 10^{-3}\,\text{m};$

$d = 0{,}4\text{m}.$

$L = ?$

Yechish: O'tkazgichning induktivligini

$$L = \frac{\Phi}{I}. \qquad (1)$$

formuladan aniqlaymiz. Bu yerda magnit oqimi

$$d\Phi = BdS. \qquad (2)$$

Mazkur holda magnit maydon induksiyasi quyidagicha aniqlanadi;

$$B = \frac{\mu_0 I}{2\pi x}, \tag{3}$$

bunda: x – sim radiusi R dan d-R gacha o'zgaradi. Shuningdek, yuza ham x ga bog'liq bo'lib

$$dS == l \cdot 2dx = 2ldx \cdot \tag{4}$$

(3) va (4) larni (2) ga qo'ysak va integrallasak quyidagini olamiz:

$$\Phi = \int_R^{(d-R)} \frac{\mu_0 I}{2\pi x} \cdot 2ldx = \frac{\mu_0 I \cdot l}{\pi} \int_R^{(d-R)} \frac{dx}{x} = \frac{\mu_0 Il}{\pi} \ln\left|\frac{d-R}{R}\right| \tag{5}$$

(5)-ni (1) ga qo'yib induktivlik ifodasini olamiz:

$$L = \frac{\mu_0 l}{\pi} \ln\left|\frac{d-R}{R}\right|. \tag{6}$$

Kattaliklarning son qiymatlarini qo'yib, topamiz:

$$L = \frac{4\pi \cdot 10^{-7} \cdot 10^3}{\pi} \ln\left|\frac{0{,}4 - 0{,}001}{0{,}001}\right| = 2{,}4 \cdot 10^{-3}\text{H} = 2{,}4\text{mH}.$$

Javob: L=2,4 mH.

6-misol. Ko'ndalang kesimning yuzasi 5 sm^2 bo'lgan solenoidda 1200 o'ram bor. 2 A tok oqqanda magnit maydon induksiyasi 0,01 Tl bo'ladi. Solenoidning induktivligi aniqlansin.

Berilgan:
S =5sm^2=5 · 10^{-4} m^2
N =1200;
I =2 A;
B =0,01 Tl=10^{-2} Tl
L=?

Yechish: Ma'lumki, solenoidning oqim ilashuvi quyidagi ifodalar yordamida aniqlanadi:

$$\psi = LI, \ (1) \text{ va } \ \psi = N \cdot \Phi \tag{2}$$

Bulardan olamiz $L \cdot I = N\Phi$

yoki

$$L = \frac{N\Phi}{I}. \tag{3}$$

Bu yerda Φ – solenoidning bitta o'ramining magnit oqimi

$$\Phi = B \cdot S \cos\alpha. \tag{4}$$

Solenoid hamda $\vec{B}$ va S ning $\vec{n}$ normallar orasidagi burchak $\alpha = 0$, $\cos\alpha = 1$.

Demak, $$L = \frac{N \cdot B \cdot S}{I}. \qquad (5)$$

$$[L] = \frac{[B]\cdot[S]}{[I]} = \frac{1\text{Tl} \cdot 1\text{m}^2}{1\text{A}} = 1\frac{\text{N} \cdot \text{m}^2}{\text{A} \cdot \text{m} \cdot \text{A}} = 1\frac{\text{N} \cdot \text{m}}{\text{A}^2} = 1\text{H}.$$

Berilganlarni qo'yib hisoblaymiz:

$$L = \frac{1200 \cdot 10^{-2} \cdot 5 \cdot 10^{-4}}{2}\text{H} = 3 \cdot 10^{-3}\text{H} = 3\ \text{mH}.$$

Javob: L=3 mH.

Mustaqil yechish uchun masalalar

1. Yuzasi 25 sm^2 bo'lgan yassi kontur 0,04 H induksiyali bir jinsli magnit maydonida turibdi. Agar kontur tekisligi induksiya chiziqlari bilan 30^0 burchak tashkil qilsa, konturga kiradigan magnit oqimi Φ aniqlansin. [50 mkVb.]

2. Uzunligi 1 m va kesimi 16sm^2 bo'lgan solenoid 2000 ta o'ramga ega. Chulg'amdagi tok kuchi 10A bo'lganda oqim ilashuvi hisoblansin.[80,5 mVb o'ram.]

3. Solenoidning temir o'zagida induksiya 1,3 Tl. Temir o'zakni po'lat o'zak bilan almashtiradilar. O'zakdagi induksiya o'zgarmay qolishi uchun tok kuchini necha marta o'zgartirish kerak. [2,4.]

4. 200 ta cho'lg'amdan iborat 5A tok oqayotgan o'zaksiz toroidning o'qidagi magnit maydonning induksiyasi B va kuchlanganligi H aniqlansin. Toroidning tashqi diametri 30sm, ichkisi 20 sm. [1,6mTl; 1,37 kA/m.]

5. Uzunligining har bir santimetrida 8 ta o'ram bo'lgan g'altakning (o'zaksiz) ko'ndalang kesim yuzasi orqali magnit oqimi aniqlansin. Solenoidning radiusi 2 sm, undagi tok kuchi 2 A. [10,1 mk Vb.]

6. Nikelin (μ =200) o'zakli 200 o'ramdan iborat solenoidning ichidagi bir jinsli magnit maydonning kuchlanganligi 10kA/m. Ko'ndalang kesimning yuzasi 10 sm^2. 1)Solenoid ichidagi maydon magnit induksiyasi; 2)oqim ilashuvi aniqlansin. [1) 2,51 Tl; 2) 0,502 VB.].

7. Uzunligi 1m, nomagnit materialga o'ralgan solenoidning induktivligi 1,6 mH. Solenoid kesimining yuzasi 20sm^2. Solenoid uzunligining har bir santimetriga to'g'ri keluvchi o'ramlar soni aniqlansin. [8 sm^{-1}.]

8. 4 mH induktivlikli solenoidda 600 ta o'ram bor. Agar o'ramlaridan oqayotgan tok kuchi 12 A bo'lsa, magnit oqimi Φ aniqlansin. [80 mk Vb.]

21-§. Elektromagnit induksiya hodisasi

Asosiy formulalar

Magnit maydonda tokli konturni ko'chirishda bajarilgan ish:

$$A = I \cdot \Delta\Phi,$$

bunda: $\Delta\Phi$ — kontur bilan chegaralangan sirtga singuvchi magnit oqimining o'zgarish; I — konturdagi tok kuchi.

Elektromagnit induksiya hodisasi uchun Faradey qonuni:

$$\varepsilon_i = -N\frac{d\Phi}{dt} = -\frac{d\psi}{dt},$$

bunda: ε_i — induksiya EYuK, N — konturdagi o'ramlar soni; ψ — oqim ilashuvi.

Xususiy hollar:

a) bir jinsli magnit maydonida ϑ tezlik bilan harakatlanayotgan l uzunlikli o'tkazgich uchlaridagi potensiallar farqi:

$$U = B \cdot l \cdot \vartheta \cdot \sin\alpha;$$

$\alpha - \vec{V}$ va $\vec{B}$ lar orasidagi burchak;

b) N ta o'ramdan iborat kesim yuzasi S bo'lgan ramkaning B induksiyali bir jinsli magnit maydonida ω burchak tezlik bilan aylanishida ramkada vujudga keladigan EYuK:

$$\varepsilon_i = B \cdot N \cdot S \cdot \omega \sin\omega t.$$

Konturdan oqayotgan zaryad miqdori:

$$Q = \frac{\Delta\Psi}{R}$$

bunda: R — konturning qarshiligi, $\Delta\Psi$ — oqim ilashuvining o'zgarishi.

Yopiq konturda vujudga keladigan o'zinduksiya EYuK:

$$\varepsilon_i = -L\frac{dI}{dt}, \quad \text{yoki} \quad \langle\varepsilon_i\rangle = -L\frac{\Delta I}{\Delta t},$$

$\frac{dI}{dt}$ yoki $\frac{\Delta I}{\Delta t}$ — tok kuchining o'zgarishi, L — konturning induktivligi.

Aktiv qarshilik R va induktivlik L ga ega zanjirdagi tok kuchi I ning oniy qiymati:

a) zanjir ulanishidan keyin

$$I = \frac{\varepsilon}{2}(1 - e^{-\frac{R}{L}t});$$

b) zanjir uzilishidan keyin

$$I = I_0 e^{-\frac{R}{L}t},$$

bunda: ε – EYuK I_0 – boshlang'ich tok kuchi t – vaqt.

Masala yechishga misollar

1-misol. 0,01 Tl induksiyali bir jinsli magnit maydonida, induksiya chiziqlariga perpendikulyar ravishda 8 sm uzunlikli to'g'ri o'tkazgich turibdi. O'tkazgichdan 2 A tok oqadi. O'tkazgich maydon kuchlari ta'sirida 5 sm masofaga siljidi. Maydon kuchlarining ishi topilsin.

Berilgan:

$B = 0{,}01\text{Tl} = 10^{-2}\text{Tl};$

$l = 8\text{sm} = 8 \cdot 10^{-2}\text{ m};$

$I = 2\text{A};$

$d = 5\text{sm} = 5 \cdot 10^{-2}\text{ m}.$

$A = ?$

Yechish: Magnit maydondagi tokli o'tkazgichning siljishida maydon kuchlari bajargan ish:

$$A = I\Delta\Phi \qquad (1)$$

ifodadan topiladi. $\Delta\Phi$ o'tkazgich siljishida u kesib o'tadigan yuzaga singuvchi magnit oqimi:

$$\Delta\Phi = B \cdot S \cdot \cos\alpha = B \cdot l \cdot d \cdot \cos\alpha. \qquad (2)$$

Bu yerda $S = l \cdot d$ – o'tkazgich kesib o'tadigan yuza. Shu bilan birga S ning normali va B vektor orasidagi burchak $\alpha = 0$ ekanligidan $\cos\alpha = 1$. Unda (1) quyidagi ko'rinishni oladi:

$$A = I \cdot B \cdot l \cdot d, \qquad (3)$$

$$[A] = [I][B][l][d] = 1\text{A} \cdot 1\text{Tl} \cdot 1\text{m} \cdot 1\text{m} = 1\text{A} \cdot \frac{\text{N}}{\text{A} \cdot \text{m}}\text{m}^2 = 1\text{N} \cdot \text{m} = 1\text{J}.$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$A = 2\cdot 10^{-2}\cdot 8\cdot 10^{-2}\cdot 5\cdot 10^{-2}\,\text{J} = 80\cdot 10^{-6}\,\text{J} = 80\text{mkJ}$.

Javob: A=80 mkJ.

2-misol. Induksiyasi 0,4 Tl bo‘lgan bir jinsli magnit maydonining kuch chiziqlariga perpendikulyar tekislikda 10 sm uzunlikni tayoqcha aylanmoqda. Aylanish o‘qi tayoqcha uchlarining biridan o‘tadi. Agar aylanish chastotasi 16 s^{-1} bo‘lsa, tayoqcha uchlaridagi potensiallar farqi aniqlansin.

Berilgan:

$B = 0{,}4\text{Tl}$;

$l = 10\text{sm} = 0{,}1\text{m}$;

$n = 16\text{s}^{-1}$.

$u = ?$

Yechish: Elektromagnit induksiya hodisasiga asosan bir jinsli magnit maydonida harakatlanayotgan o‘tkazgich uchlarida hosil bo‘ladigan potensiallar farqi quyidagicha aniqlanadi:

$$u = \text{B}\cdot l\cdot \vartheta\sin\alpha \cdot \qquad (1)$$

Masalaning shartiga ko‘ra, $\vec{V}\perp\vec{B}$ ya’ni $\alpha = \frac{\pi}{2}$, $\sin\alpha = 1$.

Tayoqchaning chiziqli tezligini quyidagicha aniqlaymiz:

$$\vartheta = \omega\cdot l = 2\pi nl \cdot \qquad (2)$$

Bu yerda aylanish o‘qi tayoqcha uchidan o‘tganligi va $\omega = 2\pi n$ e’tiborga olindi. Shunday qilib,

$$u = 2\pi nl^2\text{B} \qquad (3)$$

ni hosil qilamiz.

$$[u] = [n][l]^2[\text{B}] = 1\text{s}^{-1}\cdot 1\text{m}^2\cdot 1\text{Tl} = 1\frac{\text{m}^2}{\text{s}}\frac{\text{N}}{\text{A}\cdot\text{m}} = 1\frac{\text{m}\cdot\text{N}\cdot\text{s}}{\text{s}\cdot\text{C}} = 1\frac{\text{N}\cdot\text{m}}{\text{C}} = 1\frac{\text{J}}{\text{C}} = 1\text{V}$$

Kattaliklarning son qiymatlarini (3) ga qo‘yib topamiz:

$$u = 2\cdot 3{,}14\cdot 16\cdot (0{,}1)^2\cdot 0{,}4\text{V} \approx 0{,}4\text{V}\,.$$

Javob: u =0,4V.

3-misol. Massasi 1 g bo‘lgan ingichka mis sim kvadrat ko‘rinishga keltirilib, uchlari ulangan va induksiyasi 0,1 Tl bo‘lgan bir jinsli magnit maydonida, tekisligi maydon induksiya chiziqlariga perpendikulyar qilib joylashtirilgan. Agar kvadratning qarama-qarshi tomonlaridan tortib to‘g‘ri o‘tkazgich holatiga keltirilsa, undan oqib o‘tadigan zaryad miqdori Q aniqlansin.

Berilgan:

$m = 1g = 10^{-3}$ kg;

$B = 0{,}1$Tl;

$\rho = 1.7\cdot 10^{-8}\,\Omega\cdot\text{m}$;

$D = 8{,}8\cdot 10^{3}\,\dfrac{\text{kg}}{\text{m}^3}$.

$Q = ?$

Yechish: Konturdan oqadigan zaryad miqdori

$$Q = \frac{\Delta\psi}{R} \qquad (1)$$

ifodadan aniqlanadi. Bu yerda $\Delta\psi = N\Delta F$ – oqim ilashuvi. Mazkur masalada $N = 1$. $\Delta F = F_2 - F_1 = F_2 = F$. Chunki kvadrat to‘g‘ri chiziq holatiga keltirilganda $(S = 0)$, oqim nolga teng bo‘ladi. Shunday qilib,

$$Q = \frac{\Phi}{R}\,. \qquad (2)$$

O‘tkazgichning qarshiligini esa quyidagicha aniqlaymiz:

$$R = \rho\frac{l}{S} = \rho\frac{l^2}{V} = \rho l^2\,\frac{D}{m}. \qquad (3)$$

Bu yerda: $V = S\cdot l$ – o‘tkazgichning hajmi, $D = \dfrac{m}{V}$ – o‘tkazgich materialining zichligi, ρ – solishtirma qarshiligi

(3) ni (2) ga qo‘yib olamiz

$$Q = \frac{\Phi m}{\rho l^2 D} = \frac{\text{B}\cdot S\cdot m}{\rho 16a^2 D} = \frac{m\text{B}}{16\rho D}\,. \qquad (4)$$

Ifodani hosil qilishda $l = 4a;\ a^2 = S$, $\Phi = B\cdot S$ ekanligi nazarda tutildi. Chunki $\vec{n}\,\|\,\vec{\text{B}},\ \ \alpha = 0,\ \cos\alpha = 1.$

$$[Q] = \frac{[m][\text{B}]}{[\rho][D]} = \frac{1\text{kg}\cdot 1\text{Tl}}{1\Omega\cdot\text{m}\cdot 1\dfrac{\text{kg}}{\text{m}^3}} = 1\frac{\text{m}^2\cdot\text{N}}{\Omega\cdot\text{A}\cdot\text{m}} = 1\frac{\text{N}\cdot\text{m}}{\text{B}} = 1\frac{\text{J}\cdot\text{C}}{\text{J}} = 1\text{C}.$$

Kattaliklarning son qiymatlarini (4) ga qo‘yib olamiz:

$$Q=\frac{10^{-3}\cdot 0{,}1}{16\cdot 1{,}7\cdot 10^{-8}\cdot 8{,}8\cdot 10^{3}}\text{C}=41{,}8\cdot 10^{-3}\text{C}=41{,}8\text{mC}.$$

Javob: $Q=41{,}8$ mC.

4-misol. Solenoidda 1000 ta o‘ram bor. O‘zagining kesim yuzasi 10 sm². O‘ramlardan 1,5 Tl induksiya hosil qiluvchi tok oqadi. Agar $\Delta t=500$ mks davomida tok nolgacha kamaysa, solenoidda vujudga keladigan o‘zinduksiya EYuK ning o‘rtacha qiymati topilsin.

Berilgan:

$N=1000=10^{3};$

$S=10\ \text{sm}^{2}=10^{-3}\text{m}^{2};$

$\text{B}=1{,}5\text{Tl};$

$\Delta t=500\text{ms}=5\cdot 10^{-4}\,\text{s}.$

$\langle\varepsilon_i\rangle=?$ 1

Yechish: Solenoidda vujudga keladigan o‘zinduksiya EYuK ning o‘rtacha qiymatini quyidagicha aniqlaymiz:

$$\langle\varepsilon_i\rangle=-N\frac{\Delta F}{\Delta t}. \qquad (1)$$

Bu yerda

$$\Delta F=F_2-F_1=(0-B\cdot S\cdot\cos\alpha)=-B\cdot S\cdot\cos\alpha.$$

Chunki $F_2=0$,tok nolgacha kamayadi. $\vec{n}\,\|\,\vec{\text{B}}$, $\alpha=0$, $\cos\alpha=1$.
Shunday qilib, o‘zinduksiya EYuK uchun quyidagini olamiz:

$$\langle\varepsilon_i\rangle=\frac{NBS}{\Delta t}. \qquad (2)$$

$$[\varepsilon]=\frac{[\text{B}][S]}{[t]}=\frac{1\text{N}\cdot 1\text{m}^{2}}{1\text{s}}=1\frac{\text{N}\cdot\text{m}^{2}}{\text{A}\cdot\text{m}\cdot\text{s}}=1\frac{\text{N}\cdot\text{m}\cdot\text{s}}{C\cdot\text{s}}=1\frac{\text{J}}{\text{C}}=1\text{V}.$$

Kattaliklarning qiymatlarini qo‘yib olamiz

$$\langle\varepsilon_i\rangle=\frac{10^{3}\cdot 1{,}5\cdot 10^{-3}\text{V}}{5\cdot 10^{-4}}=0{,}3\cdot 10^{4}\text{V}=3\cdot 10^{3}\text{B}=3\text{kV}$$

Javob: $\langle\varepsilon_i\rangle=3\text{kV}$.

5-misol. Induktivligi 0,2 H va qarshiligi 1,64 Ω bo'lgan g'altak berilgan. EYuK uzilib, g'altak qisqa tutashtirilgandan 0,05 s o'tgach g'altakdagi tok kuchi necha marta kamayadi?

Berilgan: L=0,2H; R=1,64 Ω; t= 0,05 s.	**Yechish:** Manbadan ajratilgandan keyin o'zinduksiya tufayli tok kuchining kamayishi quyidagi ifodaga muvofiq ro'y beradi:
$\frac{I_0}{I} = ?$	$I = I_0 e^{-\frac{R}{L}t}$. (1)

(1) dan so'ralgan munosabatni hosil qilamiz:

$$\frac{I_0}{I} = e^{\frac{R}{L}t}. \qquad (2)$$

Kattaliklarning qiymatlarini (2) ga qo'yib quyidagini olamiz:

$$\frac{I_0}{I} = e^{\frac{1,64}{0,2}\cdot 0,05} = e^{0,41} = 1,5.$$

Javob: $\frac{I_0}{I} = 1,5$ marta.

Mustaqil yechish uchun masalalar

1. Yuzasi 300 sm^2 bo'lgan yassi kontur 0,01 Tl induksiyali bir jinsli magnit maydonida turibdi. Kontur yuzasi induksiya chiziqlariga tik yo'nalgan va undagi 10 A tok o'zgarmaydi. Tokli konturni fazoning magnit maydoni bo'lmagan sohasiga ko'chirishdagi tashqi kuchlarning ishi aniqlansin. [3 mJ.]

2. 20 A tok oqayotgan halqa 16 m Tl induksiyali bir jinsli magnit maydonida erkin o'rnashgan. Halqaning diametri 10 sm. Halqaning diametri bilan mos keluvchi o'qqa nisbatan sekingina $\alpha = \frac{\pi}{2}$ burchakka burish uchun qanday ish bajarish kerak? [2,5 mJ.]

3. Uzunligi 40 sm bo'lgan to'g'ri sim 5m/s tezlik bilan induksiya chiziqlariga tik ravishda bir jinsli magnit maydonda harakatlanmoqda. Sim uchlaridagi potensiallar farqi 0,6V. Magnit maydon indukiyasi B hisoblansin. [0,3 Tl.]

4. 1000 ta o'ramdan iborat ramka 0,1 Tl induksiyali bir jinsli magnit maydonida $10s^{-1}$ chastota bilan bir tekis aylanadi. Ramkaning yuzasi 150 sm^2. Ramkaning 300 burilish burchagiga mos keluvchi EYuK ning oniy qiymati aniqlansin. [1V.]

5. Yuzasi 100 sm^2 bo'lgan ramka 12 Ω qarshilikli 1000 ta o'ramdan iborat. Cho'lg'am uchlariga 20 Ω tashqi qarshilik ulangan. Agar ramka induksiyasi 0,1Tl bo'lgan bir jinsli magnit maydonida 8 s^{-1} chastota bilan aylanayotgan bo'lsa, zanjirdagi tokning maksimal quvvati topilsin. [78,9 W.]

6. 1 kA tok oqayotgan uzun to'g'ri o'tkazgichdan 1 m masofada 1 sm radiusli halqa turibdi. Halqa unga singadigan magnit oqimi eng ko'p bo'ladigan holatda joylashgan. Agar halqaning qarshiligi 10 Ω bo'lsa, to'g'ri o'tkazgichdagi tok uzilganda halqadan oqadigan elektr miqdori aniqlansin. [62,8 mC.]

7. Uzun to'g'ri o'tkazgichdan tok oqmoqda. O'tkazgich yonida 0,02 Ω qarshilikli ingichka simdan qilingan kvadrat ramka joylashgan. O'tkazgich ramka tekisligida yotibdi va uning 10 sm, 20 sm masofadagi ikki tomoniga parallel. Agar tok o'tkazgichga tok ulanganda ramka orqali 693 mkC zaryad miqdori oqib o'tsa, o'tkazgichdagi tok kuchi topilsin. [1000A.]

8. Reostat yordamida g'altakdagi tok kuchi 1 s davomida 0,1 A dan bir tekisda orttiriladi. G'altakning induktivligi 0,01H. O'zinduksiya EYuK ning o'rtacha qiymati topilsin. [1 mV.]

9. Tok manbaini 1Gn induktivlikli va 10 Ω qarshilikli g'altakka tutashtirdilar. Qancha vaqt o'tgandan keyin tutashtirish toki, tok kuchining 0,9 chegaraviy qiymatiga erishadi. [0,23 s.]

10. Zanjir 10 Ω qarshilikli va 1 H induktivlikli g'altakdan iborat. Tok manbaini zanjirni uzmasdan turib ajratish mumkin. Tok kuchi qancha Δt vaqt o'tgandan keyin boshlang'ich qiymatining 0,001 qismigacha kamayadi. [0,69 s.]

22-§. Magnit maydon energiyasi

Asosiy formulalar

I tokning L induktivli yopiq konturda hosil qiladigan magnit maydonining energiyasi: $W = \frac{1}{2}LI^2$.

Bir jinsli magnit maydonining hajmiy energiya zichligi:

$$\omega = \frac{\mu_0 \mu H^2}{2} = \frac{B^2}{2\mu_0 \mu} = \frac{B \cdot H}{2}.$$

Masala yechishga misollar

1-misol. G'altakning induktivligi 0,1 mH. Tok kuchining qanday qiymatida magnit maydon energiyasi 100 mkJ ga teng bo'ladi?

Berilgan:

$L = 0{,}1\text{mH} = 10^{-4}\text{H}$;

$W = 100\text{mkJ} = 10^{-4}\text{J}$.

$I = ?$

Yechish: L induktivli konturda I tok hosil qiladigan magnit maydon energiyasi quyidagi formula yordamida hosil qilinadi:

$$W = \frac{1}{2}LI^2. \qquad (1)$$

Bu ifodadan tok kuchini aniqlasak,

$$I = \sqrt{\frac{2W}{L}}. \qquad (2)$$

$$[I]^2 = \frac{[W]}{[L]} = \frac{1\text{J}}{1\text{H}} = 1\frac{\text{J} \cdot \text{A}^2}{\text{J}} = 1\text{A}^2; \quad [I] = 1\text{A}.$$

Berilganlar yordamida olamiz. $I = \sqrt{\frac{2 \cdot 10^{-4}}{10^{-4}}}\text{A} = \sqrt{2}\text{A} = 1{,}41\text{A}$.

Javob: I=1,41A.

2-misol. Temir halqaga 200 ta o'ram bir qatlam qilib o'ralgan. O'ramlardan 2,5 A tok oqqanda temirdagi magnit oqimi 0,5 mVb bo'ladi. Magnit maydonining energiyasi W aniqlansin.

Berilgan:	**Yechish:** Magnit maydonning energiyasini
$N = 200$;	$$W = \frac{1}{2}LI^2, \qquad (1)$$
$I = 2{,}5\,\text{A}$;	ifodadan aniqlaymiz. Toroidning
$\Phi = 0{,}5\text{mVb} = 5\cdot 10^{-4}\text{Vb}$.	induktivligini quyidagicha topiladi:
$W = ?$	

$$L = \mu\mu_0\left(\frac{N}{l}\right)^2 V = \mu\mu_0\left(\frac{N}{l}\right)^2 l\cdot S = \mu\mu_0\frac{N^2}{l}\cdot S. \qquad (2)$$

Bu yerda: $V = l\cdot S$ ligi e'tiborga olindi. l – toroidning uzunligi, S – ko'ndalang kesim yuzasi.

Ikkinchi tomondan $\Phi = \mu\mu_0\left(\frac{N}{l}\right)I\cdot S$.

Bundan $S = \dfrac{\Phi\cdot l}{\mu\mu_0 NI}$ (3)

(2) va (3) lar yordamida (1) ni qayta yozib olamiz:

$$W = \frac{1}{2}\frac{\text{N}\Phi}{\text{I}}\text{I}^2 = \frac{1}{2}\text{NI}\Phi \qquad (4)$$

$$[W] = [I][\Phi] = 1\,\text{A}\cdot 1\text{Vb} = 1\,\text{A}\cdot\text{Tl}\cdot\text{m}^2 = 1\frac{\text{A}\cdot\text{m}^2\cdot\text{N}}{\text{A}\cdot\text{m}} = 1\text{J}.$$

Berilganlarni qo'yib olamiz:

$$W = \frac{1}{2}200\cdot 2{,}5\cdot 5\cdot 10^{-4}\,\text{J} = 0{,}125\text{J}.$$

Javob: W= 0,125J.

3-misol. O'zagi nomagnit bo'lgan $(\mu = 1)$ toroidning har bir santimetriga 10 tadan o'ram to'g'ri keladi. Agar toroid cho'lg'amidan $16A$ tok oqayotgan bo'lsa, magnit maydon energiyasining zichligi ω aniqlansin.

Berilgan:

$n = 10\frac{1}{\text{sm}} = 10^3\frac{1}{\text{m}}$;

$I = 16\text{A}$;

$\mu = 1$.

$\omega = ?$

Yechish: Magnit maydon energiyasining zichligini quyidagicha aniqlaymiz:

$$\omega = \frac{W}{V} = \frac{LI^2}{2V}. \qquad (1)$$

Bu yerda L – toroidning induktivligi, V – hajmi

$$L = \mu\mu_0 n^2 V. \qquad (2)$$

(2) ni (1) ga qo‘yib olamiz.

$$\omega = \frac{1}{2}\mu\mu_0 n^2 I^2. \qquad (3)$$

$$[\omega] = [\mu_0][n]^2[I]^2 = 1\frac{\text{N}}{\text{A}^2}\cdot 1\frac{1}{\text{m}^2}\cdot 1\text{A}^2 = 1\frac{\text{N}\cdot\text{m}}{\text{m}^3} = 1\frac{\text{J}}{\text{m}^3}.$$

Kattaliklarning qiymatlarini (3) ga qo‘yib hisoblaymiz:

$$\omega = \frac{1}{2}4\pi\cdot 10^{-7}\cdot(10^3)^2\cdot(16)^2\frac{\text{J}}{\text{m}^3} = 161\frac{\text{J}}{\text{m}^3}.$$

Javob: $\omega = 161\frac{\text{J}}{\text{m}^3}$.

4-misol. Nomagnit o‘zakli $(\mu = 1)$ toroid chulg‘amining har bir santimetriga 10 ta o‘ram to‘g‘ri keladi. Chulg‘amdan oqadigan tok kuchining qanday qiymatida magnit maydon energiyasining zichligi $\omega = 1\frac{\text{J}}{\text{m}^3}$ bo‘ladi?

Berilgan:

$n = 10\frac{1}{\text{sm}} = 10^3\frac{1}{\text{m}}$;

$\mu = 1$;

$\omega = 1\frac{\text{J}}{\text{m}^3}$.

$I = ?$

Yechish: Magnit maydon energiyasining zichligini quyidagi ifodadan aniqlaymiz:

$$\omega = \frac{W}{V} = \frac{1}{2}\frac{LI^2}{V} \qquad (1)$$

V – toroidning hajmi, L – induktivligi.
(1) dan tok kuchini topamiz

$$I = \sqrt{\frac{2\omega V}{L}}. \qquad (2)$$

Toroidning induktivligini quyidagi ifodadan aniqlaymiz

$$L = \mu\mu_0 n^2 V. \qquad (3)$$

(3) ni (2) ga qo'yamiz: $$I = \sqrt{\frac{2\omega}{\mu\mu_0 n^2}}. \qquad (4)$$

$$[I]^2 = \frac{[\omega]}{[\mu_0][n]} = \frac{1\frac{\text{J}}{\text{m}^3}}{1\frac{\text{N}}{\text{A}^2}\cdot\frac{1}{\text{m}^2}} = 1\frac{\text{N}\cdot\text{m}^3\cdot\text{A}^2}{\text{m}^3\cdot\text{N}} = 1\text{A}^2; \quad [I] = 1\text{A}\cdot$$

Kattaliklarning qiymatlarini (4) ga qo'yamiz.

$$I = \sqrt{\frac{2\cdot 1}{1\cdot 4\pi\cdot 10^{-7}\cdot(10^3)^2}}\text{A} = 1{,}26\text{A}.$$

Javob: $I = 1{,}26\text{A}$.

Mustaqil yechish uchun masalalar

1. Solenoidda 1000 ta o'ram bor. Uning chulg'amidagi tok kuchi 1A. Solenoidning ko'ndalang kesimi orqali magnit oqimi 0,1mVb bo'lsa, magnit maydon energiyasi hisoblansin. [50 mJ.]

2. Magnit maydon induksiyasi 1Tl bo'lganda temirdagi magnit maydon energiyasining zichligi 200J/m^3. Shu shartlarda temirning magnit singdiruvchanligi μ aniqlansin. [$2\cdot 10^3$.]

3. Po'lat o'zakli toroid magnit maydonining kuchlanganligi 200A/m dan 800A/m gacha ortdi. Magnit maydon energiyasining hajmiy zichligi necha marta o'zgarganligi aniqlansin. [10,5 marta.]

4. Tok kuchining biror qiymatida o'zaksiz solenoid magnit maydoni energiyasining zichligi ω=0,2J/m^3. Agar solenoidga temir o'zak qo'yilsa, tok kuchining shu qiymatida maydon energiyasining zichligi necha marta ortadi? [$1{,}6\cdot 10^3$ marta.]

Magnit singdiruvchanlikni aniqlash uchun 75-rasmdagi grafikdan foydalanilsin. Gisterezis hodisasi hisobga olinmasin.

23-§. Moddalarning magnit xossalari

Asosiy formulalar

Elektronning orbital magnit $\overrightarrow{P_m}$ va orbital mexanik $\overrightarrow{L_e}$ momentlari orasidagi bog'lanish:

$$\overrightarrow{P_m} = -g\overrightarrow{L_e} = -\frac{e}{2m_e}\overrightarrow{L_e}$$

bu yerda: $e = 1{,}61 \cdot 10^{-19}\,C$ elektronning zaryadi, $m_e = 9{,}11 \cdot 10^{-31}$ kg massasi $g = \dfrac{e}{2m_e}$ – orbital momentlar uchun giromagnit munosabat.

Magnitlanganlik

$$\vec{J} = \frac{\overrightarrow{P_m}}{V} = \frac{\sum \overrightarrow{P_a}}{V},$$

$\overrightarrow{P_m} = \sum \overrightarrow{P_a}$ – magnetikning magnit momenti bo'lib, alohida molekulalar magnit momentlarining vektorial yig'indisiga teng.

Izotrop magnetiklarda magnitlanganlik $\vec{J}$ va magnit maydon kuchlanganligi $\vec{H}$ orasidagi bog'lanish:

$$\vec{J} = \chi\vec{H},$$

χ – magnit qabulchanlik (o'lchamsiz kattalik).

Solishtirma magnit qabulchanlik χ_{sol}, molyar magnit qabulchanlik χ_m va atom magnit qabulchanlik χ_a, magnit qabulchanlik χ bilan quyidagicha bog'langan.

$$\chi_{sol} = \frac{\chi}{\rho}, \qquad \chi_m = \frac{M}{\rho}\chi \quad \text{va} \quad \chi_a = \frac{A}{\rho}\chi,$$

ρ – moddaning zichligi, M – molyar massasi, A – atom og'irligi.

Bor magnitoni (elektronning magnit momenti):

$$\mu_B = \frac{e\hbar}{2m_e},$$

bunda: e — elementar zaryad, $\hbar = \frac{h}{2\pi}$, $h = 6{,}63 \cdot 10^{-34}$ J · s — Plank doimiysi, m_e — elektronning massasi.

$\vec{B}, \vec{H}$ va $\vec{J}$ lar orasidagi munosabat:

$$\vec{B} = \mu_0(\vec{H} + \vec{J}),$$

μ_0 – magnit doimiysi.

Magnit singdiruvchanlik va magnit qabulchanlik orasidagi munosabat:

$$\mu = 1 + \chi.$$

Izotrop paramagnetikning magnitlanganligi:

$$J = n\mu_M L(a),$$

$$\mu_M = B << kT \text{ da} \qquad \chi = \mu_0 \frac{n\mu^2{}_m}{3kT},$$

bunda: n — molekulalarning konsentratsiyasi, μ_m — molekulaning magnit momenti, $L(a)$ – Lanjevan funksiyasi, k – Bolsman doimiysi.

Ferromagnetiklarning magnit singdiruvchanligi μ, undagi maydonning induksiya B va magnitlovchi maydonning kuchlanganligi H orasidagi munosabat:

$$\mu = \frac{B}{\mu_0 H}.$$

Ferromagnetikda maydonning induksiyasi B va magnitlovchi maydonning kuchlanganligi H orasidagi bog'lanish grafik usulda tasvirlanadi.

Moddalardagi magnit maydoni uchun to'la tok qonuni:

$$\oint_L \vec{B} d\vec{l} = \oint_L B_e dl = \mu_0 (\sum I + \sum I').$$

$\sum I$ va $\sum I'$ lar mos ravishda L kontur o'rab turgan makrotoklarning va mikrotoklarning (molekular toklarning) algebraik yig'indisi.

Magnit maydon kuchlanganligi vektorining sirkulyatsiyasi quyidagicha aniqlanadi:

$$\oint_L \vec{H} \cdot \vec{dl} = \oint_L H_e \cdot d_e = \sum I ,$$

$\sum I - L$ kontur qamrab oladigan toklarning yig'indisi.

Masala yechishga misollar

1-misol. Vodorod atomidagi elektron 52,8 pm radiusli aylanma orbita bo'ylab $6{,}58 \cdot 10^{15}$ Hz chastota bilan harakatlanadi deb hisoblab quyidagilar aniqlansin: 1) teng kuchli tokning aylanma magnit momenti P_m; 2) elektronning orbital mexanik momenti L_e; 3) olingan natijalar yordamida orbital momentlar uchun giromagnit munosabat va uning bu qiymatining universal qiymat bilan mos kelishi.

Berilgan:

$r = 52{,}8\,\text{pm} = 52{,}8 \cdot 10^{-12}\,\text{m};$

$\nu = 6{,}58 \cdot 10^{15}\,\text{Hz}.$

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1) $P_m = ?$

2) $L_e = ?$

3) $\dfrac{P_m}{L_e} = ?$

quyidagicha aniqlaymiz:

$$P_m = I \cdot S , \qquad (1)$$

bu yerda: $I = e\nu$ – tok kuchi, ν – elektronning aylanish chastotasi, $S = \pi r^2$ – orbitaning yuzasi. Demak,

$$P_m = \pi e \nu r^2 . \qquad (2)$$

Yechish: 1) Elektronning harakatiga teng kuchli tokning magnit momentini

$$[P_m] = [e][\nu][r]^2 = 1\text{C} \cdot 1\text{s}^{-1} \cdot 1\text{m}^2 = 1\text{A} \cdot \text{m}^2$$

Kattaliklarning son qiymatlarini qo'yamiz $(e = 1{,}61 \cdot 10^{-19} C)$:

$$P_m = 3{,}14 \cdot 1{,}61 \cdot 10^{-19} \cdot 6{,}58 \cdot 10^{15} \cdot (52{,}8 \cdot 10^{12})\text{A} \cdot \text{m}^2 = 9{,}25 \cdot 10^{-24}\,\text{A} \cdot \text{m}^2 .$$

2) Elektronning orbital mexanik momenti

$$L_e = m_e \vartheta r , \qquad (3)$$

bu yerda: $\vartheta = 2\pi\nu\, r$ – elektronning orbitadagi tezligi.

$$L_e = 2\pi m_e \nu r^2 = 2m_e \nu S. \qquad (4)$$

$$[L] = [m][\nu][S] = 1\text{kg} \cdot 1\text{s}^{-1} \cdot 1\text{m}^2 = 1\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}.$$

Son qiymatlarini qo‘yib topamiz ($m_e = 9{,}11 \cdot 10^{-31}$kg):

$$L_e = 2 \cdot 3{,}14 \cdot 9{,}11 \cdot 10^{-31} \cdot 6{,}58 \cdot 10^{15} \cdot (52{,}8 \cdot 10^{-12})\text{kg} \cdot \frac{\text{m}^2}{\text{s}} = 1{,}05 \cdot 10^{-34}\text{kg} \cdot \text{m}^2/\text{s}.$$

3) orbital momentlar uchun giromagnit munosabat quyidagicha aniqlanadi:

$$g = \frac{|\vec{P}_m|}{|\vec{L}_e|} = \frac{e}{2m}. \qquad (5)$$

Dastlab g ni olingan natijalar yordamida:

$$g = \frac{|\vec{P}_m|}{|\vec{L}_e|} = \frac{9{,}25 \cdot 10^{-24}}{1{,}05 \cdot 10^{-34}} \frac{\text{C}}{\text{kg}} = 8{,}81 \cdot 10^{10} \frac{\text{C}}{\text{kg}};$$

so‘ngra esa universal qiymatini olamiz:

$$g = \frac{e}{2m} = \frac{1{,}61 \cdot 10^{-19}}{2 \cdot 9{,}11 \cdot 10^{-31}} \frac{\text{C}}{\text{kg}} = 8{,}83 \cdot 10^{10} \frac{\text{C}}{\text{kg}}.$$

Natijalar o‘ndan birgacha aniqlikda mos keladi.

Javob: 1) $P_m = 9{,}25 \cdot 10^{-24}\,\text{A} \cdot \text{m}^2$; 2) $L_e = 1{,}05 \cdot 10^{-34}\text{kg} \cdot \text{m}^2/\text{s}$;

3) $g \approx 8{,}80 \cdot 10^{10} \frac{\text{C}}{\text{kg}}$.

2-misol. Marganesning magnit qabulchanligi $1{,}21 \cdot 10^{-4}$. Kuchlanganligi 100 kA/m bo‘lgan magnit maydonidagi marganesning magnitlanganligi J, solishtirma magnitlanganligi J_{sol} va molyar magnitlanganligi J_m hisoblansin. Marganesning zichligi $7{,}3 \cdot 10^3$ kg/m^3, atom og‘irligi (molyar massasi) $A = 55 \cdot 10^{-3} \frac{\text{kg}}{\text{mol}}$.

Berilgan:

$\chi = 1{,}21 \cdot 10^{-4}$;

$H = 100\,\mathrm{kA/m} = 10^5\,\dfrac{\mathrm{A}}{\mathrm{m}}$;

$\rho = 7{,}3 \cdot 10^3\,\mathrm{kg/m^3}$;

$A = 55 \cdot 10^{-3}$ kg/mol.

$J = ?$

$J_{sol} = ?$

$J_m = ?$

Yechish: Magnitlanganlik J va magnit maydon kuchlanganligi H orasidagi munosabatdan foydalanamiz

$$J = \chi H, \qquad (1)$$

$$[J] = [\mathrm{H}] = 1\frac{A}{M}.$$

Solishtirma magnitlanganlikni quyidagicha aniqlaymiz:

$$J_{sol} = \frac{J}{\rho} = \frac{\chi}{\rho}\mathrm{H}; \qquad (2)$$

$$[J_{sol}] = \frac{[H]}{[\rho]} = 1\frac{\mathrm{A}}{\mathrm{m}}\frac{\mathrm{m}^3}{\mathrm{kg}} = 1\frac{\mathrm{A \cdot m^2}}{\mathrm{kg}}.$$

Molyar magnitlanganlik (mazkur holda atom magnitlanganlik) quyidagicha aniqlanadi:

$$J_m = \frac{A}{\rho}J = \frac{A}{\rho}\chi H. \qquad (3)$$

$$[J_m] = \frac{[A]}{[\rho]}[H] = \frac{1\frac{\mathrm{kg}}{\mathrm{mol}}}{1\frac{\mathrm{kg}}{\mathrm{m^3}}} \cdot 1\frac{\mathrm{A}}{\mathrm{m}} = 1\frac{\mathrm{A \cdot m^2}}{\mathrm{mol}}.$$

Kattaliklarning son qiymatlarini (1), (2) va (3) larga qo'yib hisoblaymiz:

$$J = 1{,}21 \cdot 10^{-4} \cdot 10^5\,\frac{\mathrm{A}}{\mathrm{m}} = 12{,}1\frac{\mathrm{A}}{\mathrm{m}};$$

$$J_{sol} = \frac{1{,}21 \cdot 10^{-4}}{7{,}3 \cdot 10^3} \cdot 10^5\,\frac{\mathrm{A \cdot m^2}}{\mathrm{kg}} = 1{,}66 \cdot 10^{-3}\,\frac{\mathrm{A \cdot m^2}}{\mathrm{kg}};$$

$$J_m = \frac{55 \cdot 10^{-3}}{7{,}3 \cdot 10^3} \cdot 1{,}21 \cdot 10^{-4} \cdot 10^5\,\frac{\mathrm{A \cdot m^2}}{\mathrm{mol}} = 9{,}1 \cdot 10^{-6}\,\frac{\mathrm{A \cdot m^2}}{\mathrm{mol}}.$$

Javob: $J = 12{,}1\,\mathrm{A/m}$; $J_{sol} = 1{,}66 \cdot 10^{-3}\,\dfrac{\mathrm{A \cdot m^2}}{\mathrm{kg}}$; $J_m = 9{,}1 \cdot 10^{-6}\,\dfrac{\mathrm{A \cdot m^2}}{\mathrm{mol}}$.

3-misol. Alyuminiyning molyar magnit qabulchanligi $1{,}65\cdot10^{-8}\dfrac{\text{m}^3}{\text{kg}\cdot\text{mol}}$ bo'lsa, uning magnit qabulchanligi c va magnit singdiruvchanligi μ aniqlansin.

Berilgan:

$\chi_m=1{,}65\cdot10^{-8}\dfrac{\text{m}^3}{\text{kg}\cdot\text{mol}};$

$\rho=2{,}7\cdot10^3\,\text{kg}/\text{m}^3;$

$M=A=27\cdot10^{-3}\dfrac{\text{kg}}{\text{mol}}.$

---

x=?

$\mu=?$

Yechish: Magnetiklarning molyar magnit qabulchanligi quyidagicha aniqlanadi:

$$\chi_m=\frac{M}{\rho}\chi.$$

Bundan qabulchanlikni aniqlasak,

$$\chi=\frac{\rho}{M}\chi_m. \qquad (1)$$

$$[\chi]=\frac{[\rho]}{[M]}[\chi_m]=\frac{1\frac{\text{kg}}{\text{m}^3}}{1\frac{\text{kg}}{\text{mol}}}\cdot1\frac{\text{m}^3}{\text{mol}}=1.$$

Magnit singdiruvchanlik va qabulchanlik orasidagi munosabatdan foydalanamiz:

$$\mu=1+\chi. \qquad (2)$$

Berilganlarni (1) va (2) larga qo'yamiz:

$$\chi=\frac{2{,}7\cdot10^3}{27\cdot10^{-3}}\cdot1{,}65\cdot10^{-8}=1{,}65\cdot10^{-3}=0{,}00165;$$

$$\mu=1+0{,}00165=1{,}00165.$$

Javob: $\chi=0{,}00165;\ \mu=1{,}00165.$

4-misol. Agar kislorod molekulalarining magnit momenti $2{,}8\mu_V$ ekanligi ma'lum bo'lsa, normal sharoitdagi kislorod gazining solishtirma paramagnet qabulchanligi aniqlansin.

Berilgan:

$\mu_m=2{,}8\mu_V;$

$\mu_V=9{,}27\cdot10^{-24}\dfrac{\text{J}}{\text{Tl}};$

$M=32\cdot10^{-3}\dfrac{\text{kg}}{\text{mol}};$

Yechish: Solishtirma magnit qabulchanlik quyidagicha aniqlanadi:

$$\chi_{sol}=\frac{\chi}{\rho}, \qquad (1)$$

ρ – zichlik.

$T = 273{,}15K.$	Paramagnetiklarning magnit qabulchanligini:
$\chi_{sol} = ?$	$$\chi = \frac{\mu_0 n \mu^2{}_m}{3kT} \qquad (2)$$ kabi aniqlaymiz .

Bu yerda: $k = 1{,}38 \cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$ – Bolsman doimiysi, $\mu_0 = 4\pi \cdot 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2}$ – magnit doimiysi, n – molekulalar konsentratsiyasi bo'lib, quyidagicha aniqlanadi:

$$n = \frac{N_A}{V_m} = \frac{\rho N_A}{M}; \qquad (3)$$

bunda: $N_A = 6{,}02 \cdot 10^{23}\,\mathrm{mol}^{-1}$ – Avogadro soni, $V_m = \frac{M}{\rho}$ – molyar hajmi, M – molyar massa.

(3) ni e'tiborga olib (2) ni (1) ga qo'ysak quyidagini olamiz:

$$\chi_{sol} = \frac{\mu_0 N_A \mu^2 m}{3kTM}. \qquad (4)$$

$$[\chi_{sol}] = \frac{[\mu_0][\mathrm{N}_A][\mu_m]^2}{[k][T][M]} = \frac{1\frac{\mathrm{N}}{\mathrm{A}^2}\cdot \mathrm{mol}^{-1}\cdot 1\frac{\mathrm{J}^2}{\mathrm{Tl}^2}}{1\frac{\mathrm{J}}{\mathrm{K}}\cdot 1\mathrm{K}\cdot 1\frac{\mathrm{kg}}{\mathrm{mol}}} = 1\frac{\mathrm{N}\cdot\mathrm{J}}{\mathrm{A}^2\mathrm{kg}\cdot\mathrm{Tl}^2} =$$

$$= 1\frac{\mathrm{N}\cdot\mathrm{J}}{\mathrm{A}^2\cdot\mathrm{kg}}\,\frac{\mathrm{A}^2\cdot\mathrm{m}^2}{\mathrm{N}^2} = 1\frac{\mathrm{m}^3}{\mathrm{kg}};$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$\chi_{sol} = \frac{4\cdot 3{,}14\cdot 10^{-7}\cdot 6{,}02\cdot 10^{23}\cdot(2{,}8\cdot 9{,}27\cdot 10^{-24})^2}{3\cdot 1{,}38\cdot 10^{-23}\cdot 273{,}15\cdot 32\cdot 10^{-3}} = \frac{\mathrm{m}^3}{\mathrm{kg}} = 5{,}26\cdot 10^{-6}\,\frac{\mathrm{m}^3}{\mathrm{kg}}.$$

Javob: $\chi_{sol} = 5{,}26\cdot 10^{-6}\,\frac{\mathrm{m}^3}{\mathrm{kg}}.$

5-misol. Uzunligi 20 sm, radiusi 2 sm bo'lgan solenoid chulg'amidagi o'ramlar soni 300. Agar solenoidning induktivligi 2mH va diamagnit muhitda turganida solenoid ichidagi magnitlanganlik $20\,A/m$ ga teng bo'lsa, chulg'amdan oqayotgan tok kuchi aniqlansin.

Berilgan:
l =20 sm=0,2m;
r =2 sm=$2\cdot10^{-2}$ m;
N =300;
L =2 mH=$2\cdot10^{-3}$ H
$J=20\dfrac{A}{m}$.

I=?

Yechish: Magnit maydon kuchlanganligining induksiyasi uchun ushbu ifodani yozamiz:

$$\oint_L \vec{H}d\vec{l}=\oint_L H_e dl=\sum_{i=1}^{N} I_i .$$

Mazkur hol uchun

$$\oint_L H_e dl = H\oint_L dl = H\cdot l=\sum_{i=1}^{N} I_i = N\cdot I \quad (1)$$

Chunki chulg'amning barcha o'ramlaridan bir xil tok oqadi. (1) dan tok kuchi ifodasini topamiz:

$$I=\frac{H\cdot l}{N} \qquad (2)$$

Magnit maydon kuchlanganligi va magnitlanganlik orasida quyidagi munosabat mavjud:

$$I=\chi H . \qquad (3)$$

Shuningdek,

$\mu=1+\chi$, yoki $\chi=\mu-1$.

Demak,

$$J=(\mu-1)H . \qquad (4)$$

(4) dan H ni topib (2) ga qo'ysak quyidagini olamiz:

$$I=\frac{J\cdot l}{N(\mu-1)} . \qquad (5)$$

Endi solenoidning induktivligi uchun quyidagini ifodani yozamiz:

$$L=\mu_0\mu(\frac{N}{l})^2 S\cdot l=\mu_0\mu\frac{N^2}{l}\pi r^2 ,$$

va undan μ ni aniqlaymiz:

$$\mu = \frac{L \cdot l}{\mu_0 N^2 \pi r^2}. \qquad (6)$$

(6) ni (5) ga qo‘yib topamiz:

$$I = \frac{J \cdot l}{N\left(\frac{L \cdot l}{\mu_0 N^2 \pi r^2} - 1\right)}. \qquad (7)$$

$$\frac{[L][l]}{[\mu_0][r]^2} = \frac{1Gn \cdot 1m}{1\frac{Gn}{m} \cdot 1m^2} = 1; \quad [I] = [J][l] = 1\frac{A}{m} \cdot 1m = 1A.$$

Kattaliklarning qiymatlarini (7)ga qo‘yib olamiz $(\mu_0 = 4\pi \cdot 10^{-7}\frac{\text{H}}{\text{m}})$:

$$I = \frac{20 \cdot 0{,}2}{300\left(\frac{2 \cdot 10^{-3} \cdot 0{,}2}{4 \cdot 3{,}14 \cdot 10^{-7} \cdot (300)^2 \cdot 3{,}14 \cdot (2 \cdot 10^{-2})} - 1\right)}\text{A} = \frac{4}{300(2{,}82 - 1)}\text{A} = 7{,}3 \cdot 10^{-3}\text{ A} = 7{,}3\text{mA}.$$

Javob: I=7,3 mA.

Mustaqil yechish uchun masalalar

1. Agar har bir atomining magnit momenti bor magnetoni μ_B ga va atomlarining konsenratsiyasi $6 \cdot 10^{28}$ m^{-3} ga teng bo‘lsa, to‘yinishda jismning magnitlanganligi aniqlansin. [556 kA/m.]

2. Agar platinaning solishtirma magnit qabulchanligi $1{,}3 \cdot 10^{-9}$m^3/kg bo‘lsa, uning magnit qabulchanlik va molyar magnit qabulchanligi aniqlansin. [10^{-5}; 10^{-10} m^3/mol.]

3. Misdagi magnit maydonining kuchlanganligi 1MA/m. Agar misning solishtirma magnit qabulchanligi $-1{,}1 \cdot 10^{-9}$m^3/mol bo‘lsa, uning magnitlanganligi va induksiyasi aniqlansin. [-9,8A/m; 1,26 Tl.]

4. Xrom oksidi Cr_2O_3 ning molyar magnit qabulchanligi $5{,}8 \cdot 10^{-8}$m^3/mol. Agar harorat 300 K bo‘lsa, xrom oksidi molekulasining magnit momenti (bor magnitonlarida) aniqlagsin. [3,34 μ_B.]

5. 300 K harorat va 0,5 Tl magnit induksiyasida paramagnetikning

ma'lum magnitlanganligi J ga erishildi. Agar harorat 450 K gacha ko'tarilsa, shu magnitlanganlik saqlanishi uchun zarur bo'lgan magnit induksiyasi B_2 aniqlansin. [0,75 Tl.]

6. Elektron harakatlanayotgan har qanday orbita uchun uning orbital momentlarining giromagnit munosabati bir xil ekanligi ko'rsatilsin. [e/2m.]

7. Platinadagi bir jinsli magnit maydonining kuchlanganligi 5A/m. Agar platinaning magnit qabulchanligi $3,6 \cdot 10^{-4}$ ga teng bo'lsa, molekular toklar ta'sir qiladigan maydonning magnit induksiyasi aniqlansin. [2,26 nTl.]

8. Suyuq kislorodga botirilgan 40 sm radiusli aylanma konturdan 1A tok oqmoqda. Agar suyuq kislorodning magnit qabulchanligi $3,4 \cdot 10^{-3}$bo'lsa, kontur markazidagi magnitlanganlik aniqlansin. [4,25 mA/m.]

IV BOB. TEBRANISHLAR VA TO'LQINLAR

24-§. Garmonik tebranishlar va to'lqinlar. Akustika

Asosiy formulalar

Garmonik tebranish tenglamasi:

$$S = A \cdot \cos(\omega_0 t + \varphi),$$

bunda: S – muvozanat vaziyatidan chetlanish; A – tebranish amplitudasi; $\omega_0 = \frac{2\pi}{T} = 2\pi\nu$ siklik chastota; $\nu = \frac{1}{T}$ – chastota, T – tebranish davri, φ_0 – boshlang'ich faza.

Garmonik tebranayotgan nuqtaning tezligi va tezlanishi:

$$\frac{dS}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi) = A\omega_0 \cos\left(\omega_0 t + \varphi + \frac{\pi}{2}\right);$$

$$\frac{d^2S}{dt^2} = -A\omega_0^2 \cos(\omega_0 t + \varphi) = -\omega_0^2 S.$$

Tebranayotgan m massali nuqtaning kinetik energiyasi:

$$T = \frac{m\vartheta^2}{2} = \frac{mA^2\omega_0{}^2}{2}\sin^2(\omega_0 t + \varphi);$$

potensial energiyasi:

$$\Pi = \frac{mA^2\omega_0^2}{2}\cos^2(\omega_0 t + \varphi).$$

va to'la energiyalari:

$$E = T + \Pi = \frac{1}{2}m \cdot A^2\omega_0^2).$$

m massali nuqta garmonik tebranishining differensial tenglamasi

$$m\ddot{x} = -kx, \quad \text{yoki} \quad \ddot{x} + \omega_0^2 x = 0,$$

k – elastiklik koeffitsienti $(k = m\omega_0^2)$.

Prujinali mayatnikning tebranish davri:

$$T = 2\pi\sqrt{\frac{m}{k}},$$

bunda: m – mayatnikning massasi, k – qattiqligi.

Fizik mayatnikning tebranish davri:

$$T = 2\pi\sqrt{\frac{J}{mgl}} = 2\pi\sqrt{\frac{L}{g}},$$

bunda: J – mayatnikning tebranish o‘qiga nisbatan inersiya momenti, l – osilish nuqtasidan massa markazigacha bo‘lgan masofa, $L = J/(me)$ – fizik mayatnikning keltirilgan uzunligi, g – og‘rlik kuchi tezlanishi.

l uzunlikli matematik mayatnikning tebranish davri:

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

Bir xil chastotali va bir yo‘nalishda tarqalayotgan ikkita garmonik tebranishlarning qo‘shilishi natijasida hosil bo‘ladigan tebranishning amplitudasi:

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1).$$

Bu yerda: A_1 va A_2 –qo‘shiladigan tebranishlarning amplitudalari, φ_1 va φ_2 – ularning fazalari.

Boshlang‘ich fazasi:

$$\mathrm{tg}\varphi = \frac{A_1 \cdot \sin\varphi_1 + A_2 \cdot \sin\varphi_2}{A_1 \cdot \cos\varphi_1 + A_2 \cdot \cos\varphi_2}.$$

Tepish (bienie) davri:

$$T = \frac{2\pi}{\Delta\omega}.$$

Yassi to‘lqin tenglamasi:

$$\varepsilon = (x_1 t) = A\cos(\omega t - kx + \varphi_0),$$

$\varepsilon = (x_1 t)$ − x kordinatali muhit nuqtalarining t vaqtdagi siljishi; A − amplituda; $k = \frac{2\pi}{\lambda} = \frac{2\pi}{\vartheta T} = \frac{\omega}{\vartheta}$ − to‘lqin soni; λ − to‘lqin uzunligi; T − davr, ϑ − fazoviy tezlik; φ_0 − boshlang‘ich faza.

$$\lambda = \vartheta T; \qquad \vartheta = \lambda \nu .$$

Fazalar farqi $\Delta\varphi$ va yo‘l farqi Δ orasidagi bog‘lanish:

$$\Delta\varphi = 2\pi \frac{\Delta}{\lambda}.$$

To‘lqinlar qo‘shilishida (interferensiyasida) maksimum va minimumlar sharti:

$$\Delta_{max} = \pm 2m\frac{\lambda}{2}; \qquad \Delta_{min} = \pm(2m+1)\frac{\lambda}{2},$$

bu yerda $m = 0,1,2,...,....$

Fazoviy ϑ va gruppaviy u tezliklar hamda ular orasidagi munosabat

$$\vartheta = \frac{\omega}{k}; \qquad u = \frac{d\omega}{dk}; \qquad u = \vartheta - \lambda\frac{d\vartheta}{d\lambda}.$$

Turg‘un to‘lqin tenglamasi:

$$\varepsilon,(x,t) = 2A\cos\frac{2\pi}{\lambda}x\cos\omega t = 2A\cos kx \cdot \cos\omega t.$$

Qavariqlik va tugunlarning koordinatalari:

$$x_{\kappa} = \pm m\frac{\lambda}{2}; \qquad x_{t} = \pm(m+\frac{1}{2})\frac{\lambda}{2}; \qquad m = 0,1,2,....$$

Tovush intensivligining darajasi:

$$L = \lg\left(\frac{I}{I_0}\right).$$

Bunda: I – tovush intensivligi; I_0 – eshitish chegarasida tovush intensivligi:

$$I_0 = 1\,\mathrm{pW}/\mathrm{m}^2$$

Gazlarda tovush to‘lqinlarining tezligi:

$$\vartheta = \sqrt{\gamma \frac{RT}{M}}$$

bunda: R – gaz molyar doimiysi, M – molyar massa, $\gamma = \frac{C_p}{C_v}$, T – termodinamik harorat.

Akustikada Dopler effekti:

$$\nu = \frac{(\vartheta \pm \vartheta_k)\nu_0}{\vartheta \mp \vartheta_M},$$

bunda: ν – harakatlanayotgan qabul qiluvchi uchun tovush chastotasi, ν_0 –manba chiqaradigan tovush chastotasi, ϑ_k – qabul qiluvchining tezligi, ϑ_m – manbaning harakat tezligi, ϑ – tovushning tarqalish tezligi. Agar manba va qabul qiluvchilar bir-birlariga yaqinlashayotgan bo‘lsalar yuqoridagi ishora, uzohlashayotgan bo‘lsalar pastdagi ishora olinadi.

Masala yechishga misollar

1-misol. Moddiy nuqta garmonik tebranadi. Tebranish davri 2 s, amplitudasi 50 mm, boshlang‘ich fazasi $\varphi_0 = 0$. Nuqtaning muvozanat vaziyatidan 25 mm siljigan paytdagi tezligini toping.

Berilgan:

$T = 2\mathrm{s};$

$A = 50\mathrm{mm} = 5 \cdot 10^{-2}\,\mathrm{m};$

$\varphi_0 = 0;$

$x = 25\mathrm{mm} = 2_05 \cdot 10^{-2}\,\mathrm{m}.$

$\vartheta = ?$

Yechish: Garmonik tebranish tenglamasini quyidagi ko‘rinishda yozamiz:

$$x = A \cdot \cos(\omega_0 t + \varphi_0), \qquad (1)$$

yoki $\omega_0 = \frac{2\pi}{T}$ ni e’tiborga olsak,

$$x = A \cdot \cos\left(\frac{2\pi}{T} \cdot t + \varphi_0\right). \qquad (2)$$

Tebranayotgan nuqtaning tezligi:

$$\vartheta = \frac{dx}{dt} = -A \cdot \frac{2\pi}{T} \cdot \sin\left(\frac{2\pi}{T} \cdot t + \varphi_0\right). \qquad (3)$$

Nuqta x =25 mm da bo'lgan vaqtini topish uchun boshlang'ich shartlarni (2) ga qo'yamiz:

$$2{,}5 \cdot 10^{-2}\,\text{m} = 5 \cdot 10^{-2}\,\text{m} \cdot \cos\left(\frac{2\pi}{T} \cdot t + 0\right);$$

$$\cos\frac{2\pi}{T} \cdot t = \frac{1}{2}; \quad \frac{2\pi}{T} \cdot t = \frac{\pi}{3}; \quad t = \frac{T}{6} = \frac{2}{6}\,\text{s} = \frac{1}{3}\,\text{s}.$$

Natijani (3) ga qo'yib olamiz

$$\vartheta = -\frac{2 \cdot 3{,}14 \cdot 5 \cdot 10^{-2}}{2}\sin\left(\frac{2 \cdot 3{,}14 \cdot \frac{1}{3}}{2} + 0\right)\frac{\text{m}}{\text{s}} = -15{,}70 \cdot 10^{-2} \cdot \sin\frac{\pi}{3}\frac{\text{m}}{\text{s}} = -0{,}136\,\frac{\text{m}}{\text{s}};$$

$$|\vartheta| = 0{,}136\ \frac{\text{m}}{\text{s}}.$$

Javob: $|\vartheta| = 0{,}136\ \frac{\text{m}}{\text{s}}$.

2-misol. Massasi 10 g bo'lgan moddiy nuqta $x = A\sin(\omega t + \varphi_0)$ tenglamaga muvofiq harakatlanadi. Nuqtaga ta'sir etuvchi maksimal kuchni va tebranayotgan nuqtaning tola energiyasini toping. A=5sm, $\omega = \frac{\pi}{5}\text{s}^{-1}$.

Berilgan:

$m = 10\text{g} = 10^{-2}\,\text{kg};$

$A = 5\text{sm} = 5 \cdot 10^{-2}\,\text{m};$

$\omega - \frac{\pi}{5}\text{s}^{-1}.$

$F_{\max} = ?$

$E = ?$

Yechish: Moddiy nuqtaning harakat tenglamasi:

$$x = A \cdot \sin(\omega t + \varphi_0), \qquad (1)$$

tezligi

$$\vartheta = \frac{dx}{dt} = A\omega\cos(\omega t + \varphi_0), \qquad (2)$$

tezlanishi

$$a=\frac{d\vartheta}{dt}=\frac{d^2x}{dt^2}=-A\omega^2\sin(\omega t+\varphi_0)=A\omega^2\cos(\omega t+\varphi_0+\pi) \quad (3)$$

Nyutonning ikkinchi qonuniga muvofiq:

$$F=m\cdot a=m\cdot A\omega^2\cos(\omega t+\varphi_0+\pi),$$

F – o'zining eng katta qiymatiga $\cos(\omega t+\varphi_0+\pi)=1$ da erishishini nazarda tutsak,

$$F_{max}=mA\omega^2.$$

$$[F]=[m][A][\omega]^2=1\text{kg}\cdot 1\text{m}\cdot 1\text{s}^{-2}=1\text{kg}\cdot\frac{\text{m}}{\text{s}^2}\cdot=1\text{N}.$$

Shuningdek, to'la energiya uchun:

$$E=T+\Pi=\frac{1}{2}mA^2\cdot\omega_0^2. \quad (5)$$

$$[E]=[m][A]^2[\omega]^2=1\text{kg}\cdot 1\text{m}^2\cdot 1\text{s}^{-2}=1\text{kg}\cdot\frac{\text{m}}{\text{s}^2}\cdot\text{m}=1\text{N}\cdot\text{m}=1\text{J}$$

Kattaliklarning son qiymatlarini (4) va (5) larga qo'yib olamiz:

$$F_{max}=10^{-2}\cdot 5\cdot 10^{-2}\cdot\left(\frac{3{,}14}{5}\right)^2\text{N}=19{,}7\cdot 10^{-5}\text{N}=197\text{mkN}.$$

$$E=\frac{1}{2}\cdot 10^{-2}\cdot(5\cdot 10^{-2})^2\cdot\left(\frac{3{,}14}{5}\right)^2\text{J}=4{,}93\cdot 10^{-6}\text{J}=4{,}93\text{mkJ}.$$

Javob: $F_{max}=197\text{mN}$, $E=4{,}93\text{mkJ}$.

3-misol. Davrlari $T_1=T_2=8$ s va amplitudalari $A_1=A_2=2$ sm bo'lgan bir tomonga yo'nalgan ikkita garmonik tebranishlarning qo'shilishi natijasida hosil bo'ladigan tebranish tenglamasi yozilsin. Tebranishlar fazalarining farqi $\Delta\varphi=\varphi_2-\varphi_1=\frac{\pi}{4}$ va birining boshlang'ich fazasi nolga teng.

Berilgan:

$T_1 = T_2 = 8s;$

$A_1 = A_2 = 2sm = 2 \cdot 10^{-2}$ m;

$\Delta\varphi = \varphi_2 - \varphi_1 = \frac{\pi}{4};$

$\varphi_1 = 0.$

$x=?$

Yechish: Ikkita garmonik tebranishlarning qo'shilishi natijasida hosil bo'ladigan harakat ham garmonik tebranish bo'lib uning tenglamasi quyidagicha bo'ladi:

$$x = A\cos(\omega t + \varphi). \qquad (1)$$

Bir tomonga yo'nalgan teng davrli tebranishlarning qo'shilishidan hosil bo'lgan tebranishning davri:

$$T = T_1 = T_2.$$

Demak,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8s} = \frac{\pi}{4} s^{-1}. \qquad (2)$$

Amplitudasi:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_1 - \varphi_2)} =$$

$$= \sqrt{(2\cdot10^{-2})^2 + (2\cdot10^{-2})^2 + 2\cdot2\cdot10^{-2}\cdot2\cdot10^{-2}\cdot\cos\frac{\pi}{4}}\,\text{m} =$$

$$= 3{,}7\cdot10^{-2}\,\text{m}. \qquad (3)$$

Boshlang'ich fazasi:

$$\operatorname{tg}\varphi = \frac{A_1\cdot\sin\varphi_1 + A_2\cdot\sin\varphi_2}{A_1\cdot\cos\varphi_1 + A_2\cos\varphi_2} = \frac{2\cdot10^{-2}\cdot\sin o + 2\cdot10^{-2}\sin\frac{\pi}{4}}{2\cdot10^{-2}\cdot\cos o + 2\cdot10^{-2}\cos\frac{\pi}{4}} = \frac{\sin\frac{\pi}{4}}{\cos o + \cos\frac{\pi}{4}} = 0{,}414$$

$$\varphi = \operatorname{arctg}(0{,}414) = \frac{\pi}{8}. \qquad (4)$$

(2), (3) va (4) larni (1) ga qo'yib olamiz:

$$x = 3{,}7\cdot10^{-2}\cdot\cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)\text{m}.$$

Javob: $x = 3{,}7 \cdot 10^{-2} \cdot \cos\left(\frac{\pi}{4}t + \frac{\pi}{8}\right)$,m.

4-misol. Tinch turgan qabul qiluvchi, yaqinlashayotgan va 360 Hz chastota bilan to‘lqin chiqarayotgan manba tarqatuvchi to‘lqinlarni 400 Hz tovush to‘lqinlari sifatida qayd qiladi. Havoning harorati 290 K, molyar massa 0,29kg/mol bo‘lsa, tovush manbaining tezligi topilsin.

Berilgan:

$\nu_0 = 360\text{Hz};$

$\nu = 400\text{Hz};$

$T = 290\text{K};$

$M = 0{,}029\,{}^{\text{kg}}\!/_{\text{mol}}.$

$\vartheta_M = ?$

Yechish: Dopler effektining akustikadagi ifodasini qabul qiluvchi tinch turgan ($\vartheta_k = 0$) manba yaqinlashayotgan hol uchun yozamiz:

$$\nu = \frac{\vartheta \cdot \nu_0}{\vartheta - \vartheta_M}, \qquad (1)$$

bu yerda: ϑ – tovushning tarqalish tezligi. (1) dan ϑ_m uchun:

$$\vartheta_m = \vartheta\left(1 - \frac{\nu_0}{\nu}\right). \qquad (2)$$

Gazlarda tovush to‘lqinlarining tarqalish tezligi

$$\vartheta = \sqrt{\gamma \frac{RT}{M}} \qquad (3)$$

ekanligidan foydalansak,

$$\vartheta_\text{M} = \left(1 - \frac{\nu_0}{\nu}\right)\sqrt{\gamma \frac{RT}{\text{M}}}. \qquad (4)$$

Bu yerda: $R = 8{,}31 \frac{\text{J}}{\text{K} \cdot \text{mol}}$ – molyar gaz doimiysi, $\gamma = \frac{i+2}{i} = 1{,}4$ (havo uchun molekulalarning erkinlik darajasi $i = 5$ deb olindi).

(4) dagi ayirma birliksiz ekanligini e'tiborga olib tezlikning birligini tekshirib ko'ramiz:

$$[\vartheta]^2 = \frac{[R][T]}{[M]} = \frac{1\frac{\mathrm{J}}{\mathrm{K \cdot mol}} \cdot 1\mathrm{K}}{1\frac{\mathrm{kg}}{\mathrm{mol}}} \quad 1\frac{\mathrm{N \cdot m}}{\mathrm{kg}} = 1\frac{\mathrm{kg \cdot m^2}}{\mathrm{kg \cdot s^2}} = 1\frac{\mathrm{m^2}}{\mathrm{s^2}},$$

$$[\vartheta] = 1\frac{\mathrm{m}}{\mathrm{s}}.$$

Kattaliklarning qiymatlarini (4) ga qo'yamiz:

$$\vartheta_{\mathrm{M}} = \left(1 - \frac{360}{400}\right) - \frac{\sqrt{1{,}4 \cdot 8.31 \cdot 290}}{0{,}029}\frac{\mathrm{m}}{\mathrm{s}} = 34{,}1\frac{\mathrm{m}}{\mathrm{s}}.$$

Javob: $\vartheta_{\mathrm{M}} = 34{,}1\frac{\mathrm{m}}{\mathrm{s}}$.

Mustaqil yechish uchun masalalar

1. 4 s davr va 3 sm amplituda bilan garmonik tebranayotgan nuqta tezligining va tezlanishining maksimal qiymatlari aniqlansin. [4,71sm/s; 7,4 sm/s^2.]

2. Moddiy nuqtaning harakat tenglamasi $x = \sin\frac{\pi}{6}t$ ko'rinishga ega. Moddiy nuqta tezligi va tezlanishning maksimal qiymatlariga erishiladigan vaqt aniqlansin. [3 s; 9 s; 15 s.]

3. Garmonik tebranayotgan nuqta (boshlang'ich fazasi nolga teng) muvozanat vaziyatidan 2,4 sm siljiganda tezligi 3sm/s, 2,8sm siljiganda tezligi 2sm/s bo'ladi. Tebranish amplitudasi va davri topilsin. [3,1 $\cdot$ 10^{-2}m; 4,1s.]

4. Vaqtning 1) $t = \frac{T}{6}$; 2) $t = \frac{T}{8}$ paytlari uchun garmonik tebranayotgan nuqta kinetik va potensial energiyalarining nisbati aniqlansin. Boshlang'ich faza nolga teng. [1) 1/3; 2) 1.]

5. $x_1 = 0{,}02\sin(5\pi t + \frac{\pi}{2})$ m va $x_2 = 0{,}03\sin(5\pi t + \frac{\pi}{4})$m tenglamalarga muvofiq o'zgaradigan va bir tomonga yo'nalgan tebranishlarning

qo'shilishidan hosil bo'ladigan garmonik tebranishning amplitudasi va fazasi topilsin. $[4,6\cdot 10^{-2}m;\quad 62^0 46']$

6. Nuqta bir vaqtda ikki $x = 2\sin\omega tM$ va $y = 2\cos\omega tM$ o'zaro perpendikulyar tebranishlarga qatnashadi. Nuqtaning harakat traektoriyasini toping. [$x^2+y^2=4$.]

7. Prujinaga osilgan yukning massasi 600 g ga orttirilsa, yukning tebranish davri 2 marta oshadi. Prujinaga osilgan dastlabki yukning massasi topilsin. [0,2 kg.]

8. Matematik mayatnik tebranishi so'nishining logarifmik diskrementi 0,3 ga teng. Mayatnikning bir marta to'la tebranishida tebranish amplitudasi necha marta kamayadi. [1,22.]

9. Tebranish manbaidan 4 sm masofadagi nuqta T/6 vaqtda muvozanat vaziyatidan yarim amplitudaga teng siljiydi. Yuguruvchi to'lqin uzunligi topilsin. [0,48 m.]

10. Turg'un to'lqinning birinchi va to'rtinchi qavariqlari orasidagi masofa 15 sm bo'lsa, to'lqin uzunligi aniqlansin. [0,1 m.]

11. 20^0C da molyar massa $2,9\cdot 10^{-2}$kg/mol bo'lgan gazdagi tovushning tezligi 343m/s. Gazning o'zgarmas bosimdagi va hajmdagi molyar issiqlik sig'imlarining nisbatlari aniqlansin. [1,4.]

12. Poezd 54km/soat tezlik bilan harakatsiz qabul qiluvchi yonidan o'tadi va signal beradi. Qabul qiluvchi 53 Hz chastota o'zgarishini qayd qiladi. Tovushning tezligini 340m/s deb hisoblab, poyezd signalining chastotasi aniqlansin. [599 Hz.]

25-§. O'zgaruvchan tok. Elektromagnit tebranishlar va to'lqinlar

Asosiy formulalar

Uchlariga $U = U_m \cos\omega t$ kuchlanish beriladigan, R – qarshilikli rezistor, L – induktivlikli g'altak va C – sig'imli kondensator ketma-ket ulangan elektr zanjirining o'zgaruvchan tokka to'la qarshiligi:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega s}\right)^2} = \sqrt{R^2 + (R_L - R_c)^2},$$

bunda: $R_L = \omega L$ – induktiv reaktiv qarshilik, $R_c = \dfrac{1}{\omega C}$ – sig'imiy reaktiv qarshilik.

Tok kuchi va kuchlanish orasidagi fazalar siljishi

$$\text{tg}\varphi = \frac{\omega L - 1/(\omega C)}{R}.$$

Tok kuchi va kuchlanishlarning ta'sir etuvchi (effektiv) qiymatlari

$$I = \frac{I_m}{\sqrt{2}}; \quad U = \frac{U_m}{\sqrt{2}}.$$

I_m va U_m lar tok kuchi va kuchlanishlarning amplitudaviy qiymatlari.

O'zgaruvchan tok zanjirida ajraladigan o'rtacha quvvat:

$$\langle P \rangle = \frac{1}{2} I_m U_m \cos\varphi,$$

bu yerda:

$$\cos\varphi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

Elektr zaryadining konturdagi erkin garmonik tebranishlarining differensial tenglamasi va uning yechimi:

$$\ddot{Q}+\frac{1}{LC}Q=0; \quad Q=Q_m\cdot\cos(\omega_0 t+\varphi).$$

Bunda: Q_m — zaryadning tebranish amplitudasi; $\omega_0=\frac{1}{\sqrt{LC}}$ — konturning xususiy chastotasi.

Tomson formulasi:

$$T=2\pi\sqrt{LC}\cdot$$

Yassi elektromagnit to'lqin tenglamasi:

$$\vec{E}=\vec{E_0}\cos(\omega t-kx+\varphi); \quad \vec{H}=\vec{H_0}\cos(\omega t-kx+\varphi),$$

bu yerda: $\vec{E_0}$ va $\vec{H_0}$ — mos ravishda to'lqin elektr va magnit maydon kuchlanganliklarining amplitudalari, ω — siklik chastota, $k=\frac{\omega}{\vartheta}$ — to'lqin soni, φ — $x=0$ koordinatali nuqtalarda tebranishning boshlang'ich fazaviy tezligi:

$$\vartheta=\frac{1}{\sqrt{\varepsilon\mu_0}}\cdot\frac{1}{\sqrt{\varepsilon\mu}}=\frac{C}{\sqrt{\varepsilon\mu}},$$

bu yerda: $C=\frac{1}{\sqrt{\varepsilon\mu_0}}$ — yorug'likning bo'shliqda tarqalish tezligi, ε_0 va μ_0 lar mos ravishda elektr va magnit doimiylari. ε va μ lar mos ravishda muhitning elektr va magnit kirituvchanliklari.

Elektromagnit to'lqinning elektr *(E)* va magnit *(H)* maydon kuchlanganliklarining oniy qiymatlari orasidagi munosabat

$$\sqrt{\varepsilon_0\varepsilon}E=\sqrt{\mu_0\mu}H.$$

Elektromagnit maydon energiyasining hajmiy zichligi

$$\omega=\frac{\varepsilon_0\varepsilon E^2}{2}+\frac{\mu_0\mu H^2}{2}.$$

Elektromagnit maydon energiyasi oqimining zichligi — Umov-Poynting vektori:

$$\vec{S}=\left[\vec{E}\cdot\vec{H}\right].$$

Masala yechishga misollar

1-misol. G'altak cho'lg'ami ko'ndalang kesim yuzasi 1mm[2] bo'lgan 500 ta mis sim o'ramidan iborat. G'altakning uzunligi 50 sm va diametri 5 sm. G'altakning to'la qarshiligi uning aktiv qarshiligidan ikki marta katta bo'lishi uchun g'altakka qanday chastotali o'zgaruvchan tok ulanishi kerak?

Berilgan:

$S = 1\text{mm}^2 = 10^{-6}\,\text{m}^2$;

$N = 500$;

l =50 sm=0,5 m;

$d = 5\text{sm} = 5\cdot 10^{-2}\,\text{m}$;

$\frac{Z}{R} = 2$;

$\rho = 1{,}7\cdot 10^{-8}\,\Omega\cdot\text{m}$.

$\nu = ?$

Yechish: Masalada berilgan holda $(R_C = 0)$ zanjirning to'la qarshiligi quyidagicha aniqlanadi:

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad (1)$$

bu yerda $\omega = 2\pi\upsilon$, ω – o'zgaruvchan tok chastotasi. L– g'altakning induktivligi

$$L = \mu_0\left(\frac{N}{l}\right)^2 V = \mu_0\left(\frac{N}{l}\right)^2 l\cdot\pi\frac{d^2}{4} = \pi\mu_0 d^2\frac{N^2}{4\cdot l}. \qquad (2)$$

(1) dan ω ni aniqlab olamiz:

$$\omega = \frac{R}{L}\sqrt{\left(\frac{Z}{R}\right)^2 - 1} = \sqrt{3}\frac{R}{L}. \qquad (3)$$

(3) ni hosil qilishda $\frac{Z}{R} = 2$ ligi e'tiborga olingan. G'altakning aktiv qarshiligini quyidagicha aniqlaymiz:

$$R = \rho\frac{l_S}{S} = \rho\frac{\pi d\cdot N}{S}. \qquad (4)$$

Bu yerda mis simning uzunligi $l_S = \pi d\cdot N$ ekanligidan foydalanilgan. (2) va (4) larni (3) ga qo'yib olamiz:

$$\omega = \frac{4\cdot\sqrt{3}\rho\cdot l}{\mu_0 dS\cdot N}, \qquad \text{yoki} \qquad \nu = \frac{2\sqrt{3}\rho\cdot l}{\pi\mu_0 dS\cdot N}. \qquad (5)$$

$$[\omega] = \frac{[\rho][l]}{[\mu_0][d][S]} = \frac{1\Omega\cdot \mathrm{m}\cdot 1\mathrm{m}}{1\frac{\mathrm{H}}{\mathrm{m}}\cdot 1\mathrm{m}\cdot 1\mathrm{m}^2} = 1\frac{\Omega\cdot \mathrm{A}}{\mathrm{V}\cdot \mathrm{s}} = 1\mathrm{s}^{-1}.$$

Kattaliklarning son qiymatlarini (5) ga qo‘yib olamiz:

$$\left(\mu_0 = 4\pi\cdot 10^{-7}\frac{\mathrm{H}}{\mathrm{m}}\right)$$

$$\nu = \frac{2\cdot\sqrt{3}\cdot 1{,}7\cdot 10^{-8}\cdot 0{,}5}{4\pi^2\cdot 10^{-7}\cdot 5\cdot 10^{-2}\cdot 10^{-6}\cdot 500}\mathrm{s}^{-1} = 240\mathrm{Hz}.$$

Javob: $\nu = 240\mathrm{Hz}$.

2-misol. 220 B kuchlanishli va 50 Hz chastotali o‘zgaruvchan tok zanjiriga 35,4 mkF sig‘imli kondensator, $100\,\Omega$ aktiv qarshilik va 0,7H induktivlikli g‘altak ketma-ket ulangan. Zanjirdagi tok kuchini, hamda kondensator, aktiv qarshilik va induktivlik g‘altakdagi kuchlanish tushushini toping.

Berilgan:

$U_m = 220\mathrm{N}$;

$\nu = 50\mathrm{Hz}$;

$C = 35{,}4\mathrm{mkF} = 35{,}4\cdot 10^{-6}\,\mathrm{F}$;

$R = 100\,\Omega$;

$L = 0{,}7\mathrm{H}$.

$I_m = ?$

$U_C = ?$

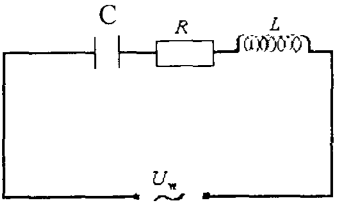
$U_L = ?$

$U_R = ?$

Yechish: Tok kuchini o‘zgaruvchan tok uchun Om qonunining ifodasidan topamiz.:

$$I_m = \frac{U_m}{Z} = \frac{U_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

yoki $\omega = 2\pi\upsilon$ ligidan:



54-rasm

$$I_m = \frac{U_m}{\sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}} \qquad (1)$$

Kuchlanishlarni topish uchun esa mos ravishda quyidagi ifodalardan foydalanamiz:

$$U_C = I \cdot R_S = \frac{I}{\omega C} = \frac{I}{2\pi\nu C}, \qquad (2)$$

$$U_R = I \cdot R \qquad (3)$$

$$U_L = I \cdot R_L = I \cdot \omega L = 2\pi\nu I \cdot L. \qquad (4)$$

Kattaliklarning qiymatlarini qo‘yib olamiz.

$$I_m = \frac{220}{\sqrt{(100)^2 + \left(2 \cdot 3{,}14 \cdot 50 \cdot 0{,}7 - \frac{1}{2 \cdot 3{,}14 \cdot 50 \cdot 35{,}4 \cdot 10^{-6}}\right)^2}}\,\text{A} = 1{,}34\,\text{A};$$

$$U_S = \frac{1{,}34}{2 \cdot 3{,}14 \cdot 50 \cdot 35{,}4 \cdot 10^{-6}}\,\text{V} = 121\text{V};$$

$$U_R = 1{,}34 \cdot 100\text{V} = 134\text{V};$$

$$U_L = 2 \cdot 3{,}14 \cdot 50 \cdot 1{,}34 \cdot 0{,}7\text{V} = 295\text{V}.$$

Javob: $I_m = 1{,}34\,\text{A}$; $U_S = 121\text{V}$; $U_R = 134\text{V}$; $U_L = 295\text{V}$.

3-misol. Chulg‘ami 1000 o‘ramdan iborat bo‘lgan g‘altakning uzunligi 50 sm va ko‘ndalang kesim yuzasi 3 sm^2. G‘altak qoplamalarining yuzasi 75 sm^2 dan va ular orasidagi 5mm oraliq havo bilan to‘ldirilgan kondensatorga parallel ulangan. Shunday konturning tebranish davri topilsin.

Berilgan:

$l = 50\text{sm} = 0{,}5\text{m}$;

$S_L = 3\text{sm}^2 = 3\cdot10^{-4}\text{m}^2$;

$N = 1000 = 10^3$;

$S_C = 75\text{sm}^2 = 7{,}5\cdot10^{-3}\text{m}^2$;

$d = 5\text{mm} = 5\cdot10^{-3}\text{m}$;

$\varepsilon = 1$;

$\mu = 1$.

$T = ?$

Yechish: Konturda aktiv qarshilik bo‘lmaganda, Tomson formulasiga muvofiq tebranish konturining davri quyidagicha aniqlanadi:

$$T = 2\pi\sqrt{LC}\,. \qquad (1)$$

Bu yerda L g‘altakning induktivligi

$$L = \mu_0\mu\frac{N^2}{l}S_L \qquad (2)$$

va C yassi kondensatorning sig‘imi

$$C = \frac{\varepsilon_0\varepsilon S_C}{d}\,. \qquad (3)$$

(2), (3) formulalarni (1) ga qo‘yib quyidagini olamiz:

$$T = 2\pi\sqrt{\frac{\mu_0\mu\varepsilon_0\varepsilon N^2 S_L\cdot S_C}{l\cdot d}}\,. \qquad (4)$$

Kattaliklarning son qiymatlarini qo‘yib hisoblaymiz:

$$\left(\mu_0 = 4\pi\cdot10^{-7}\,\frac{\text{H}}{\text{m}};\qquad \varepsilon_0 = 8{,}85\cdot10^{-12}\,\frac{\text{F}}{\text{m}}\right)$$

$$T = 2\cdot3{,}14\cdot\sqrt{\frac{4\cdot3{,}14\cdot10^{-7}\cdot8{,}85\cdot10^{-12}\cdot(10^3)^2 3\cdot10^{-4}\cdot7{,}5\cdot10^{-3}}{0{,}5\cdot5\cdot10^{-3}}}s = .$$

$= 6{,}28\cdot10^{-7}$ s=628 ns.

Javob: T=628 ns.

5-misol. Tebranish konturining induktivligi 0,5mH. Konturning 300 m to‘lqin uzunligida rezonans ro‘y berishi uchun konturning elektr sig‘imi qanday bo‘lishi kerak?

Berilgan:

$L = 0{,}5\text{mH} = 5\cdot 10^{-4}\,\text{H};$

$\lambda = 300\text{m}\cdot$

$S_O = ?$

Yechish: Tomson formulasiga muvofiq tebranish konturining tebranish davri quyidagicha aniqlanadi:

$$T = 2\pi\sqrt{LC_O}\,. \qquad (1)$$

(1) dan elektr sig'imini aniqlaymiz:

$$C_O = \frac{T^2}{4\pi^2 L}\,. \qquad (2)$$

Bu erda: T — konturning rezonans ro'y bergandagi tebranish davri. U rezonans to'lqin uzunligi bilan quyidagicha bog'langan.

$$\lambda = cT,$$

yoki

$$T = \frac{\lambda}{c}\,. \qquad (3)$$

(3) ni (2) ga qo'ysak olamiz. Bu yerda $c = 3\cdot 10^8\ \text{m}/\text{s}$ – elektromagnit to'lqinlarning bo'shliqdagi tezligi.

$$C_O = \frac{\lambda^2}{4\pi^2 c^2 L} \qquad (4)$$

$$[C_O] = \frac{[\lambda]^2}{[c]^2[L]} = \frac{1\text{m}^2}{1\frac{\text{m}^2}{\text{s}^2}\cdot 1\text{H}} = 1\frac{\text{s}^2}{\text{H}} = 1\frac{\text{s}^2\cdot\text{A}}{\text{V}\cdot\text{s}} = 1\frac{\text{s}\cdot\text{C}}{\text{V}\cdot\text{s}} = 1\text{F}\,.$$

Berilganlarni (4) ga qo'yib olamiz:

$$C_O = \frac{(300)^2}{4\cdot(3{,}14)^2\cdot(3\cdot 10^8)^2\cdot 5\cdot 10^{-4}}F = 0{,}051\cdot 10^{-9}F = 51nF\,.$$

Javob: $C_O = 51nF$.

Mustaqil yechish uchun masalalar

1. Zanjirning 10 mkF sig'imli kondensator va 50Ω qarshilikli rezistor parallel ulangan qismining 50 Hz chastotali o'zgaruvchan tok uchun to'la qarshiligi aniqlansin. $[49{,}4\Omega]$

2. 32kHz chastotali va kuchlanishining amplitudaviy qiymati 120V bo'lgan generator 1nF sig'imli rezonans zanjiriga ulangan. Agar zanjirning aktiv qarshiligi 5Ω bo'lsa, kondensatordagi kuchlanishning amplitudaviy qiymati aniqlansin. [119 kV.]

3. Tebranish konturining aktiv qarshiligi $0{,}4\Omega$. Konturda tok kuchining amplitudaviy qiymati 30mA bo'lgan garmonik tebranishlarni ta'minlay olishi uchun, tebranish konturi iste'mol qiladigan o'rtacha quvvat $\langle P\rangle$ aniqlansin. [18mW.]

4. Kuchlanishining ta'sir etuvchi qiymati 120V bo'lgan zanjirga aktiv qarshiligi 10Ω bo'lgan o'tkazgich va induktivligi 0,1H bo'lgan g'altak ketma-ket ulangan. Agar zanjirdagi tokning amplitudaviy qiymati 5A bo'lsa, tokning chastotasi aniqlansin. [51,6 Hz.]

5. Tebranish konturi 1mH induktivlikli g'altak va 2nF sig'imli kondensatordan tashkil topgan. Konturning qanday to'lqin uzunligiga moslashtirilgani aniqlansin. Konturning aktiv qarshiligi hisobga olinmasin. [$2{,}67\cdot10^3$ m.]

6. Tebranish konturi 1mH induktivlikli g'altakdan va qoplamalarining yuzasi 155 sm^2, orasidagi masofa 1,5mm bo'lgan kondensatordan tashkil topgan. Konturda $63\,\Omega$ to'lqin uzunligida rezonans ro'y bersa, kondensator qoplamlari orasidagi muhitning dielektrik kirituvchanligi aniqlansin. [6,11.]

7. Biror muhitda elektromagnit to'lqinlarning tarqalish tezligi 250Mm/s ni tashkil qiladi. Agar bu elektromagnit to'lqinlarning bo'shliqdagi chastotasi 1MHz ni tashkil qilsa, ularning muhitdagi to'lqin uzunligi aniqlansin.[250 m.]

8. Tebranish konturi 0,5nF sig'imli kondensator va 0,4mH induktivlikli g'altakdan iborat. Konturda hosil bo'ladigan to'lqinlarning to'lqin uzunligi aniqlansin. [843 m.]

9. Bo'shliqda x o'qi bo'ylab yassi elektromagnit to'lqinlar tarqaladi. To'lqin elektr maydon kuchlanganligining amplitudasi 10 V/m. To'lqin magnit maydon kuchlanganligining amplitudasi aniqlansin. [0,265 A/m.]

Yassi monoxromatik elektromagnit to'lqinlar x o'qi bo'ylab tarqaladi. To'lqin elektr maydon kuchlanganligining amplitudasi $5\frac{mV}{m}$, magnit maydon kuchlanganligining amplitudasi $1\frac{mA}{m}$. To'lqinning x o'qiga perpendikulyar joylashgan 15 sm^2 yuzali sirt orqali 10min vaqtda olib o'tadigan energiyasi aniqlansin. To'lqin davri $T \ll t$. [2,25 mkJ.]

V BOB. OPTIKA

26-§. Geometrik optika va fotometriya elementlari

Asosiy formulalar

Sferik ko'zguning fokus masofasi:

$$f = \frac{R}{2},$$

bunda R – ko'zguning egrilik radiusi.

Sferik ko'zguning optik kuchi:

$$F = \frac{1}{f}.$$

Sferik ko'zgu formulasi (yupqa linza formulasi):

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b},$$

bunda a va b – mos ravishda ko'zgu qutbidan buyum va tasvirgacha masofalar. Agar tasvir mavhum bo'lsa b manfiy ishora bilan olinadi.

Yorug'likning sinish qonuni:

$$\sin\frac{i_1}{i_2} = n_{21},$$

bunda: i_1 – tushish burchagi, i_2 – sinish burchagi, $n_{21} = \frac{n_2}{n_1}$ – ikkinchi muhitning birinchi muhitga nisbatan sindirish ko'rsatkichi

Lupaning burchak kattalashtirishi

$$G = D/f,$$

bunda D – eng yaxshi ko'rish masofasi (D = 25 sm).

Teleskopning burchak kattalashtirishi:

$$G = f_{ob}/f_{ok},$$

f_{ob} va f_{ok} – mos ravishda obyektivning va okulyarning fokus masofalari.

Izotropik nuqtaviy yorug'lik manbayi: ω fazoviy burchak chegarasida tarqatadigan yorug'lik oqimi:

$$\Phi_\vartheta = I \cdot \omega,$$

bunda I – yorug‘lik kuchi;
To‘la yorug‘lik og‘imi:

$$\Phi_0 = 4\pi I.$$

Sirtning yoritilganligi:

$$E_\vartheta = \frac{\Phi}{S},$$

S – yorug‘lik tushayotgan tekislikning yuzasi.
Yorug‘lik tarqatuvchi sirtning ravshanligi:

$$L_\vartheta = \frac{I}{\sigma}.$$

Bunda: I – kuzatish yo‘nalishidagi yorug‘lik kuchi; σ – yorug‘lik tarqatuvchi sirtning shu yo‘nalishga tik bo‘lgan tekislikdagi proeksiyasining yuzasi.
Yorituvchanlik:

$$M_\vartheta = \frac{\Phi_\vartheta}{S}.$$

Bu yerda: Φ_ϑ – sirt chiqarayotgan yorug‘lik oqimi; S – shu sirtning yuzasi.

Masala yechishga misollar

1-misol. Yorug‘lik nuri 45^0 burchak ostida yassi parallel shisha ($n = 1{,}6$) plastinkaga tushadi. Agar plastinkadan chiqayotgan nur tushayotgan nurning davomidan $h = 2$ sm masofaga siljisa, plastinka qalinligi topilsin.

Berilgan:
n=1,6:
i=45⁰;
h=2 sm=2 · 10⁻² m.

$d = ?$

Yechish: Rasmdan ko‘rinib turibdiki plastinkadan chiqayotgan nur tushayotgan nurga parallel bo‘ladi. 55-rasmdan

$$\frac{d}{\cos r} = \frac{h}{\sin(i - r)} \qquad (1)$$

ko‘rinib turibdi. (1) ifodadan d ni aniqlasak,

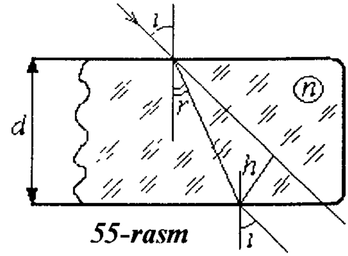
$$d = \frac{h \cdot \cos r}{\sin(i - r)} = \frac{h \cdot \cos r}{\sin i \cdot \cos r - \cos\ i \cdot \sin r}. \qquad (2)$$

Sinish qonuniga muvofiqЖ

$$\frac{\sin i}{\sin r} = n,$$

bundan

$$\sin r = \frac{\sin i}{n}, \qquad (3)$$



55-rasm

bu ifodani (2) ga qo‘ysak va burchak kosinuslarini sinuslar orqali ifodalasak, plastinka qalinlıgi uchun quyidagi ifodani olamiz:

$$d = \frac{h\sqrt{n^2 - \sin^2 i}}{\sin i(\sqrt{n^2 - \sin^2 i} - \sqrt{1 - \sin^2 i})}. \qquad (4)$$

Berilganlarni (4) ifodaga qo‘yib hisoblaymiz:

$$d = \frac{2 \cdot 10^{-2} m \cdot \sqrt{(1,6)^2 - \sin^2 45^0}}{\sin 45^0(\sqrt{(1,6)^2 - \sin^2 45^0} - \sqrt{1 - \sin^2 45^0})} \text{m} = 5,58 \cdot 10^{-2} \text{m} = 5,58 \text{sm}$$

Javob: d=5,58 sm.

2-misol. Projektor $2\theta = 40^0$ ochilish burchagi bilan kesik konus ko‘rinishidagi yorug‘lik dastasini beradi. Projektorning yorug‘lik oqimi 80 klm. Yorug‘lik oqimi konus ichida tekis taqsimlangan deb hisoblab, projektorning yorug‘lik kuchi aniqlansin.

Berilgan:

$2\theta = 40^0$;

$\Phi = 40 \text{klm} = 4 \cdot 10^4 \text{lm}.$

$I = ?$

Yechish: Izotropik manbaning yorug‘lik kuchi J yorug‘lik oqimi Φ ning yorug‘lik oqimi tarqaladigan chegaradagi fazoviy burchak ω ga nisbatiga teng, ya’ni

$$I = \frac{\Phi}{\omega}. \qquad (1)$$

Ma'lumki, elementar fazoviy burchak $d\omega$ quyidagicha aniqlanadi:

$$d\omega = 2\pi \sin\theta \cdot d\,\theta \cdot$$

Konusning 2θ ochilish burchagiga mos keluvchi fazoviy burchakni esa quyidagi integral orqali topamiz:

$$\omega = 2\pi \int_0^{\theta_0} \sin\theta d\theta,$$

buni integrallaymiz:

$$\omega = 2\pi(1 - cos\theta_0) = 4\pi \sin^2\left(\frac{\theta}{2}\right). \qquad (2)$$

(2) ni (1) ga qo'yib quyidagini olamiz:

$$I = \frac{\Phi}{4\pi \sin^2\left(\frac{\theta}{2}\right)} \qquad (3)$$

Kattaliklarning qiymatlarini (3) ga qo'ysak,

$$I = \frac{4 \cdot 10^4 \text{lm}}{4 \cdot 3{,}14 \cdot \sin^2 10^0} = 211\text{kkd}.$$

Javob: *I*=211 kkd.

3-misol. Diametri 2,5 sm va uzunligi 40 sm bo'lgan silindrsimon lyuminestsensiyaga asoslangan lampa 5 m masofada lampa o'qiga tik yo'nalishda 2 l k yoritilganlik hosil qilinadi. Lampani kosinusoidal tarqatuvchi sifatida qarab: 1) berilgan yo'nalishdagi yorug'lik kuchi *I;* 2) ravshanlik *L;* 3) lampaning yorqinligi *R* aniqlansin.

Berilgan:

$d = 2{,}5\text{sm} = 2{,}5 \cdot 10^{-2}\text{m};$

$l = 40\text{sm} = 0{,}4\text{m};$

$r = 5\text{m};$

$E_\vartheta = 2l\text{k}.$

I=?
L=?
R=?

Yechish: 1. Lampaning katta o'lchami uning uzunligi bo'lib, u yoritilganlik o'lchangan masofadan 12 marta kichik. Demak, berilgan yo'nalishdagi yorug'lik kuchini hisoblashda lampani nuqtaviy manba deb qabul qilish va quyidagi formuladan foydalanish mumkin:

$$E_\vartheta = \frac{I}{r^2} \qquad (1)$$

Bundan

$$I = E_g \cdot r^2. \qquad (2)$$

2. Ravshanlikni hisoblash uchun

$$L = \frac{I}{\sigma} \qquad (3)$$

ifodadan foydalanamiz, bunda σ – yorug'lik manbayining kuzatish yo'nalishiga tik tekislikdagi proeksiyasining yuzasi.

Silindrsimon lampa hamda proeksiya yuzasi, uning uzunligi l va kengligi d bo'lgan to'g'ri to'rtburchak shakliga ega bo'ladi. Demak, $\sigma = l \cdot d$ Shunday qilib,

$$L = \frac{I}{l \cdot d}. \qquad (4)$$

3. Lampani kosinusoidal tarqatuvchi sifatida qaralganda uning yorqinligi

$$R = \pi L \qquad (5)$$

kabi aniqlanadi.

Kattaliklarning qiymatlarini (2), (4) va (5) larga qo'yib, hisoblaymiz:

$$I = 2 \cdot 25\text{kd} = 50\text{kd};$$

$$L = \frac{50}{0{,}4 \cdot 2{,}5 \cdot 10^{-2}} \frac{\text{kd}}{\text{m}^2} = 5\frac{\text{kkd}}{\text{m}^2};$$

$$R = 3{,}14 \cdot 5\text{klk} = 15{,}70\text{klk}.$$

Javob: I=50 kd; $L = 5\frac{\text{kkd}}{\text{m}^2}$; $R = 15{,}0\text{klk}$.

4-misol. 50 kd bilan bir tekis yorituvchi yorug'lik manbayining diametri 50 sm.

1) yoritgich chiqaradigan to'la yorug'lik oqimi Φ; 2) agar yoritgichdan chiqayotgan yorug'lik oqimining 20% ekranga tushayotgan bo'lsa, uning yorqinligi R; 3) yoritilganligi E_l, yorqinligi R_l va ravshanligi B_l aniqlansin. Ekranning yuzasi 0,5m² va uning sirtidan yorug'likning qaytish koeffitsienti 0,7.

Berilgan:

$I = 500\text{kd};$

$d = 50\text{sm} = 0{,}5\text{m};$

$\frac{\Phi_1}{\Phi} = 0{,}2;$

$S_1 = 0{,}5\text{m}^2;$

$\rho = 0{,}7.$

---

1) $\Phi = ?$

2) $R = ?$

3) $E_1 = ?$

$R_1 = ?$

$B_1 = ?$

Yechish: 1) Nuqtaviy manbadan chiqayotgan to'la yorug'lik oqimi

$$\Phi = 4\pi J \qquad (1)$$

kabi aniqlanadi.

2) Yorug'lik manbayining yorqinligi:

$$R = \frac{\Phi}{S}.$$

Agar $S = 4\pi r^2 = \pi d^2$ ekanligini nazarda tutsak va (1)ni e'tiborga olsak

$$R = \frac{4\pi I}{\pi d^2} = \frac{4I}{d^2}. \qquad (2)$$

3) Ekranning yoritilganligi:

$$E_1 = \frac{\Phi_1}{S_1} = 0{,}2\frac{\Phi}{S_1}, \qquad (3)$$

yorqinligi

$$R_1 = \rho E_1, \qquad (4)$$

va ravshanligi:

$$B_1 = \frac{R_1}{\pi}. \qquad (5)$$

Kattaliklarning qiymatlarini (1) (5) larga qo'yib hisoblaymiz:

$$\Phi = 4 \cdot 3{,}14 \cdot 500\text{kd} = 6{,}28\text{klm};$$

$$\Phi = \frac{4 \cdot 500}{0{,}25}\frac{\text{klm}}{\text{m}^2} = 8\frac{\text{klm}}{\text{m}^2};$$

$$E_1 = 0{,}2\frac{6{,}28\text{klm}}{0{,}5\text{m}^2} = 2{,}51\text{klk};$$

$$R_1 = 0{,}7 \cdot 2{,}51\frac{\text{klm}}{\text{m}^2} = 1{,}76\frac{\text{klm}}{\text{m}^2};$$

$$B_1 = \frac{1{,}76}{3{,}14}\frac{\text{kd}}{\text{m}^2} = 0{,}56\frac{\text{klm}}{\text{m}^2} = 560\frac{\text{lm}}{\text{m}^2}.$$

Javob: 1) $\Phi = 6{,}28\,\text{klm}$; 2) $R = 8\dfrac{\text{klm}}{\text{M}^2}$; 3) $E_1 = 2{,}51\,\text{klk}$;

$$R_1 = 1{,}76\frac{\text{klm}}{\text{m}^2};\quad B_1 = 560\frac{\text{lm}}{\text{m}^2}.$$

Mustaqil yechish uchun masalalar

1. Botiq yumaloq ko'zgu ekranda narsa tasvirini 4 marta kattalashtirib beradi. Narsadan ko'zgugacha bo'lgan masofa 25 sm. Ko'zguning egrilik radiusi aniqlansin. [0,4 m.]

2. Botiq linzaning fokus masofasi 15 sm. Ko'zgu narsaning haqiqiy tasvirini uch marta kichraytirib beradi. Narsadan ko'zgugacha bo'lga a masofa aniqlansin. [0,6 m.]

3. Stolda qog'oz bo'lagi yotibdi. Qog'ozga 30^0 burchak ostida tushayotgan yorug'lik nuri yorug' dog' hosil qiladi. Agar qog'oz ustiga 5 sm qalinligidagi yassi parallel shisha plastinka qo'yilsa, bu dog' qanchaga siljiydi. [h=1,1 sm.]

4. Prizmaning sindiruvchi burchagi 60^0. Nurning dastlabki yo'nalishdan eng kam oqish burchagi 30^0. Prizma yasalgan shishaning sindirish ko'rsatkichi aniqlansin. [n=1,41.]

5. Egrilik radiuslari bir xil 0,5m bo'lgan ikkita soat shishasidan ikki yoqlama botiq "havo" linzasi yasalgan. Shunday linza suvda qanday optik kuchga ega bo'ladi? [−1,32 D.]

6. Havoda turgan linzaning fokus masofasi 5 sm, qand eritmasiga botirilgandan keyin esa 35 sm bo'lsa, eritmaning sindirish ko'rsatkichi aniqlansin. [1,4.]

7. Doiraviy maydoncha ustida lampa osilib turibdi. Maydonchaning o'rtasidagi yoritilganlik 40 lk, chekkasida esa 5 lk. Nur maydoncha chekkasiga qanday burchak ostida tushadi? [60^0.]

8. Oppoq cho'g' bo'lgancha qizdirilgan metall tolaning uzunligi 30sm, diametri 0,2mm. Tolaning unga tik yo'nalishdagi yorug'lik kuchi 24kd. Tolaning ravshanligi aniqlansin. [400 kd/m^2.]

9. Qorakuya qurumi qatlami bilan qoplangan sirtning yoritilganligi 150lk, ravshanligi hamma yo'nalishlarda bir xil va 1 kd/m^2 ga teng. Qurumning qaytarish koeffitsienti aniqlansin. [0,98.]

27-§. Yorug'lik interferensiyasi

Asosiy formulalar

Ikkita kogerent to'lqinning fazalari farqi:

$$d=\frac{2\pi}{\lambda_0}(L_2-L_1)=\frac{2\pi}{\lambda_0}\Delta,$$

bunda: $L=S\cdot n$ – optik yo'l uzunligi (S – yorug'lik to'lqinining muhitdagi geometrik yo'li uzunligi; n – shu muhitning sindirish ko'rsatkichi); $\Delta=L_2-L_1$ – ikkita yorug'lik to'lqinlarining optik yo'l farqi; λ_0 – vakuumda to'lqin uzunligi.

Interferension maksimumlar sharti:

$$\Delta=\pm m\lambda_0 (m=0,1,2...).$$

Interferension minimumlar sharti:

$$\Delta=\pm(2m+1)\frac{\lambda_0}{2}(m=0,1,2...).$$

Interferension yo'llar kengligi: $(l >> d)$

$$\Delta x=\frac{l}{d}\lambda_0,$$

bunda: d – ekrandan l masofada turgan ikkita kogerent manbalar orasidagi masofa.

Yupqa yassi parallel pardaning yuqori va quyi sirtlaridan qaytuvchi yorug'lik interferensiyasi uchun $(n_0=1)$:

maksimumlar

$$2d\cdot n\cdot\cos r\pm\frac{\lambda_0}{2}=2d\sqrt{n^2-\sin^2 i}\pm\frac{\lambda_0}{2}=m\lambda_0 (m=0,1,2,...)$$

va minimumlar

$$2d\cdot n\cdot\cos r\pm\frac{\lambda_0}{2}=2\cdot d\sqrt{n^2-\sin^2 i}\pm\frac{\lambda_0}{2}=(2m+1)\frac{\lambda_0}{2}(m=0,1,2,...)$$

shartlari. Bu yerda: d – pardaning qalinligi; n – uning sindirish ko'rsatkichi;

i — tushish burchagi; r — sinish burchagi. $\pm\frac{\lambda_0}{2}$ had muhitlar sirtidan yorug'likning qaytishi natijasida yarim to'lqinning yo'qolishini ko'rsatadi. Agar $n > n_0$ bo'lsa, musbat ishora, $n < n_0$ bo'lsa manfiy ishora olinadi.

Qaytgan yorug'lik uchun Nyuton yorug' halqalarining (o'tgan yorug'lik uchun qorong'i) radiuslari

$$r_m = \sqrt{\left(m - \tfrac{1}{2}\right)\lambda_0 R}, \qquad (m = 1,2,3...)$$

bunda: m — halqa nomeri; R — linzaning egrilik radiusi.

Qaytgan yorug'lik uchun Nyuton qorong'i halqalarining (o'tgan yorug'lik uchun yorug') radiuslari:

$$r_m = \sqrt{m\lambda_0 R}, \qquad (m = 1,2,3...).$$

Masala yechishga misollar

1-misol. Ikkita interferensiyaga kirishuvchi monoxromatik yorug'lik nurlarining optik yo'l farqi $0{,}3\lambda_0$. Ularning fazalar farqi topilsin.

Berilgan:

$\Delta = 0{,}3\lambda_0$.

δ=?

Yechish: Monoxromatik yorug'lik to'lqinlarining fazalar farqi:

$$\delta = \frac{2\pi}{\lambda_0}\Delta, \qquad (1)$$

ifoda yordamida aniqlanadi.

Unga kattaliklarning son qiymatlarini qo'yamiz

$$\delta = \frac{2\cdot\pi}{\lambda_0}\cdot 0{,}3\lambda_0 = 0{,}6\pi.$$

Javob: $\delta = 0{,}6\,\pi = 108^0$.

2-misol. Interferensiyaga kirishuvchi nurlarning optik yo'l farqi 1,9 mkm bo'lganda ko'zga ko'rinuvchi yorug'lik spektri (0,76 mkm dan 0,38 mkm gacha) uchun: 1) maksimal kuchaytirilgan; 2) maksimal susaytirilgan, — barcha to'lqin uzunliklari aniqlansin.

Berilgan:	**Yechish:** 1) Interferensiya natijasida maksimal kuchayuvchi yorug'lik to'lqinlari quyidagi ifodadan aniqlanadi:
$\Delta = 1{,}9\text{mkm};$ $\lambda_1 = 0{,}76\text{mkm};$ $\lambda_2 = 0{,}38\text{mkm}$	$$\Delta = \pm m\lambda, \qquad (m = 0,1,2,...). \qquad (1)$$ Bundan $$\lambda = \pm\frac{\Delta}{m}. \qquad (2)$$
1) $\lambda_{max} = ?$ 2) $\lambda_{min} = ?$	

(2) ifodaga kattaliklarning son qiymatlarini qo'yish m ning $m=3$, $m=4$, $m=5$ qiymatlaridagina to'lqin uzunliklari berilgan oraliqda yotishi ko'rinadi:

$$\lambda'_{max} = \frac{1{,}9\text{mkm}}{3} = 0{,}633\text{mkm}; \qquad (m = 3)$$

$$\lambda''_{max} = \frac{1{,}9\text{mkm}}{4} = 0{,}475\text{mkm}; \qquad (m = 4)$$

$$\lambda'''_{max} = \frac{1{,}9\text{mkm}}{5} = 0{,}38\text{mkm}. \qquad (m = 5)$$

2) Interferensiya natijasida maksimal susayuvchi yorug'lik to'lqinlari esa quyidagicha aniqlanadi:

$$\Delta = \pm(2m+1)\frac{\lambda}{2}, (m = 0,1,2,3,.....). \qquad (3)$$

Bundan $$\lambda = \pm\frac{2\Delta}{2m+1} \qquad (4)$$

Kattaliklarning qiymatlarini (4) ga qo'yish m ning $m=2$, $m=3$, $m=4$ qiymatlari uchungina to'lqin uzunliklari berilgan oraliqda yotishi ko'rinadi:

$$\lambda'_{min} = \frac{2\cdot 1{,}9\text{mkm}}{5} = 0{,}76\text{mkm}; \qquad (m = 2)$$

$$\lambda''_{min} = \frac{2\cdot 1{,}9\text{mkm}}{4} = 0{,}543\text{mkm}: \qquad (m = 3)$$

$$\lambda'''_{min} = \frac{2\cdot 1{,}9\text{mkm}}{9} = 0{,}422\text{mkm}. \qquad (m = 4)$$

Javob: 1) $\lambda'_{max} = 0{,}633\text{mkm}$; $\lambda''_{max} = 0{,}475\text{mkm}$; $\lambda'''_{max} = 0{,}38\text{mkm}$.

2) $\lambda'_{min} = 0{,}76\text{mkm}$; $\lambda''_{min} = 0{,}543\text{mkm}$; $\lambda'''_{min} = 0{,}422\text{mkm}$.

3-misol. Sindirish burchagi $40''$ bo'lgan shisha ponaga ($n=1,5$) 600 nm to'lqin uzunlikli monoxromagnit yorug'lik tik tushadi. Interferension manzarada ikkita qo'shni minimumlar orasidagi masofa aniqlansin.

Berilgan:

$n = 1{,}5$;

$\alpha = 40'' = 1{,}94 \cdot 10^{-4} rad$;

$\lambda = 600nm = 6 \cdot 10^{-7} m.$

$b = ?$

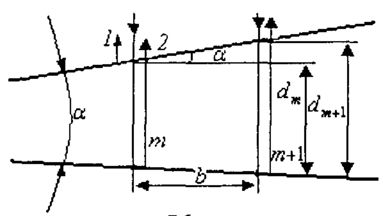
Yechish: Tushayotgan yorug'lik ponaning yuqori va quyi qirralaridan qaytadi (56-rasm). Pona burchagi kichik bo'lgani uchun 1 va 2 nurlar parallel va shu bilan birga kogerent. Shuning uchun ham interferensial manzarasi kuzatiladi.

Umumiy holda pona uchun minimumlar sharti:

$$2 \cdot d \cdot n \cdot \cos r + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}, \quad (m = 0,1,2,...) \qquad (1)$$

Bu yerda: $d - m$ -nomerga mos keluvchi qora yo'l joyidagi pona qalinligi. Masala shartiga ko'ra tushish burchagi nolga teng $(r = 0)$. Unda (1) quyidagi ko'rinishni oladi:

$$2 \cdot d \cdot n = m\lambda \cdot \qquad (2)$$



56-rasm

Bundan

$$d = \frac{m\lambda}{2n}. \qquad (3)$$

Rasmdan ko'rinib turibdiki,

$$\sin\alpha = \frac{\Delta d}{b} = \frac{d_{m+1} - d_m}{b}, \qquad (4)$$

yoki burchak kichik bo'lganida $\sin\alpha \approx \alpha$ bajarilishini e'tiborga olsak va (3) dan foydalansak,

$$\alpha = \frac{(m+1)\lambda - m\lambda}{2b \cdot n} = \frac{\lambda}{2bn}. \tag{5}$$

(5)-dan esa so'ralgan b uchun ifodani topamiz:

$$b = \frac{\lambda}{2n\alpha}. \tag{6}$$

Berilganlarni (6) ga qo'yib hisoblaymiz:

$$b = \frac{6 \cdot 10^{-7}\,\text{m}}{2 \cdot 1{,}5 \cdot 1{,}94 \cdot 10^{4}} 1{,}03 \cdot 10^{-3}\,\text{m} = 1{,}03\text{mm}.$$

Javob: b=1,03 mm.

4-misol. Qaytayotgan yorug'likda ($\lambda = 0{,}6$ mkm) kuzatilayotgan ikkinchi yoruq Nyuton halqasining radiusi $r_2 = 1{,}2mm$. Tajriba uchun olingan yassi qavariq linzaning optik kuchi aniqlansin.

Berilgan:	Yechish:
$\lambda = 0{,}6mkm = 0{,}6 \cdot 10^{-6}\,m;$ $r_2 = 1{,}2mm = 1{,}2 \cdot 10^{-3}\,m.$ $D = ?$	**Yechish:** Yassi qavariq linzaning optik kuchi quyidagicha aniqlanadi: $$D = \frac{1}{2R}. \tag{1}$$ Bu yerda R – linzaning egrilik radiusi.

Uni aniqlash uchun qaytgan yorug'likda Nyuton yorug' halqalarining radiuslarini aniqlash formulasidan foydalanamiz:

$$r_m = \sqrt{\left(m - \frac{1}{2}\right)\lambda R}. \tag{2}$$

(2) dan linzaning egrilik radiusi R ni topsak,

$$R = \frac{2 \cdot r_m^2}{(2m-1)\lambda} \tag{3}$$

va m=2 ligini hisobga olsak,

$$R = \frac{2 \cdot r_2^2}{(2 \cdot 2 - 1)\lambda} = \frac{2 \cdot r_2^2}{3\lambda}. \tag{4}$$

(4) ni (1) ga qo‘yib topamiz:

$$D=\frac{3\lambda}{4\cdot r_2^2}. \qquad (5)$$

Kattaliklarning son qiymatlarini qo‘yib hisoblaymiz:

$$D=\frac{3\cdot 0{,}6\cdot 10^{-6}\,\mathrm{m}}{4\cdot(1{,}2\cdot 10^{-3}\,\mathrm{m})^2}=\frac{1{,}8}{5{,}76\mathrm{m}}=0{,}3125\,\mathrm{D}.$$

Javob: $D=0{,}3125$ D.

5-misol. Fokus masofasi 1 m bo‘lgan yassi qavariq linza qavariq tomoni bilan shisha plastinka ustida turibdi. Qaytgan yorug‘likda kuzatilayotgan beshinchi Nyuton qorong‘u halqasining radiusi 1,1 mm. Yorug‘likning to‘lqin uzunligi aniqlansin.

Berilgan:

$f=1$ m;

$r_5=1{,}1\mathrm{mm}=1{,}1\cdot 10^{-3}$ m;

$m=5$.

$\lambda=?$

Yechish: Qaytgan yorug‘lik uchun Nyuton qorong‘u halqalarining radiusi quyidagicha aniqlanadi:

$$r_m=\sqrt{m\lambda\cdot R}. \qquad (1)$$

Bundan

$$\lambda=\frac{r_m^2}{m\cdot R}. \qquad (2)$$

Agar linzaning fokus masofasi $f=\frac{R}{2}$ ligini hisobga olsak:

$$R=2f. \qquad (3)$$

(3) ni (2) ga qo‘yamiz: $$\lambda=\frac{r_m^2}{2mf}. \qquad (4)$$

Berilganlarni (4) ga qo‘ysak,

$$\lambda=\frac{(1{,}1\cdot 10^{-3}\,\mathrm{m})^2}{2\cdot 5\cdot 1\mathrm{m}}=\frac{1{,}21}{10}\cdot 10^{-6}\,\mathrm{m}=0{,}121\mathrm{mkm}.$$

Javob: $\lambda=0{,}121$ mkm.

Mustaqil yechish uchun masalalar

1. Havoda harakatlanayotgan yorug'lik to'lqinining yo'liga qalinligi 1 mm bo'lgan shisha plastinka qo'ydilar. Agar to'lqin plastinkaga: 1) normal; 2) $i=30^0$ burchak ostida tushsa, optik yo'l uzunligi qanchaga o'zgaradi? [1) 0,5 mm; 2) 0,548 mm.]

2. 0,6 mkm to'lqin uzunlikli monoxromatik yorug'likning yo'liga qalinligi 0,1 mm bo'lgan yassi parallel shisha plastinka turibdi. Yorug'lik plastinkaga normal tushadi. Optik yo'l uzunligi L, $\lambda/2$ ga o'zgarishi uchun plastinkani qanday burchakka burish kerak? [30 mrad=1,72^0.]

3. Ikkita kogerent yorug'lik (05mkm) manbalari orasidagi masofa d=0,1 mm. Interferensiya manzarasining o'rta qismida interferensiya yo'llari orasidagi masofa 1sm. Manbalardan ekrangacha bo'lgan masofa aniqlansin. [2 m.]

4.Yung tajribasida tirqishlari orasidagi masofa 0,8 mm. Interferensiya yo'llarining kengligi 2mm bo'lishi uchun ekranni tirqishdan qanday masofada joylashtirish kerak? [2,5 m.]

5. Yupqa shisha ponaga (n=1,55) monoxromatik yorug'lik normal tushadi. Pona sirtlari orasidagi ikki qirrali burchak $\alpha = 2'$. Agar qaytgan yorug'likdagi qo'shni interferensiya maksimumlari orasidagi masofa 0,3mm bo'lsa, yorug'lik to'lqining uzunligi aniqlansin. [541 nm.]

6. Qaytgan yorug'likdagi Nyutonning ikkinchi va birinchi qora halqalari orasidagi masofa 1 mm. O'ninchi va to'qqizinchi halqalar orasidagi $\Delta r_{10,9}$ masofa aniqlansin. [0,3 mm.]

7. Optik kuchi 2D bo'lgan yassi qavariq linza yassi tomoni bilan shisha plastinkada yotibdi. O'tayotgan yorug'likda Nyutonning to'rtinchi qorong'u halqasining radiusi 0,7mm. Yorug'likning to'lqin uzunligi aniqlansin. [490 nm.]

8. Nyutonning ikkita yorug' halqalarining diametrlari mos ravishda 4,0 mm va 4,8 mm. Halqalarning tartib nomerlari aniqlanmagan, lekin ikkita ulchangan halqalar orasida yana uchta yorug' halqa joylashganligi ma'lum. Halqalar qaytuvchi yorug'likda kuzatiladi (500 nm).Tajriba uchun olingan yassi qavariq linzaning egrilik radiusi topilsin. [0,88 m.]

9. Nyuton halqalarini kuzatuvchi qurilmada qaytgan yorug'likda uchinchi qorong'u halqaning (m=3) radiusi o'lchanadi. Yassi parallel plastinka va linza orasidagi bo'shliqni suyuqlik bilan to'ldirilganlarida, shu radiusga nomeri birga katta bo'lgan halqa ega bo'ldi. Suyuqlikning sindirish ko'rsatkichi aniqlansin. [1,33.]

10. Ekranda 480 nm to'lqin uzunlikli ikkita kogerent yorug'lik dastalarining interferensiyasi kuzatiladi. Yorug'lik dastalaridan birining yo'liga kvarsdan yasalgan (1,46) yupqa plastinkani joylashtirganlarida interferensiya manzarasi 69 ta yo'lga siljidi. Kvars plastinkaning qalinligi aniqlansin. [6 sm.]

28-§. Yorug'lik difraksiyasi

Asosiy formulalar

Sferik to'lqinlar uchun m-Frenel zonasi tashqi chegarasining radiusi

$$r_m = \sqrt{\frac{a \cdot b}{a+b} m\lambda},$$

bunda: m —Frenel zonasining nomeri, λ — to'lqin uzunligi, a va b lar mos ravishda, nuqtaviy manbadan difraksion manzara kuzatiladigan ekrangacha va yumaloq tirqishli diafragmagacha bo'lgan masofalar.

Yorug'lik bitta tirqishga normal tushganda hosil bo'ladigan diafraksiya maksimumlari va minimumlarining shartlari:

$$a \cdot \sin\varphi = \pm(2m+1)\frac{\lambda}{2}, \quad a \cdot \sin\varphi = \pm 2m\frac{\lambda}{2}. \ (m=1,2,3,...)$$

bunda: a —tirqish kengligi; φ — difraksiya burchagi; m — spektrning tartib nomeri; λ — to'lqin uzunligi.

Yorug'lik difraksion panjaraga normal tushganda hosil bo'ladigan bosh maksimumlar va qo'shimcha minimularning shartlari:

$$d \cdot \sin\varphi = \pm 2m\frac{\lambda}{2}, \quad (m = 0,1,2,...)$$

$$d \cdot \sin\varphi = \pm m'\frac{\lambda}{N}, \quad (m' = \lambda 1,2,..., N-1, N+1,...,2N+1.$$

Bunda: d — difraksion panjara doimiysi; N — panjaradagi shtrixlar soni.

Difraksion panjara doimiysi

$$d = \frac{1}{N_0},$$

bunda: N_0 — panjaraning birlik uzunligiga to'g'ri keluvchi tirqishlar soni.

Fazoviy panjaradagi difraksion maksimumlar sharti (Vulf — Bregg formulasi):

$$2 \cdot d \cdot \sin\varphi = m\lambda \quad (m = 1,2,3,...),$$

bunda: $\lambda, (\lambda + \delta\lambda)$ — panjara ajrata oladigan ikkita qo'shni spektral chiziqlarning to'lqin uzunliklari; t — spektrning tartibi; m — panjaradagi shtrixlarning to'la soni.

Masala yechishga misollar

1-misol. Zich to'lqin fronti uchun to'rtinchi Frenel zonasining radiusi 3mm. Yigirma beshinchi zonaning radiusi aniqlansin.

Berilgan:
$m_1=4$;
$r_4=3\text{mm}=3\cdot 10^{-3}\text{m}$;
$m_2=25$.

$r_{25}=?$

Yechish: Yassi to'lqin fronti holida Frenel zonalarining radiusi quyidagi formula yordamida aniqlanadi:

$$r_m=\sqrt{mR_0\lambda}, \quad (1)$$

bunda: R_0 – tirqishdan kuzatish nuqtasigacha bo'lgan masofa, λ – to'lqin uzunligi.

Mos ravishda to'rtinchi va yigirma beshinchi zonalar uchun quyidagini yozamiz:

$$r_4=\sqrt{4\cdot R_0\cdot\lambda}, \quad (2)$$

$$r_{25}=\sqrt{25\cdot R_0\cdot\lambda} \quad (3)$$

va ular yordamida quyidagi munosabatni tuzamiz:

$$\frac{r_4}{r_{25}}=\frac{\sqrt{4\cdot R_0\cdot\lambda}}{\sqrt{25\cdot R_0\cdot\lambda}}=\frac{2}{5}.$$

Bundan $$r_{25}=\frac{5}{2}r_4, \quad (4)$$

r_4 – ning qiymatini (4) ga qo'yib quyidagini olamiz:

$$r_{25}=\frac{5}{2}\cdot 3\text{mm}=7{,}5\text{mm}=7{,}5\cdot 10^{-3}\,\text{m}.$$

Javob: $r_{25}=7{,}5\text{mm}=7{,}5\cdot 10^{-3}\,\text{m}$.

2-misol. Ingichka tirqishga yassi monoxromatik yorug'lik to'lqini (628 nm) normal tushadi. Agar ikkinchi difraksion maksimum $1^0 30'$ burchak ostida kuzatilsa, tirqishning kengligi aniqlansin.

Berilgan:

$\lambda = 628\text{nm} = 628 \cdot 10^{-9}\,\text{m};$

$\varphi = 1^0 30';$

$m = 2.$

$a = ?$

Yechish: Monoxromatik yorug'lik dastasining ingichka tirqishdagi difraksiyasi uchun difraksion maksimumlar sharti

$$a \cdot \sin\varphi = \pm(2m+1)\frac{\lambda}{2},$$

$$(m = 1,2,3,...)$$

Bundan

$$a = \frac{(2m+1)\cdot\lambda}{2\cdot\sin\varphi}$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$a = \frac{(2\cdot 2+1)\cdot 628\cdot 10^{-9}\,\text{m}}{2\cdot 0{,}026} = 6\cdot 10^{-5}\,\text{m} = 0{,}06\text{mm},$$

Javob: a =0,06 mm.

3-misol. Rentgen nurlarining parallel dastasi osh tuzi kristalining qirrasiga tushadi. Kristall atom tekisliklari orasidagi masofa 280pm. Qirra tekisligiga 65^0 burchak ostida birinchi tartibli difraksion maksimum kuzatilsa, Rentgen nurlarining to'lqin uzunligi λ aniqlansin.

Berilgan:

$d = 280\,\text{pm} = 280 \cdot 10^{-12}\,\text{m};$

$\theta = 65^0;$

$m = 1.$

$\lambda = ?$

Yechish: Fazoviy panjaradagi difraksion maksimumlar uchun Vulf-Bregg formulasi:

$$2\cdot d\cdot\sin\theta = m\lambda,$$

$$(m = 1,2,3,...)$$

Bundan

$$\lambda = \frac{2\cdot 280\cdot 10^{-12}\,\text{m}\cdot\sin 65}{1} = 5{,}074\cdot 10^{-7}\,\text{m};$$

Javob: $\lambda = 5{,}074\cdot 10^{-7}\,\text{m}.$

4-misol. Birinchi tartibli spektrida $\lambda_1 = 404{,}4\text{nm}$ va $\lambda_2 = 404{,}7\text{nm}$ dubletni ajrata olishi uchun davri 20 mkm bo'lgan difraksion panjaraning kengligi qanday bo'lishi kerak?

Berilgan:

$\lambda_1 = 404{,}4\text{nm} = 404{,}4 \cdot 10^{-9}\,\text{m};$

$\lambda_2 = 404{,}7\text{nm} = 404{,}7 \cdot 10^{-9}\,\text{m};$

$d = 20\text{mkm} = 2 \cdot 10^{-5}\,\text{m};$

$m = 1.$

$S = ?$

Yechish: Difraksion panjaraning kengligi quyidagi ifoda yordamida aniqlanadi:

$$S = d \cdot N, \qquad (1)$$

bunda: N – panjaraning to'la shtrixlar soni bo'lib, quyidagi ifoda yordamida aniqlanishi mumkin.

$$N = \frac{R}{m}. \qquad (2)$$

Bu yerda, m – spektr tartibi, R – panjaraning ajrata olish xususiyati

$$R = \frac{\lambda_1}{\Delta\lambda} = \frac{\lambda_1}{\lambda_2 - \lambda_1}. \qquad (3)$$

(2) ni (1) ga, natijaga esa (3) ni qo'yib quyidagini olamiz:

$$S = \frac{d \cdot R}{m} = \frac{d \cdot \lambda_1}{m(\lambda_2 - \lambda_1)}. \qquad (4)$$

Kattaliklarning son qiymatlarini o'rniga qo'yib hisoblaymiz:

$$S = \frac{2 \cdot 10^{-5} \cdot 404{,}7 \cdot 10^{-9}}{1 \cdot (404{,}7 - 404{,}4) \cdot 10^{-9}}\,\text{m} = \frac{808{,}8}{0{,}3} \cdot 10^{-5}\,\text{m} \approx 2{,}7 \cdot 10^{-2}\,\text{m} = 2{,}7\text{sm}.$$

Javob: $S = 2{,}7 \cdot 10^{-2}\,\text{m} = 2{,}7\text{sm}$.

7-misol. Kengligi 2 sm va davri 5mkm bo'lgan difraksion panjara, qizil nurlar sohasidagi (0,7 mkm) ikkinchi tartibli spektrda qanday to'lqin uzunliklarni ajrata olishi mumkin?

Berilgan:

$S = 2\text{sm} = 2 \cdot 10^{-2}\,\text{m};$

$d = 5\text{mkm} = 5 \cdot 10^{-6}\,\text{m};$

Yechish: Panjaraning ajrata olish qobiliyati quyidagicha aniqlanadi:

$$R = \frac{\lambda}{\Delta\lambda}. \qquad (1)$$

$\lambda = 0{,}7\text{mkm} = 7\cdot 10^{-7}\text{ m};$

$m = 2.$

$\Delta\lambda = ?$

Bundan

$$\Delta\lambda = \frac{\lambda}{R}. \qquad (2)$$

Ikkinchi tomondan

$$N = \frac{\lambda_1}{m(\lambda_2 - \lambda_1)}. \qquad (3)$$

Bu yerda $N = \frac{S}{d}$ ligi hisobga olindi.

$$\Delta\lambda = \frac{\lambda\cdot d}{m\cdot S}.$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$\Delta\lambda = \frac{7\cdot 10^{-7}\cdot 5\cdot 10^{-6}}{2\cdot 2\cdot 10^{-2}}\text{m} = \frac{3{,}5}{4}\cdot 10^{-10}\text{m} = 0{,}875\cdot 10^{-10}\text{m} \approx 0{,}09\text{nm}.$$

Javob: $\Delta\lambda \approx 0{,}09$ nm.

Mustaqil yechish uchun masalalar

1. Agar tasvir vaziyatini topish to'lqin frontidan 1m masofada turgan nuqta uchun bajarilayotgan bo'lsa, yassi to'lqin fronti uchun (0,5 mkm) beshinchi Frenel zonasining radiusi hisoblansin. [1,58 mm.]

2. 4 mm diametrli dumaloq tirqishli diafragmaga monoxromatik yorug'lik (0,5 mkm) nurlarining parallel dastasi tikka tushadi. Kuzatish nuqtasi tirqish o'qida yotib, undan 1m masofada joylashgan. Tirqishda nechta Frenel zonasi joylashgan? Agar kuzatish joyiga ekran joylashtirilsa, difraksiya manzarasining markazida qanday dog' hosil bo'ldi, qorami yoki yorug'mi? [8 ta, qora.]

3. Kengligi 0,05 mm bo'lgan tirqishga monoxromatik yorug'lik (0,6mkm) normal tushadi. Yorug'lik dastasining dastlabki yo'nalishi va to'rtinchi qorong'u difraksiya yo'lidagi yo'nalishi orasidagi burchak aniqlansin. $[2^0 45'.]$

4. Agar monoxromatik yorug'lik (0,6 mkm) holida kuzatilayotgan difraksiyada beshinchi tartibli maksimum 180 burchakka og'gan bo'lsa, difraksion panjaraning har bir millimetrida nechta shtrix bor. [103.]

5. Difraksion panjara normal tushayotgan monoxromatik yorug'lik bilan yoritilgan. Agar difraksion manzaradagi ikkinchi tartibli maksimum

14^0 burchakka og‘gan bo‘lsa, uchinchi tartibli maksimumning og‘ish burchagi aniqlansin. $[21^0 17'.]$

6. Davri 10 mkm bo‘lgan difraksion panjaraga 30^0 burchak ostida 600 nm to‘lqin uzunlikli monoxromatik yorug‘lik tushadi. Ikkinchi bosh maksimumga mos keluvchi difraksiya burchagi aniqlansin. $[38{,}3^0.]$

7. 30^0 difraksiya burchagi va 600 nm to‘lqin uzunligi uchun difraksion panjaraning burchak dispersiyasi aniqlansin. $\left[9{,}62 \cdot 10^5 \frac{\text{rad}}{\text{m}}.\right]$

8. Ikkinchi tartibli spektrda 500 nm yorug‘lik uchun difraksion panjaradagi burchak dispersiyasi $4{,}08 \cdot 10^5$rad/m ga teng. Difraksion panjara doimiysi aniqlansin. [5 mkm.]

9. 600 nm to‘lqin uzunlikli monoxromatik yorug‘lik difraksion panjaraga normal tushadi. Agar difraksion panjaraning doimiysi 2mkm bo‘lsa, shu panjara yordamida olinishi mumkin bo‘lgan eng yuqori tartibli spektr aniqlansin. [3.]

10. To‘lqin uzunligi 0,5mkm bo‘lgan monoxromatik yorug‘lik uchun to‘rtinchi tartibli maksimumda 30^0 burchak mos keladigan bo‘lsa, difraksion panjaraning har 1mm ga to‘g‘ri keluvchi shtrixlar soni aniqlansin. [250 mm^{-1}.]

11. Doimiysi 5 mkm bo‘lgan difraksion panjaraga 30^0 burchak ostida 0,5 mkm to‘lqin uzunlikli monoxromatik yorug‘lik tushadi. Uchinchi tartibli bosh maksimum uchun difraksiya burchagi aniqlansin. $[53^0 8'.]$

12. Osh tuzi kristalining qirrasiga rentgen nurlarining (147 nm) parallel dastasi tushadi. Agar ikkinchi tartibli difraksion maksimum, nurlar kristall sirtiga $31^0 30'$ burchak ostida tushganda kuzatilsa, kristalning atom tekisliklari orasidagi masofa aniqlansin. [0,28.]

13. Kadmiyning qizil chizig‘i (644nm) uchun, uzunliklari bir xil $(l_1 = l_2 = l = 5\text{mm})$, lekin davrlari turlicha $(d_1 = 4\text{mkm}, d_2 = 8\text{mkm})$ bo‘lgan ikkita difraksion panjaralarning eng katta ajrata olish xususiyatlari solishtirilsin. [$R_{1max} = R_{2max} = 7500$.]

14. Uzunligi 2,5sm bo‘lgan difraksion panjaraning doimiysi 5mkm. Ikkinchi tartibli spektrda to‘lqin uzunligi 0,5 mkm bo‘lgan yorug‘lik uchun shu panjara ajrata oladigan to‘lqin uzunliklar farqi aniqlansin. [50 pPa.]

29-§. Elektromagnit to'lqinlarning muhit bilan ta'sirlashuvi. Yorug'likning qutblanishi

Asosiy formulalar

Nurning prizmadan o'tishda og'ish burchagi φ va prizmaning sindirish burchagi A orasidagi bog'lanish:

$$\varphi = A(n-1),$$

bunda: $n = \sqrt{E}$ – prizmaning sindirish ko'rsatkichi, e – muhitning dielektrik kirituvchanligi.

Yorug'likning moddalarda yutilish qonuni (Buger qonuni):

$$I = I_0 e^{-\alpha x}.$$

I va I_0 – lar mos ravishda, qalinligi x – bo'lgan yutuvchi modda qatlamiga kiruvchi va undan chiquvchi yassi monoxromatik yorug'lik to'lqinlarining intensivligi; α – yutish koeffitsienti.

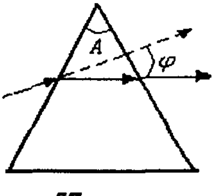
Vakuumdagi elektromagnit to'lqinlar uchun Dopler effekti:

$$\nu = \nu_0 \frac{\sqrt{1 - \vartheta^2/c^2}}{1 + \frac{\vartheta}{c}\cos\theta},$$

bunda: ν va ν_0 – mos ravishda manba chiqaradigan va qabul qiluvchi qabul qiladigan elektromagnit to'lqinlarning chastotalari; ϑ – manbaning qabul qiluvchiga nisbatan tezligi; c – yorug'likning bo'shliqdagi tezligi; θ – tezlik vektori $\vec{v}$ va kuzatuvchiga bog'langan sanoq sistemasidagi kuzatish yo'nalishi orasidagi burchak.

Vakuumdagi elektromagnit to'lqinlar uchun Doplerning ko'ndalang effekti $\left(\theta = \frac{\pi}{2}\right)$

$$\nu = \nu_0 \sqrt{1 - \frac{\vartheta^2}{c^2}}$$



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Vavilov – Cherenkov effekti:

$$\cos\theta = \frac{c}{(n \cdot \vartheta)},$$

bunda: θ – nurlanishning tarqalish yo‘nalishi va zarra tezligining vektori orasidagi burchak; n – muhitning sindirish ko‘rsatkichi.

Yorug‘likning qutblanish darajasi:

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}},$$

bunda: I_{max} va I_{min} – mos ravishda analizatordan o‘tuvchi qisman qutblangan yorug‘likning maksimal va minimal intensivliklari:

Malyus qonuni:

$$I = I_0 \cos^2 \alpha,$$

bunda: I – analizatordan o‘tgan yassi qutblangan yorug‘likning intensivligi,

I_0 – analizatorga tushayotgan yassi qutblangan yorug‘likning intensivligi,

α – qutblagich va analizatorlarning bosh tekisliklari orasidagi burchak.

Bryuster qonuni:

$$tgi_B = n_{21},$$

i_B – dielektrikdan qaytuvchi nur yassi qutblangan bo‘lishi uchun tushish burchagi,

n_{21} – nisbiy sindirish ko‘rsatkichi.

Kerr yacheykasidagi l yo‘l uzunligida oddiy va g‘ayrioddiy nurlar orasida hosil bo‘ladigan optik yo‘l farqi:

$$\Delta = l(n_0 - n_e) = k \cdot l \cdot E^2,$$

bunda: n_0 va n_e – mos ravishda, optik o‘qqa perpendikulyar yo‘nalishda oddiy va g‘ayrioddiy nurlar uchun sindirish ko‘rsatkichlari. E – elektr maydon kuchlanganligi, k – moddani xarakterlovchi doimiylik.

Qutblanish burchagining burilish burchagi:

optik aktiv kristallar va sof suyuqliklar uchun:

$$\varphi = \alpha d;$$

optik aktiv eritmalar uchun:

$$\varphi = [\alpha]Cd,$$

Bunda: d – yorug'likning optik aktiv moddada o'tuvchi yo'lining uzunligi.

$\alpha, [\alpha]$ – solishtirma burilish, C – optik aktiv moddaning suyuqlikdagi konsentrasiyasi.

Masala yechishga misollar

1-misol. 550 nm to'lqin uzunlikli monoxromatik yorug'lik chiqarayotgan manba $0{,}2c$ tezlik bilan kuzatuvchi tomonga harakatlanadi. Kuzatuvchining qabul qiluvchisi qayd etadigan to'lqinning uzunligi aniqlansin.

Berilgan:

$\lambda_0 = 550\text{nm} = 5{,}5 \cdot 10^{-10}\,\text{m};$

$\vartheta = 0{,}2\text{c};$

$c = 3 \cdot 10^8\ \text{m}/\text{s};$

$\Theta = \pi.$

$\lambda = ?$

Yechish: Vakuumdagi elektromagnit to'lqinlar uchun Dopler effekti quyidagi ifoda bilan aniqlanadi.

$$\nu = \nu_0 \frac{\sqrt{1 - \vartheta^2/c^2}}{1 + \frac{\vartheta}{c}\cos\Theta} \qquad (1)$$

bu erda: ν_0 va ν – mos ravishda, manba chiqaradigan va qabul qiluvchi qabul qiladigan elektromagnit nurlanishning chastotalari; ϑ – manbaning qabul qiluvchiga nisbatan tezligi; α – kuzatuvchi bilan bog'langan sanoq sistemasida o'lchanuvchi tezlik vektori $\vec{V}$ va kuzatish yo'nalishi orasidagi burchak.

Masalaning shartiga ko'ra $\Theta = \pi$ $(\cos\Theta = -1)$ ligidan va $\nu = \dfrac{c}{\lambda}$ ekanligini hisobga olsak (1) ni quyidagicha yozishimiz mumkin:

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\frac{\sqrt{1 - \beta^2}}{(1 - \beta)}. \qquad (2)$$

Bundan qabul qiluvchi qayd etadigan to'lqin uzunligi:

$$\lambda = \lambda_0 \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}. \qquad (3)$$

$\beta = \frac{\vartheta}{c} = 0{,}2$ ekanligini hisobga olib λ_0 ning qiymatini (3) ga qo'ysak:

$$\lambda = 5{,}5 \cdot 10^{-7} \frac{\sqrt{1-0{,}2}}{\sqrt{1+0{,}2}} = 4{,}49 \cdot 10^{-7}\,\text{m} = 449\text{nm}.$$

ni topamiz.

Javob: $\lambda = 449$ nm.

2-misol. Agar elektronning minimal impulsi $2{,}44 \cdot 10^{-22}\,\text{kg} \cdot \text{m/s}$ bo'lsa, Vavilov-Cherenkov effekti kuzatiladigan muhitning sindirish ko'rsatkichi aniqlansin.

Berilgan:

$m_0 = 9{,}11 \cdot 10^{-31}\,\text{kg};$

$P_{min} = 2{,}44 \cdot 10^{-22}\,\text{kg} \cdot \text{m/s}.$

$n = ?$

Yechish: Vavilov – Cherenkov effekti zaryadlangan relyativistik zarralarning muhitdagi o'zgarmas tezligi ϑ, yorug'likning shu muhitdagi fazoviy tezligidan katta bo'lgandagina hosil bo'ladi, ya'ni quyidagi shart bajarilishi kerak:

$$\vartheta > \frac{c}{n}. \qquad (1)$$

Agar $\beta = \frac{\vartheta}{c}$ ligini e'tiborga olsak (1) ni quyidagicha yozishimiz mumkin

$$\beta \cdot n > 1 \quad . \qquad (2)$$

Relyativistik zarraning impulsi quyidagicha aniqlanadi

$$P = \frac{m_0 \vartheta}{\sqrt{1-\beta^2}} = \frac{m_0 \beta c}{\sqrt{1-\beta^2}}. \qquad (3)$$

Impulsning minimal qiymati P_{min} ga $\beta_{min} = \frac{1}{n}$ mos keladi. Unda

$$P_{min} = \frac{m \cdot c}{\sqrt{n^2 - 1}}. \qquad (4)$$

Bundan so'ralayotgan muhitning sindirish ko'rsatkichi:

$$n = \sqrt{\frac{m_0^2 c^2}{P_{min}^2} + 1} \qquad (5)$$

Kattaliklarning son qiymatlarini qo'yib topamiz

$$n = \sqrt{\frac{(9{,}11 \cdot 10^{-31})^2 \cdot (3 \cdot 10^{10})^2}{(2{,}44 \cdot 10^{-22})^2} + 1} = 1{,}5.$$

Javob: $n = 1{,}5$.

3-misol. Agar har biriga, tushayotgan yorug'lik oqimining k=10% yo'qotilsa, qutblanish tekisliklari $\alpha = 30^0$ tashkil etadigan ikkita nikeldan o'tayotgan yorug'lik necha marta susayadi?

Berilgan:

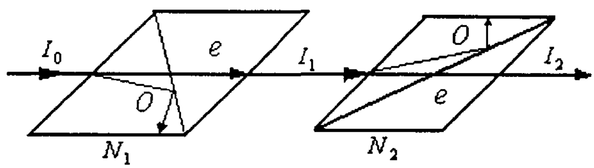
$\alpha = 30^0$;

$k = 10\%$;

yoki k=0,1

$\frac{I_0}{I_2} = ?$

Yechish:



58-rasm

Nikol prizmasining qirrasiga tushayotgan tabiiy yorug'lik ikkita nurga ajraladi: Oddiy va g'ayrioddiy. Har ikkala nurning ham intensivliklari teng va to'la qutblangan.

Birinchi prizmadan o'tayotgan yorug'likning intensivligi:

$$I_1 = \frac{1}{2} I_0 (1-k). \qquad (1)$$

Ikkinchi prizmadan o'tayotgan g'ayrioddiy (e) nurning intensivligi I_2 (oddiy nur (0) yutiladi) Malyus qonuni yordamida quyidagicha aniqlanadi:

$$I_2 = I_1 \cdot \cos^2 \alpha . \qquad (2)$$

Agar ikkinchi prizmadagi yutilishini ham hisobga olsak,

$$I_2 = I_1 (1-k) \cos^2 \alpha . \qquad (3)$$

Agar (1) ni e'tiborga olsak (3) quyidagi ko'rinishni oladi:

$$I_2 = \frac{1}{2} I_0 (1-k)^2 \cos^2 \alpha . \qquad (4)$$

So'ralgan munosabatni tuzamiz:

$$\frac{I_0}{I_2} = \frac{I_0}{\frac{1}{2} I_0 (1-k)^2 \cdot \cos^2 \alpha} = \frac{2}{(1-k)^2 \cos^2 \alpha}. \qquad (5)$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$\frac{I_0}{I_2} = \frac{2}{(1-0{,}1)^2 \cdot \cos^2 30^0} = \frac{2}{0{,}81 \cdot 0{,}75} = 3{,}3 .$$

Javob: $\frac{I_0}{I_2} = 3{,}3$.

4-misol. Yorug'likning havodan osh tuzi kristaliga tushishidagi Bryuster burchagi $i_B = 57^0$. Yorug'likning shu kristaldagi tezligi aniqlansin.

Berilgan:

$i_B = 57^0$;

$n_1 = 1$.

$\vartheta_2 = ?$

Yechish: Bryuster qonuniga muvofiq

$$\mathrm{tg} i_B = n_{21} = \frac{n_2}{n_1}. \qquad (1)$$

Bu yerda: $n_1 = \frac{c}{\vartheta_1}$ birinchi va $n_2 = \frac{c}{\vartheta_2}$ – ikkinchi muhitlarning sindirish ko'rsatkichlari. c – yorug'likning muhitdagi tezligi $c = 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$. ϑ_1 va ϑ_2 lar mos ravishda yorug'likning birinchi va ikkinchi muhitdagi tezliklari. Shu bilan birga $n_1 = 1$.

Shunday qilib, $$\text{tg}\,i_B = \frac{n_2}{n_1} = \frac{n_2}{1} = n_2 \;. \qquad (2)$$

Yoki $$\text{tg}\,i_B = \frac{c}{\vartheta_2} \;. \qquad (3)$$

(3) dan $$\vartheta_2 = \frac{c}{\text{tg}\,i_B} \;. \qquad (4)$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$\vartheta_2 = \frac{3 \cdot 10^8}{\text{tg}57^0}\frac{\text{m}}{\text{s}} = 2 \cdot 10^8 \frac{\text{m}}{\text{s}}.$$

Javob: $\vartheta_2 = 2 \cdot 10^8 \frac{\text{m}}{\text{s}}$.

5-misol. Uzunligi 8 sm bo'lgan shisha naychada saqlanayotgan nikotin (sof suyuqlik) natriy sariq yorug'ligining qutblanish tekisligini 136,6 grad burchakka buradi. Nikotinning zichligi $1{,}01 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$. Nikotinning solishtirma buralishi aniqlansin.

Berilgan:
l =8 sm=8 · 10^{-2}m;
φ=136,6 grad;
$\rho = 1{,}01 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$

$[\alpha] = ?$

Yechish: Sof suyuqliklarda monoxromatik yorug'lik qutblanish tekisligining burilish burchagi quyidagi ifoda yordamida aniqlanadi:

$$\varphi = [\alpha]\rho \cdot l \;. \qquad (1)$$

Bundan

$$[\alpha] = \frac{\varphi}{\rho \cdot l}. \qquad (2)$$

Kattaliklarning son qiymatlarini qo'yib olamiz:

$$[\alpha] = \frac{136,6}{1,01 \cdot 10^3 \cdot 8 \cdot 10^{-2}} \frac{\text{grad} \cdot \text{m}^3}{\text{kg} \cdot \text{m}} = 1,69 \frac{\text{grad} \cdot \text{m}^2}{\text{kg}}.$$

Javob: $[\alpha] = 1,69 \frac{\text{grad} \cdot \text{m}^2}{\text{kg}}$.

6-misol. Uzunligi 20 sm va konsentrasiyasi 0,25g/sm^3 bo'lgan shakar eritmasi monoxramatik yorug'likning qutblanish tekisligini $33^0 20'$ ga buradi. Uzunligi 15 sm bo'lgan boshqa eritma esa shu yorug'likning qutblanish tekisligini 20^0 ga buradi. Ikkinchi eritmadagi shakarning konsentrasiyasi aniqlansin.

Berilgan:

$S_1 = 0,25 \, g/_{sm^3} = 250 \, kg/_{m^3}$;

$l_1 = 20\text{sm} = 0,2\text{m}$;

$\varphi_1 = 33^0 20' = 33,3\text{grad}$

$l_2 = 15\text{sm} = 0,15\text{m}$;

$\varphi_2 = 20^0$.

$S_2 = ?$

Yechish: Eritmalarda monoxromatik yorug'lik qutblanish tekisligining burilish burchagi quyidagicha aniqlanadi:

$$\varphi = [\alpha] S \cdot l. \qquad (1)$$

Bunda $[\alpha]$ – shakarning solishtirma buralishi

Birinchi eritma uchun:

$$\varphi_1 = [\alpha] \cdot C_1 \cdot l_1. \qquad (2)$$

Ikkinchi eritma uchun:

$$\varphi_2 = [\alpha] S_2 \cdot l_2. \qquad (3)$$

va quyidagi nisbatni tuzamiz:

$$\frac{\varphi_1}{\varphi_2} = \frac{[\alpha] S_1 \cdot l_1}{[\alpha] S_2 l_2} = \frac{S_1 \; l_1}{S_2 \cdot l_2}.$$

Bundan

$$S_2 = \frac{S_1 \cdot l_1 \cdot \varphi_2}{l_2 \cdot \varphi_1}. \quad (4)$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$S_2 = \frac{250 \cdot 0{,}2 \cdot 20}{0{,}15 \cdot 33_0 3} \frac{kg}{m^3} = 200 \frac{kg}{m^3}.$$

Javob: $S_2 = 200 \frac{kg}{m^3}$.

Mustaqil yechish uchun masalalar

1. Shisha prizmaning qirrasiga (n=1,5) yorug'lik nuri tik tushadi. Agar prizmaning sindirish burchagi 30^0 bo'lsa, nurning og'ish burchagi aniqlansin. $[18^0 36'.]$

2. Yorug'lik nuri shisha prizmaga (n=1,5) qanday burchak bilan tushgan bo'lsa, shunday burchak bilan chiqadi. Agar prizmaning sindirish burchagi 60^0 bo'lsa nurning prizmadan og'ish burchagi aniqlansin. ($37^0 11$.)

3. Ma'lum to'lqin uzunlikli monoxromatik yorug'lik uchun berilgan moddaning yutish koeffitsienti 0,1 sm^{-1}. Yorug'likni : 1) 2 marta; 2) 5 marta – susaytirish uchun modda qatlamining qalinliklari x_1 va x_2 lar qancha bo'lishi aniqlansin. Yorug'likning qaytishidagi yo'qotish hisobga olinmasin. [1) x_1=6,93sm; 2) x_2=16,1 sm.]

4. To'lqin uzunligi 0,5 mkm bo'lgan monoxromatik yorug'lik manbai kuzatuvchi tomonga 0,15 c tezlik bilan harakatlanmoqda (c-yorug'likning bo'shliqdagi tezligi). Kuzatuvchining qabul qiluvchisi qanday to'lqin uzunlikli yorug'likni qayd qilishi aniqlansin. [430 nm.]

5. Ma'lum tumanlikning bizdan uzoqlashishida vodorod nurlanishning chizig'i (λ=653,3 nm) uning spektrida qizil tomonga 2,5nm ga siljiydi. Tumanlikning uzoqlashish tezligi aniqlansin. [11,4 Mm/s.]

6. 100 keV kinetik energiyali vodorod atomlari dastasiga nisbatan to'g'ri burchak ostida kuzatilganda, atomar vodorod spektral chiziqlari uchun (l=486,1nm) dopler siljish aniqlansin. [51,7 pm.]

7. Sindirish ko'rsatkichi 1,54 bo'lgan muhitda elektron harakat yo'nalishiga 30^0 burchak ostida Cherenkov nurlanishi ro'y berishi uchun, elektronlarning tezligi qanday bo'lishi kerak. Elektronning tezligi yorug'lik tezligi ulushlarida ifodalansin. [0,75 c.]

8. Sindirish ko'rsatkichi 1,5 bo'lgan muhitda cherenkov nurlanish vujudga kelishi uchun elektron qanday eng kichik tezlashtiruvchi potensiallar farqidan o'tishi kerakligi aniqlansin. [175 kV.]

9. Yorug'likning havodan osh tuzi kristaliga tushishdagi Bryuster burchagi 57^0. Shu kristaldagi yorug'lik tezligi aniqlansin. [194 Mm/s.]

10.Analizator, polyarizatordan chiqayotgan yorug'lik intensivligini 2 marta kamaytiradi. Polyarizator va analizatorlarning o'tkazish tekisliklari orasidagi burchak aniqlansin. Analizatorda yorug'lik intensivligining yo'qotilishi hisobga olinmasin. [45^0.]

11.Polyarizator va analizatorlarning o'tkazish tekislikari orasidagi burchak 45^0. Agar burchak 60^0 gacha orttirilsa, analizatordan chiqayotgan yorug'lik intensivligi necha marta kamayadi? [2 marta.]

12. Agar har bir nikolda unga tushayotgan yorug'likning o'n foizi yo'qotilsa, o'tkazish tekisliklari $\alpha = 30^0$ burchak hosil qiladigan ikkita nikol orqali o'tayotgan yorug'likning intensivligi necha marta kamayadi. [3,3 marta.]

13. Qisman qutblangan yorug'likning darajasi 0,5 ga teng. Analizator orqali o'tkazilayotgan yorug'likning maksimal intensivligi minimalidan necha marta farq qiladi. [3 marta.]

14. Nikolga qisman qutblangan yorug'lik dastasi tushadi. Nikolning qandaydir holatida undan o'tadigan yorug'lik intensivligi minimal bo'ladi. Nikolning o'tkazish tekisligini oldingi holatiga nisbatan 45^0 burchakka burganlarida yorug'lik intensivligi 1,5 marta ortdi. Yorug'likning qutblanish darajasi aniqlansin. [0,348.]

15. Uzunligi 8 sm bo'lgan shisha naychada saqlanayotgan nikotin (toza suyuqlik) natriy sariq yorug'ligining qutblanish tekisligini 137^0 burchakka buradi. Nikotinning zichligi $1{,}01 \cdot 10^3 kg/m^3$. Nikotinning solishtirma burashi $[\alpha]$ aniqlansin. [169 grad $\cdot$ m^2/kg.]

16. Qand eritmasi solingan naychadan o'tganda natriy sariq yorug'ligi qutblanish tekisligining burilish burchagi 40^0. Naychaning uzunligi 15sm. Qandning solishtirma burashi $1{,}17 \cdot 10^{-2} rad \cdot m^3/(m \cdot kg)$. Eritmaning zichligi ρ aniqlansin. [0,4 g/sm^3.]

VI BOB. NURLANISHNING KVANT NAZARIYASI

30-§. Issiqlik nurlanish qonunlari

Asosiy formulalar

Stefan—Bolsman qonuni:

$$R_e = \sigma T^4,$$

bunda: R_e — qora jismning energetik yorituvchanligi (nurlanuvchanligi);

$\sigma = 5{,}67 \cdot 10^{-8} \mathrm{W/(m^2 \cdot K^4)}$ — Stefan — Bolsman doimiysi; T — termodinamik harorat.

Qora jismning energetik yorituvchanligi R_e va energetik yorituvchanlikning spektral zichligi $r_{\nu,T}$ $(r_{\lambda,T})$ orasidagi bog'lanish:

$$R_e = \int_0^\infty r_{\nu,T}\, d\nu = \int_0^\infty r_{\lambda,T}\, d\lambda.$$

Kulrang jismning energetik yorituvchanligi:

$$R_T^K = A_T \sigma T^4,$$

bunda: A_T — kulrang jismning yutish xususiyati.

Vinning siljish qonuni:

$$\lambda_{\max} = \frac{b}{T},$$

bunda: $\lambda_{\max}$ — qora jism energetik yorituvchanligi spektral zichligining maksimal qiymatiga mos keluvchi to'lqinning uzunligi; $b = 2{,}90 \cdot 10^{-3} \mathrm{m \cdot K}$ — Vin doimiysi.

Qora jism energetik yorituvchanligi maksimal spektral zichligining haroratga bog'liqligi:

$$r_{\lambda,T} = CT^5,$$

bu yerda $C = 1{,}30 \cdot 10^{-5} \mathrm{W/(m^3 \cdot K^5)}$.

Qora jism energetik yorituvchanligining spektral zichligi uchun:
Reley-Djins formulasi:

$$r_{\nu,T} = \frac{2\pi\nu^2}{c^2} kT ,$$

bu yerda: ν – nurlanish chastotasi, $k = 1{,}38 \cdot 10^{-23}$ J/K – Bolsman doimiysi, $c = 3 \cdot 10^8$ m/s – yorug'likning bo'shliqdagi tezligi;

Plank formulasi:

$$r_{\nu,T} = \frac{2\pi\nu^2}{c^2} \frac{h\nu}{e^{h\nu/(kT)} - 1},$$

$$r_{\lambda,T} = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/(kT\lambda)} - 1}.$$

Bunda: $h = 6{,}63 \cdot 10^{-34}\, J \cdot s$ – Plank doimiysi, λ – nurlanishning to'lqin uzunligi.

Kvant energiyasi:

$$E_0 = h\nu = \frac{hc}{\lambda}.$$

Radiatsion T_p va haqiqiy T haroratlar orasidagi bog'lanish:

$$T_T = \sqrt[4]{A_T} \cdot T,$$

bunda: A_T – kulrang jismning yutish qobiliyati.

Masala yechishga misollar

1-misol. Qora jismning dastlabki harorati 500K. Jismni qizdirgandan keyin nurlanuvchanligi 5 marta orsa, uning keyingi harorati qanday bo'ladi?

Berilgan:
T_1=500K,
n =5.
T_2=?

Yechish: Qora jismning nurlanuvchanligi (energetik yorituvchanligi) Stefan-Bolsman qonuniga muvofiq aniqlanadi. Qonunni har ikkala holat uchun ham yozamiz:

$$R_{e1} = \sigma T_1^4, \qquad (1)$$

va $R_{e2} = \sigma T_2^4$, (2)

Tegishli munosabatni tuzsak,

$$n = \frac{R_{e2}}{R_{e1}} = \left(\frac{T_2}{T_1}\right)^4. \quad (3)$$

Bundan

$$T_2 = \sqrt[4]{n} T_1. \quad (4)$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

$$T_2 = \sqrt[4]{5} \cdot 500\text{K} \approx 1{,}5 \cdot 500\text{K} = 750\text{K}.$$

Javob: $T_2 = 750\text{K}$.

3-misol. Agar qora jism energetik nurlanishi to'lqin uzunligining maksimal qiymati 600 nm bo'lsa, uning harorati va energetik nurlanishi aniqlansin.

Berilgan:

$\lambda_{max} = 600\text{nm} = 6 \cdot 10^{-7}\,\text{m}.$

$T = ?$

$R_e = ?$

Yechish: Vinning siljish qonuniga muvofiq,

$$\lambda_{max} = \frac{b}{T}, \quad (1)$$

bundan

$$T = \frac{в}{\lambda_{max}}. \quad (2)$$

Stefan-Bolsman qonuniga muvofiq qora jismning energetik nurlanishi:

$$R_e = \sigma T^4. \quad (3)$$

λ_{max} ning qiymatini (2) ga, T ning qiymatini (3) ga qo'yib va $b = 2{,}9 \cdot 10^{-3}\,\text{m} \cdot \text{K}$ – Vin doimiysi, $\sigma = 5{,}67 \cdot 10^{-8} \frac{W}{\text{m}^2 \cdot \text{K}^4}$ – Stefan-Bolsman doimiyliklaridan foydalanib olamiz

$$T = \frac{2{,}9 \cdot 10^{-3}\,\text{m} \cdot \text{K}}{6 \cdot 10^{-7}\,\text{m}} = 4833{,}3\text{K};$$

$$R_e = 5{,}67 \cdot 10^{-8} \frac{\mathrm{N}}{\mathrm{m}^2 \cdot \mathrm{K}^4} (4833{,}3 \cdot \mathrm{K})^4 = 3{,}1 \cdot 10^7 \frac{\mathrm{W}}{\mathrm{m}^2}.$$

Javob: $T = 4833{,}3\mathrm{K}$; $R_e = 3{,}1 \cdot 10^7 \frac{\mathrm{W}}{\mathrm{m}^2}$.

4-misol. Pechning ko'rish tirqishidan $4 \frac{\mathrm{kJ}}{\min}$ energiya oqimi nurlanadi. Agar tirqishning yuzasi 8 sm^2 bo'lsa, pechning harorati aniqlansin.

Berilgan:

$\overline{F}_e = 4 \frac{\mathrm{kJ}}{\min} = 66{,}7 \mathrm{J/s}$;

$S = 8\ \mathrm{sm}^2 = 8 \cdot 10^{-4} \mathrm{m}^2$;

$T = ?$

Yechish: Energiya oqimi quyidagi formula bilan aniqlanadi:

$$\overline{F}_e = R_e \cdot S. \quad (1)$$

Bu yerda: S – tirqish yuzasi. R_e – esa Stefan-Bolsman qonunidan aniqlanadi:

$$R_e = \sigma \mathrm{T}^4, \quad (2)$$

(2) ni (1) ga qo'yib quyidagini olamiz:

$$F_e = \sigma \cdot \mathrm{T}^4 \cdot S. \quad (3)$$

Bundan

$$T = \sqrt[4]{\frac{\overline{F}_e}{\sigma \cdot S}}. \quad (4)$$

Berilganlarni va Stefan-Bolsman – doimiysi $\sigma = 5{,}67 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}$ ni hisobga olib topamiz:

$$T = \sqrt[4]{\frac{66{,}7}{5{,}67 \cdot 10^{-8} \cdot 8 \cdot 10^{-4}}} \mathrm{K} = 1{,}1 \cdot 10^3 \mathrm{K} = 1100 \mathrm{K}.$$

Javob: $T = 1100$ K.

4-misol. Agar platinaning yutish xususiyati 0,8 bo'lsa, eritilgan platinaning 50 sm^2 sirti 1min vaqtda qancha issiqlik miqdori yo'qotishi aniqlansin. Platinaning erish harorati 1770°C.

Berilgan:
A_l=0,8;
S=50 sm² =5 . 10^{-3} m²;
t=1 min =60 s;
T= t+273=2043 K.
Q=?

Yehish: Platina yo'qotadigan issiqlik miqdori uning qizigan sirtidan nurlanadigan energiyaga teng:

$$Q = W = A_T \cdot R_e \cdot S \cdot t, \quad (1)$$

bu yerda: S – nurlatadigan sirt; t – vaqt, R_e – energetik nurlanish.
Stefan-Bolsman qonuniga muvofiq

$$R_e = \sigma T^4, \qquad (2)$$

bu erda: $\sigma = 5{,}67 \cdot 10^{-8}\, \text{W}/(\text{m}^2 \cdot \text{K}^4)$ – Stefan-Bolsman doimiysi, T – harorat.

(2) ni (1) ga qo'ysak:

$$Q = A_T \cdot \sigma \cdot T^4 \cdot S \cdot t. \qquad (3)$$

Kattaliklarning qiymatlarini (3) ga qo'yib yo'qotilgan issiqlik miqdorini topamiz:

$$Q = 0{,}8 \cdot 5{,}67 \cdot 10^{-8} \cdot (2043)^4 \cdot 5 \cdot 10^{-3} \cdot 60\text{J} = 2{,}37 \cdot 10^5\,\text{J} = 237\text{kJ}$$

Javob: $Q = 237$ kJ.

5-misol. 3500 K haroratda volfram tolasining yutish xususiyati 0,35. Tolaning radiatsion harorati T_c aniqlansin.

Berilgan:
T =3500 K;
A_T=0,35.
T_r=?

Yechish: Tolaning radiatsion harorati quyidagi ifoda yordamida aniqlanadi:

$$T_r = \sqrt[4]{A_T} \cdot T.$$

Kattaliklarning qiymatlarini qo'yib olamiz:

$$T_r = \sqrt[4]{0{,}35} \cdot 3500\text{K} = 2690\text{K} = 2{,}69\text{kK}$$

Javob: T_r=2,69 kK.

6-misol. Plank formulasidan foydalanib:
1) Reley-Djins; 2) Stefan-Bolsman va 3) Vinning siljish qonunlari formulalarini hosil qilinsin.

Yechish: Plank formulasini yozamiz:

$$r_{\nu,\mathrm{T}} = \frac{2\pi\nu^2}{c^2}\frac{h\nu}{e^{h\nu/(KT)} - 1} \quad . \qquad (1)$$

1. Kichik chastotalarda, ya'ni $h\nu << k\mathrm{T}$ da (kvantning energiyasi $h\nu$ issiqlik harakati energiyasi $k\mathrm{T}$ dan juda kichik) (1) – dan Reley – Djins formulasini hosil qilish mumkin. Buning uchun eksponensial funksiyani qatorga yoyamiz va ikkita had bilan chegaralanamiz:

$$e^{h\nu/(k\mathrm{T})} \approx 1 + \frac{h\nu}{k\mathrm{T}},$$

Yoki

$$e^{h\nu/(k\mathrm{T})} - 1 \approx \frac{h\nu}{k\mathrm{T}}. \qquad (2)$$

(2) ni (1) ga qo'yib quyidagini olamiz:

$$r_{\nu,\mathrm{T}} \approx \frac{2\pi h\nu^3}{c^2}\frac{1}{h\nu/(kt)} = \frac{2\pi\nu^2}{c^2}k\mathrm{T}. \qquad (3)$$

2) Plank formulasidan Stefan-Bolsman formulasini hosil qilish uchun

$$R_e = \int_0^\infty r_{\nu,\mathrm{T}}d\nu \qquad (4)$$

ifodadan foydalanamiz. (1) ni (4) ga qo'yib olamiz:

$$R_e = \int_0^\infty \frac{2\pi h\nu^3}{S^2}\frac{1}{e^{h\nu/(k\mathrm{T})} - 1}d\nu . \qquad (5)$$

(5) ni integrallash uchun $x = \frac{h\nu}{\mathrm{kT}}$ o'lchamsiz o'zgaruvchini kiritamiz.

Unda

$$dx = \frac{h}{\mathrm{kT}}d\nu; \quad d\nu = \frac{\mathrm{kT}}{h}dx . \qquad (6)$$

(6) dan foydalanib (5) ni qayta yozsak,

$$R_e = \frac{2\pi k^4}{c^2 h^3} T^4 \int_0^\infty \frac{x^3 dx}{e^x - 1} = \sigma T^4 . \qquad (7)$$

Bu yerda: $\sigma = \frac{2\pi k^4}{S^2 h^3} \int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{2\pi^5 k^4}{15 c^2 h^3}$

belgilash kiritilgan, va $\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}$ ligidan foydalanilgan.

3) Plank formulasidan Vinning siljish qonunini hosil qilish uchun

$$r_{\lambda,T} = \frac{c}{\lambda^2} r_{\nu,T} \qquad (8)$$

ifodadan foydalanamiz. Demak, (1) ni (8) ga qo'ysak,

$$r_{\lambda,1} = \frac{c}{\lambda^2} r_{\nu,T} = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/(kT\lambda)} - 1} . \qquad (9)$$

Funksiya o'zining maksimumiga erishgandagi λ_{max} qiymatni topish uchun (9) ni λ bo'yicha differensiallaymiz va uni nolga tenglashtiramiz:

$$\frac{dr_{\lambda,T}}{d\lambda} = \frac{2\pi c^2 h}{\lambda^6 \left(e^{hc/(kT\lambda)} - 1 \right)} \left(\frac{\frac{hc}{kT\lambda} e^{hc/(kT\lambda)}}{e^{hc/(kT\lambda)} - 1} - 5 \right) = 0 .$$

$x = \frac{hc}{(kT\lambda_{max})}$ o'zgaruvchini kiritamiz:

$$xe^x - 5(e^x - 1) = 0 .$$

(10) tenglamani ketma-ket yaqinlashish usuli bilan yechish $x = 4{,}965$ natijani beradi.

Demak, $\dfrac{hc}{kT\lambda_{max}} = 4{,}965$.

Bundan $T \cdot \lambda_{max} = \dfrac{h \cdot c}{4{,}965\text{K}} = b$ ni hosil qilamiz.

Mustaqil yechish uchun masalalar

1. Qora jismning energetik yorituvchanligi 16 marta kamayishi uchun uning haroratini necha marta kamaytirish kerakligi aniqlansin. [2 marta]

2. Qora jismning energetik yorituvchanligi 10kW/m^2. Shu jism energetik yorituvchanligi spektral zichligining maksimumiga to‘g‘ri keluvchi to‘lqin uzunligi aniqlansin.[4,47 mkm.]

3. Sirius yulduzining yuqori qatlamlaridagi harorat 10kK. Shu yulduzning 1km^2 yuzali sirtidan sochilayotgan energiya oqimi topilsin. [56,7 GW.]

4. Qora jismning dastlabki harorati 3kK. U soviganda energetik yorituvchanlik spektral zichligining maksimumiga to‘g‘ri keluvchi to‘lqin uzunligi 8mkm ga o‘zgardi. Qora jismning keyingi harorati aniqlansin. [323].

5. Qora jismni 600K haroratdan 2400 K haroratgacha qizitishdi. 1) uning energetik yorituvchanligi necha marta ortganligi; 2) energetik yorituvchanlik spektral zichligining maksimumiga mos keluvchi to‘lqin uzunligi qanday o‘zgarishi aniqlansin. [1) 256 marta; 2) 3,62 mkm ga kamayadi.]

6. Quyoshning yuqori qatlamlarining harorati 5,3 kK ga teng. Quyoshni qora jism sifatida qabul qilib, uning energetik yorituvchanligi spektral zichligining maksimumiga mos keluvchi to‘lqin uzunligi aniqlansin. [547.]

7. Arktur yorqin yulduzining energetik yorqinligi spektral zichligining maksimal qiymatiga 580nm to‘lqin uzunligi mos keladi. Yulduzni qora jismdek nur sochadi deb qabul qilib, uning sirtining harorati T aniqlansin. [4,98kK.]

8. Nikelenni qora jism deb hisoblab, sirtining yuzasi 0,5sm^2 bo‘lgan erigan nikelenning haroratini 1453^{0}C da tutib turish uchun zarur bo‘lgan quvvat aniqlansin. Energiya yo‘qotilishi inobatga olinmasin. [25,2W.]

9. Quyoshni qora jism deb hisoblab va energetik yorituvchanligi spektral zichligining maksimal qiymatiga to‘g‘ri keluvchi to‘lqin uzunligi 500nm ga to‘g‘ri keladi deb olib: 1) quyosh sirtining harorati; 2) 10 min davomida quyosh sirtidan elektromagnit to‘lqinlar sifatida nurlanadigan energiya; 3) shu vaqt davomida nurlanish natijasida quyosh yo‘qotadigan massa aniqlansin.

$[1)T = 5{,}8kK;\ 2)W = 2{,}34 \cdot 10^{29} J;\ 3)\Delta m = 2{,}6 \cdot 10^{12} kg.]$

10. Atrof-muhitning harorati 23^{0}C bo‘lganda, nurlatadigan energiyasi yutadiganidan 10 marta ko‘p bo‘ladigan jismning harorati aniqlansin. [533K.]

31-§. Fotoeffekt. Yorug'likning bosimi. Kompton effekti

Asosiy formulalar

Tashqi fotoeffekt uchun Eynshteyn formulasi:

$$E = h\nu = A + T_{max}.$$

Bunda: $E = h\nu$ metall sirtiga tushayotgan fotonning energiyasi, A– elektronning metaldan chiqish ishi; T_{max} – fotoelektronlarning maksimal kinetik energiyasi. Agar elektron norelyativistik, ya'ni $h\nu < 5\text{keV}$ bo'lsa,

$$T_{max} = \frac{1}{2} m_0 \vartheta_{max}^2,$$

bunda: m_0 – elektronning tinchlikdagi massasi,

$\frac{1}{2} m_0 \vartheta_{max}^2 = eU_0$ (U_0 – tutuvchi kuchlanish).

Agar elektron relyativistik, ya'ni $h\nu >> 5\text{keV}$ bo'lsa,

$$T_{max} = (m - m_0)c^2 = m_0 c^2 \left(\frac{1}{\sqrt{1 - \vartheta_{max}^2 / c^2}} - 1 \right).$$

Bunda: m – relyativistik elektronning massasi.

Berilgan metall uchun fotoeffektning "Qizil chegarasi" quyidagicha aniqlanadi:

$$\nu_0 = \frac{A}{h}, \qquad \lambda_0 = \frac{hc}{\lambda}.$$

Bunda: λ_0– fotoeffekt ro'y beradigan nurlanishning maksimal to'lqin uzunligi (ν_0 – mos ravishda minimal chastota). Fotonning massasi va impulsi:

$$m_\gamma = \frac{E}{c^2} = \frac{h\nu}{c^2}; \qquad P_\gamma = \frac{h\nu}{c}.$$

Sirtga tik tushayotgan yorug'likning bosimi:

$$P = \frac{F_e}{c}(1+\rho) = \omega(1+\rho).$$

Bunda: $F_e = Nh\nu$ sirtning nurlatilganligi, ya'ni birlik sirtga vaqt birligida tushadigan barcha fotonlarning energiyasi; ρ – qaytarish koeffitsienti; ω – nurlanish energiyasining hajmiy zichligi.

Kompton sochilishida elektromagnit to'lqinlar to'lqin uzunliklarining o'zgarishi:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta) = \frac{2h}{m_0 c}\sin^2\frac{\vartheta}{2} = 2\lambda_c \sin^2\frac{\vartheta}{2}.$$

Bunda: λ va λ' lar mos ravishda tushayotgan va sochilgan nurlarning to'lqin uzunliklari; m_0 – elektronning massasi; ϑ – sochilish burchagi; $\lambda_c = \frac{h}{m_0 c}$ – kompton to'lqin uzunligi.

Masala yechishga misollar

1-misol. Agar fotoelektronlarning maksimal tezligi 3000 km/s bo'lsa, platina plastinkaga tushayotgan fotonlarning to'lqin uzunligi aniqlansin.

Berilgan:

$\vartheta_{max} = 3000\,\text{km/s} = 3\cdot10^6\,\frac{\text{m}}{\text{s}}$;

$A = 10^{-18}$ J.

$\lambda = ?$

Yechish: Tashqi fotoeffekt uchun Eynshteyn formulasini yozamiz

$$h\nu = A + \frac{m\vartheta_{max}^2}{2}, \qquad (1)$$

chunki $\frac{m\vartheta_{max}^2}{2} - 5{,}1\text{eV} < 5\text{keV}$.

Agar $\nu = \frac{c}{\lambda}$ ligini e'tiborga olsak:

$$h\frac{c}{\lambda} = A + \frac{m\vartheta_{max}^2}{2}. \qquad (2)$$

Bundan

$$\lambda = \frac{2 \cdot h \cdot c}{2A + m\vartheta_{max}^2}. \qquad (3)$$

Bu yerda: $h = 6{,}625 \cdot 10^{-34}\ \text{J} \cdot \text{s}$ – Plank doimiysi;

$c = 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$ – yorug'likning bo'shliqdagi tezligi;

$m = 9{,}11 \cdot 10^{-31}\,\text{kg}$ – elektronning massasi.

Kattaliklarning qiymatlarini (3) ga qo'yib olamiz

$$\lambda = \frac{2 \cdot 6{,}625 \cdot 10^{-34} \cdot 3 \cdot 10^8}{2 \cdot 10^{-18} + 9{,}1 \cdot 10^{-31} \cdot 9 \cdot 10^{12}}\text{m} = 3{,}9 \cdot 10^{-8}\,\text{m} = 39\text{nm}$$

Javob: $\lambda = 39\text{nm}$.

2-misol. Katodi litiy bo'lgan fotoelementga 200 nm to'lqin uzunlikli yorug'lik tushadi. Fototok to'xtashi uchun fotoelementga qo'yilishi kerak bo'lgan tutuvchi potensiallar farqining eng kichik qiymati aniqlansin.

Berilgan:

$\lambda = 200\text{nm} = 2 \cdot 10^{-7}\,\text{m};$

$A = 3{,}7 \cdot 10^{-19}\ \text{J}.$

$U_0 = ?$

Yechish: Tashqi fotoeffekt uchun Eynshteyn formulasiga muvofiq:

$$h\nu = A + \frac{m\vartheta^2}{2}. \qquad (1)$$

Bu yerda $\frac{m\vartheta^2}{2} = eU_0$ (2)

elektronning kinetik energiyasi. Unda

$$h\nu = A + eU_0. \qquad (3)$$

(3) dan U_0 ni aniqlasak va $\nu = \frac{c}{\lambda}$ ekanligini nazarda tutsak,

$$U_0 = \frac{h \cdot c - A \cdot \lambda}{\lambda \cdot e}. \qquad (4)$$

Bu yerda: $h = 6{,}625 \cdot 10^{-34}$ J $\cdot$ s – Plank doimiysi, $c = 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$ – yorug'likning bo'shliqdagi tezligi, $e = 1{,}6 \cdot 10^{-19} C$ – elektronning zaryadi.

Kattaliklarning son qiymatlarini (4) ga qo'yib hisoblaymiz:

$$U_0 = \frac{6{,}625 \cdot 10^{-34} \cdot 3 \cdot 10^8 - 3{,}7 \cdot 10^{-19} \cdot 2 \cdot 10^{-7}}{2 \cdot 10^{-7} \cdot 1{,}6 \cdot 10^{-19}} \text{V} = 3{,}9\text{V}.$$

Javob: $U_0 = 3{,}9\text{V}$.

3-misol. Rux uchun fotoeffektning qizil chegarasi 310 nm. Agar rux plastinkaga 200 nm to'lqin uzunlikli yorug'lik tushayotgan bo'lsa, fotoelektronlarning maksimal kinetik energiyasi elektron-voltiarda aniqlansin.

Berilgan:

$\lambda_0 = 310\text{nm} = 3{,}1 \cdot 10^{-7}\text{m};$

$A = 6{,}4 \cdot 10^{-19}\text{J};$

$\lambda = 200\text{nm} = 2 \cdot 10^{-7}\text{m}.$

$T_{max} = ?$

Yechish: Fotoeffekt uchun Eynshteyn formulasidan T_{max} ni topamiz:

$$T_{max} = h\nu - A. \qquad (1)$$

Agar $A_0 = h\nu_0$, va $\nu = \frac{c}{\lambda}$ ligini hisobga olsak,

$$T_{max} = h\nu - h\nu_0 = h \cdot c\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right). \qquad (2)$$

$h = 6{,}625 \cdot 10^{-34}$ J $\cdot$ s, $c = 3 \cdot 10^8 \,\text{m}/\text{s}$ va kattaliklarning son qiymatlarini (2) ga qo'ysak

$$T_{max} = 6{,}625 \cdot 10^{-34} \cdot 3 \cdot 10^8 \left(\frac{1}{2 \cdot 10^{-7}} - \frac{1}{3{,}1 \cdot 10^{-7}}\right)\text{J} = 3{,}53 \cdot 10^{-19}\text{J} = 2{,}2\text{eV}.$$

Javob: $T_{max} = 2{,}2eV$.

4-misol. To'lqin uzunligi 600 nm bo'lgan yorug'lik yassi yaltiroq sirtga tik tushib 4 mkPa bosim vujudga keltiradi. Shu sirtning $1mm^2$ yuzasiga 10 s davomida tushayotgan fotonlarning soni *N* aniqlansin.

Berilgan:

$\lambda = 600nm = 6 \cdot 10^{-7}$ m;

$P = 4mkPa = 4 \cdot 10^{-6} \dfrac{N}{m^2}$;

$S = 1mm^2 = 10^{-6} m^2$;

$t = 10s$.

$N = ?$

Yechish: *S* yuzaga *t* vaqtda tushayotgan fotonlarning soni quyidagicha aniqlanadi:

$$N = t \cdot S \cdot n \cdot \qquad (1)$$

Bu yerda,

$$n = \frac{E_e}{\varepsilon_f}, \qquad (2)$$

vaqt birligida birlik yuzaga tushadigan fotonlar soni. E_e – sirtning nurlatilganligi. Uni

$$p = \frac{E_e}{c}(\rho + 1)$$

ifodadan aniqlaymiz va yassi yaltiroq sirt uchun $\rho = 1$ ligini e'tiborga olamiz. Unda (2) quyidagi ko'rinishni oladi:

$$n = \frac{P \cdot c}{2 \cdot \varepsilon_f} = \frac{P \cdot \lambda}{2h}. \qquad (3)$$

(3) ni (1) ga qo'yib, *N* uchun quyidagini topamiz:

$$N = \frac{P \cdot \lambda \cdot t \cdot S}{2 \cdot h}, \qquad (4)$$

bu yerda: $h = 6{,}625 \cdot 10^{-34}$ J · s – Plank doimiysi.

Kattaliklarning son qiymatlarini (4) ga qo'yib olamiz

$$N = \frac{4 \cdot 10^{-6} \cdot 6 \cdot 10^{-7} \cdot 10 \cdot 10^{-6}}{2 \cdot 6{,}625 \cdot 10^{-34}} = 1{,}8 \cdot 10^{16}.$$

Javob: $N = 1{,}8 \cdot 10^{16}$ ta foton.

5-misol. 0,4 MeV energiyali foton erkin elektronda 90^0 burchak ostida sochildi. Sochilgan fotonning energiyasi va elektron olgan kinetik energiya aniqlansin.

Berilgan:

$\varepsilon_1 = 0{,}4MeV = 4\cdot 10^5 eV;$

$\theta = 90^0.$

$\varepsilon_2 = ?$

$T = ?$

Yechish: Kompton formulasiga muvofiq:

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1-\cos\theta) = \frac{2h}{m_0 c}\sin^2\frac{\theta}{2}. \qquad (1)$$

Agar $\lambda = \dfrac{hc}{\varepsilon}$ ligini nazarda tutsak,

$$\frac{hc}{\varepsilon_2} - \frac{hc}{\varepsilon_1} = \frac{2\cdot h\cdot c}{m_0 c^2}\sin^2\frac{\theta}{2}. \qquad (2)$$

Barcha hadlarni hc ga bo'lib ε_2 ni topsak,

$$\varepsilon_2 = \frac{\varepsilon_1\cdot m_0 c^2}{2\cdot\varepsilon_1\cdot\sin^2\frac{\theta}{2} + m_0 c^2}. \qquad (3)$$

Elektron oladigan kinetik energiya esa fotonning dastlabki va ochilgandan keyingi energiyalarining farqiga teng:

$$T = \varepsilon_1 - \varepsilon_2. \qquad (4)$$

$E_0 = m_0 c^2 = 0{,}511\cdot 10^6 eV$ elektronning tinchlikdagi energiyasi ekanligini nazarda tutib, (3) va (4) lar yordamida quyidagini olamiz:

$$\varepsilon_2 = \frac{4\cdot 10^5\cdot 0{,}511\cdot 10^6}{2\cdot 4\cdot 10^5\cdot 0{,}5 + 0{,}511\cdot 10^6} eV = 2{,}24\cdot 10^5 \text{ eV};$$

$$T = 4\cdot 10^5 eV - 2{,}24\cdot 10^5 eV = 1{,}76\cdot 10^5 \text{ eV}.$$

Javob: $\varepsilon_2 = 2{,}24\cdot 10^5$ eV; $T = 1{,}76\cdot 10^5$ eV.

Mustaqil yechish uchun masalalar

1) Agar 3,7 V tutuvchi potensial qo'yilganda fototok to'xtasa, metall sirtidan chiqayotgan fotoelektronlarning maksimal tezligi aniqlansin. [1,14 mm/s.]

2) Biror metall uchun fotoeffektning "Qizil chegarasi" 500 nm. Foton energiyasining fotoeffekt vujudga keltiradigan eng kichik qiymati aniqlansin. [22,48 eV.]

3) Metall sirtidan uzilib chiqadigan elektronlar 3 V tutuvchi kuchlanish quyilganda to'la tutib qolinadi. Shu metall uchun fotoeffekt, tushayotgan monoxromatik yorug'lik chastotasining $6\cdot10^{14}s^{-1}$ qiymatidan boshlanadi. 1) shu metall uchun elektronlarning chiqish ishi; 2) qo'llanilayotgan nurlanish chastotasi aniqlansin? $\left[1)2{,}48eV; \quad 2)1{,}32\cdot10^{15}s^{-1}.\right]$

4) Agar volfram uchun fotoeffektning "Qizil chegarasi" 275 nm bo'lsa, elektronlarning chiqish ishi aniqlansin. [4,52 eV.]

5) Kaliy uchun elektronlarning chiqish ishi 2,2 eV ga teng va 400 nm to'lqin uzunlikli monoxromatik yorug'lik bilan yoritilmoqda. Fototokni to'xtatish uchun qo'yilishi zarur bo'lgan eng kichik tutuvchi kuchlanishning qiymati aniqlansin. [0,91 V.]

6) Platina plastinka uchun (A_2=6,3 eV) tutuvchi kuchlanish 3,7 V ni tashkil qiladi. Shu sharoitlarda boshqa plastinka uchun tutuvchi kuchlanish 5,3 V bo'lsa, shu plastinka uchun elektronlarning chiqish ishi aniqlansin. [4,7 eV.]

7) 5 eV energiyali fotonlar, chiqish ishi 4,7 eV bo'lgan metalldan fotoelektronlarni uzib chiqarmoqda. Elektronlarning uchib chiqishi natijasida shu metall sirtiga beriladigan maksimal impuls aniqlansin.

$$\left[2{,}96\cdot10^{-25}kg\cdot\frac{m}{s}.\right]$$

8) To'lqin uzunligi 0,5 mkm fotonning: 1) energiyasi; 2) impulsi; 3) massasi aniqlansin. $\left[1)2{,}48eV; \quad 2)1{,}33\cdot10^{-27}kg\cdot\frac{m}{s}; \quad 3)4{,}43\cdot10^{-36}kg.\right]$

9. Impulsi, to'lqin uzunligi $\lambda = 0{,}5$mkm bo'lgan fotonning impulsiga teng bo'lishi uchun, elektronning qanday tezlik bilan harakatlanishi kerakligi aniqlansin. [1,45 km/s.]

10) Impulsi, 0,8V potensiallar farqidan o'tgan elektronning impulsiga teng bo'lgan, fotonning to'lqin uzunligi aniqlansin. [392 pm.]

11) 500 nm to'lqin uzunlikli monoxromatik yorug'lik nurining o'ziga perpendikulyar joylashgan, qoraytirilgan sirtga ko'rsatadigan bosimi 0,12

mkPa. Har sekundda, 1 m^2 sirtiga tushadigan fotonlarning soni aniqlansin. $[9{,}05\cdot 10^{19}.]$

12) To'lqin uzunligi 600 nm bo'lgan monoxromatik yorug'likning, qoraytirilgan va tushayotgan nurlanishga perpendikulyar joylashgan sirtga ko'rsatayotgan bosimi 0,1 mkPa. 1) yorug'lik dastasidagi fotonlarning konsentrasiyasi; 2) 1 m^2 sirtga har sekundda tushayotgan fotonlarning soni aniqlansin. $[1)3{,}02\cdot 10^{11}\,m^{-3};\quad 2)9{,}06\cdot 10^{19}.]$

13) Kompton effekti natijasida 60^0 burchak ostida sochilgan nurning to'lqin uzunligi 57 pm bo'lsa, tushayotgan rentgen nurning to'lqin uzunligi aniqlansin. [56,9 pm.]

14) 5 mm to'lqin uzunlikli foton tinch turgan elektrondan 90^0 burchak ostida kompton sochilishiga uchradi. 1) sochilish natijasida to'lqin uzunligining o'zgarishi; 2) elektron olgan energiya; 3) elektron olgan turtki impulsi aniqlansin. $\left[1)2{,}43\,\text{pm};\quad 2)81{,}3\,\text{keV};\quad 3)1{,}6\cdot 10^{-22}\,\text{kg}\cdot\frac{\text{m}}{\text{s}}.\right]$

15) 0,3 MeV energiyali foton 180^0 burchak ostida erkin elektrondan sochildi. Sochilgan fotonning energiyasi tushayotgan foton energiyasining qanday qismini tashkil qiladi? [0,461.]

VII BOB. KVANT MEXANIKASI ELEMENTLARI

32-§. Bor nazariyasi

Asosiy formulalar

Borning birinchi postulati (statsionar holatlar postulati):

$m_e \cdot \vartheta_n \cdot r_n = n\hbar$, $\quad (n = 1,2,3,...)$

Bunda: m_e – elektronning massasi, ϑ_n – elektronning r_n radiusli n – orbitadagi tezligi, $\hbar = \dfrac{h}{2\pi}$, h – Plank doimiysi.

Borning ikkinchi postulati (chastotalar qoidasi): $h\nu = E_n - E_m$.

Bunda: E_n va E_m lar mos ravishda atomning nurlanishgacha (yutilishgacha) va keyingi statsionar holatlarining energiyalari;
n – statsionar orbitadagi elektronning energiyasi:

$$E_n = -\frac{1}{n^2}\frac{Z^2 m_e \cdot e^4}{8h^2\varepsilon_0^2}, \qquad (n = 1,2,3,...)$$

bunda: Z – elementning Mendeleyev davriy sistemasidagi tartib raqami,

e – elementar zaryad, ε_0 – elektrostatik doimiy.

Vodorodsimon atomda elektron statsionar orbitasining radiusi:

$$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi \cdot m_e \cdot Ze^2}, \qquad (n = 1,2,3,...)$$

Vodorod spektridagi seriyalar uchun Balmer formulasi:

$$\nu = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right),$$

bunda: ν – vodorod atomi spektridagi spektral chiziqlarning chastotasi; $R = 3{,}29 \cdot 10^{15}\ s^{-1}$ – Ridberg domiysi; m – seriyani aniqlaydi (m=1,2,3,...); n – mos seriyadagi alohida chiziqlarni aniqlaydi $(n = m+1, m+2,...)$; m=1 (Layman seriyasi), m=2 (Balmer seriyasi), m=3 (Pashen seriyasi), m=4 (Breket seriyasi), m=5 (Pfund seriyasi), m=6 (Xemfri seriyasi).

Mozli formulasi $\nu = R(Z-\sigma)^2\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$.

Bunda: Z – elementning tartib nomeri, σ – ekranlash doimiysi.

K – seriyaning α – chizig'i uchun $(\sigma = 1, m = 1, n = 2)$

$$\nu = R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right).$$

Rentgen spektrining qisqa to'lqin uzunlikli chegarasi quyidagicha aniqlanadi:

$$\lambda_{min} = \frac{c}{\nu_{max}} = \frac{ch}{E_{max}} = \frac{ch}{eU},$$

bunda: E_{max} – rentgen nurlarining energiyasi, c – yorug'likning bo'shlig'dagi tezligi, U – rentgen trubkasiga qo'yilgan kuchlanish.

Masala yechishga misollar

1-misol. Bor nazariyasiga muvofiq vodorod atomi ikkinchi statsionar orbitasining radiusi va undagi elektronning tezligi hisoblansin.

Berilgan:	Yechish:
$n=2$, H_2.	Vodorod atomi elektroni statsionar orbitasining radiusi quyidagi ifoda yordamida aniqlanadi:
$r_2 = ?$	$$r_n = \frac{n^2 \cdot h^2 \varepsilon_0}{\pi \cdot m_e \cdot Ze^2}, \quad (1)$$
$\vartheta_2 = ?$	bunda: $h = 6{,}63 \cdot 10^{-34}$ J·s – Plank doimiysi,

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\,\frac{\text{F}}{\text{m}}$ – elektrostatik doimiylik, $e = 1{,}6 \cdot 10^{-19} C$ – elektronning zaryadi.

Borning birinchi postulatiga muvofiq:

$$m_e \cdot \vartheta_n \cdot r_n = n\frac{h}{2\pi}. \quad (2)$$

Bundan $$\vartheta_n = \frac{nh}{2\pi \cdot m_e \cdot r_n}. \quad (3)$$

Agar (1) ni e'tiborga olib, (3) ni qayta yozsak,

$$\vartheta_n = \frac{Z \cdot e^2}{2\varepsilon_0 \cdot n \cdot h}. \qquad (4)$$

Berilgan masalada $Z = 1$ va $n = 2$ ligini nazarga olib, kattaliklarning qiymatlarini (1) va (4) larga qo'ysak

$$r_2 = \frac{2^2 \cdot (6{,}63 \cdot 10^{-34})^2 \cdot 8{,}85 \cdot 10^{-12}}{3{,}14 \cdot 9{,}11 \cdot 10^{-31} \cdot 1 \cdot (1{,}6 \cdot 10^{-19})^2} \text{m} = 2{,}12 \cdot 10^{-10} \text{m} = 212 \, \text{pm}.$$

$$\vartheta_2 = \frac{1 \cdot (1{,}6 \cdot 10^{-19})^2}{2 \cdot 8{,}85 \cdot 10^{-12} \cdot 2 \cdot 6{,}63 \cdot 10^{-34}} \frac{\text{m}}{\text{s}} = 1{,}1 \cdot 10^6 \frac{\text{m}}{\text{s}} = 1{,}1 \frac{\text{Mm}}{\text{s}}.$$

Javob: $r_2 = 212 \, \text{pm}$; $\vartheta_2 = 1{,}1 \frac{\text{Mm}}{\text{s}}$.

2-misol. Vodorod atomidagi elektronning uchinchi statsionar orbitadan ikkinchisiga o'tishida chiqarilgan fotonning energiyasi aniqlansin.

Berilgan:
$Z=1$,
$n=3$,
$m=2$

$\varepsilon_F = ?$

Yechish: Borning ikkinchi postulatiga muvofiq vodorod atomidagi elektronning n – statsionar orbitadan m–ga o'tishida chiqariladigan fotonning energiyasi quyidagicha aniqlanadi:

$$\varepsilon_F = E_n - E_m. \qquad (1)$$

Bu yerda elektronning statsionar orbitadagi energiyalari:

$$E_n = -\frac{1}{n^2} \frac{Z^2 m_e e^4}{8h^2 \varepsilon_0^2}, \qquad (2)$$

$$E_m = -\frac{1}{m^2} \frac{Z^2 m_e \cdot e^4}{8h^2 \varepsilon_0^2}. \qquad (3)$$

(2) va (3) larni (1) ga qo'ysak,

$$\varepsilon_F = \frac{Z^2 \cdot m \cdot e^4}{8 \cdot h^2 \varepsilon_0^2} \left(\frac{1}{m^2} - \frac{1}{n^2} \right). \qquad (4)$$

Berilgan masalada $Z = 1, n = 3, m = 2$ ligini va standart kattaliklarning qiymatlarini e'tiborga olib (4) dan quyidagini olamiz:

$$\varepsilon_L = \frac{1^2 \cdot 9{,}1 \cdot 10^{-31} \cdot (1{,}6 \cdot 10^{-19})^4}{8 \cdot (6{,}63 \cdot 10^{-34})^2 \cdot (8{,}85 \cdot 10^{-12})^2} \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \text{J} = \frac{10{,}75}{36} \cdot 10^{-18}\ \text{J} = 1{,}89\text{eV}.$$

Javob: $\varepsilon_L = 1{,}89\text{eV}$.

3-misol. Bor nazariyasiga muvofiq, n=2 bosh kvant soni bilan aniqlanuvchi g'alayonlangan holatda bo'lgan vodorod atomidagi elektronning aylanish davri hisoblansin.

Berilgan:
$Z = 1$;
$n = 2$.

$T = ?$

Yechish: Elektronning aylanish davri quyidagi ifoda yordamida aniqlanadi:

$$T = \frac{2\pi r_n}{\vartheta_n}. \qquad (1)$$

Bu yerda: r_n – elektron orbitasining radiusi,

$$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi \cdot m_e \cdot Ze^2}, \qquad (2)$$

ϑ_n – elektronning tezligi (1-misolga qarang)

$$\vartheta_n = \frac{Z \cdot e^2}{2\varepsilon_0 \cdot n \cdot h}. \qquad (3)$$

(2) va (3) larni (1) ga qo'ysak topamiz:

$$T = \frac{4 \cdot n^3 \cdot h^3 \varepsilon_0^2}{m_e (Ze^2)^2}, \qquad (4)$$

masala shartlarini va standart kattaliklarning qiymatlarini (4) ga qo'yib, quyidagini olamiz:

$$T = \frac{4 \cdot (2)^3 \cdot (6{,}63 \cdot 10^{-34})^3 \cdot (8{,}85 \cdot 10^{-12})^2}{9{,}11 \cdot 10^{-31} \cdot (1)^2 \cdot (1{,}6 \cdot 10^{-19})^4}\ \text{s} = 1{,}2 \cdot 10^{-15}\ \text{s}.$$

Javob: $T = 1{,}2 \cdot 10^{-15}$ s.

4-misol. Vodorod atomi nurlanish spektri Balmer seriyasidagi fotonning maksimal energiyasi aniqlansin.

Berilgan:	**Yechish:** nurlangan fotonning energiyasi
$Z=1$;	$\varepsilon = h \cdot \nu$ (1)
$m=2$;	ifodadan aniqlanadi. Bu yerda: h – Plank
$n=\infty$	doimiysi, ν – nurlanish chastotasi.
$\varepsilon_{max}=?$	Balmer seriyasi uchun

$$\nu = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right). \qquad (2)$$

Chiqarilgan fotonning energiyasi $n=\infty$ da eng katta bo'ladi. Shu shartni hisobga olsak va (2) ni (1) ga qo'ysak olamiz:

$$\varepsilon_{max} = h\frac{R}{4}. \qquad (3)$$

h va Ridberg doimiysi R larning qiymatlarini (3) ga qo'yib quyidagini olamiz:

$$\varepsilon_{max} = \frac{1}{4} 6{,}63 \cdot 10^{-34} \cdot 3{,}29 \cdot 10^{15}\,\mathrm{J} = 5{,}45 \cdot 10^{-19}\,\mathrm{J} = 3{,}4\mathrm{eV}.$$

Javob: $\varepsilon_{max} = 3{,}4\mathrm{eV}$.

5-misol. G'alayonlangan vodorod atomidagi elektron ikkinchi energetik sathga o'tganida uning orbitasining radiusi 9 marta kamaygan bo'lsa, chiqarilgan fotonning chastotasi aniqlansin.

Berilgan:	**Yechish:** Balmerning umumlashgan formulasiga muvofiq chiqarilgan fotonning chastotasi
$m=2$;	
$\frac{r_n}{r_m} = \frac{r_n}{r_2} = 9.$	quyidagicha aniqlanadi:
$\nu = ?$	$\nu = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right).$ (1)

Vodorod atomidagi elektron orbitasining radiusini topish ifodasidan

$$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi \cdot m_e \cdot Ze^2} \qquad (2)$$

foydalanib $\left(\dfrac{r_n}{r_m}\right)$ nisbatni tuzamiz:

$$\frac{r_n}{r_m} = \frac{n^2}{m^2} = 9. \qquad (3)$$

(3) dan $\qquad n^2 = 9m^2. \qquad (4)$

(4) ni (1) ga qo'ysak,

$$\nu = R\left(\frac{1}{m^2} - \frac{1}{9m^2}\right) = \frac{R}{m^2}\left(1 - \frac{1}{9}\right) = \frac{2}{9}R. \qquad (5)$$

Ridberg doimiysining qiymatini (5) ga qo'yamiz:

$$\nu = \frac{2}{9} \cdot 3{,}29 \cdot 10^{15}\,\mathrm{s}^{-1} = 7{,}31 \cdot 10^{14}\,\mathrm{s}^{-1}.$$

Javob: $\nu = 7{,}31 \cdot 10^{14}\,\mathrm{s}^{-1}$.

6-misol. Qandaydir elementning chiziqli rentgen spektri o'rganilganda K− seriya α − chizig'ining to'lqin uzunligi 76pm ekanligi aniqlandi. Bu qanday elementligi aniqlansin.

Berilgan:

$\lambda_\alpha = 76\,\mathrm{pm} = 76 \cdot 10^{-12}\,\mathrm{m}.$

$Z = ?$

Yechish: Rentgen nurlari K-seriyasi α-chizig'ining chastotasi quyidagicha aniqlanadi:

$$\nu = \frac{3}{4}R(Z-1)^2, \qquad (1)$$

yoki $\nu = \dfrac{c}{\lambda}$ ligidan $\qquad \dfrac{c}{\lambda_\alpha} = \dfrac{3}{4}R(Z-1)^2. \qquad (2)$

Bundan Z ni topib olsak, $\qquad Z = \sqrt{\dfrac{4}{3}\dfrac{c}{\lambda_\alpha \cdot R}} + 1. \qquad (3)$

Kattaliklarning qiymatlarini (3) ga qo'yib olamiz:

$$Z = \sqrt{\frac{4}{3}\frac{3 \cdot 10^8}{76 \cdot 10^{-12} \cdot 3{,}29 \cdot 10^{15}}} + 1 = 41.$$

Javob: $Z=41$, niobiy.

7-misol. Volfram atomidagi elektron *M*-qatlamdan *L*-qatlamga o'tdi. $\sigma = 5{,}5$ deb hisoblab chiqarilgan fotonning energiyasi aniqlansin.

Berilgan:	**Yechish:** Fotonning energiyasini
$Z=74$,	$$\varepsilon_{\Phi} = h\nu, \qquad (1)$$
$n=3$, $m=2$,	ifodadan aniqlaymiz. Fotonning chastotasini esa Mozli formulasi yordamida aniqlaymiz
$\sigma=5{,}5$.	$$\nu = R(Z-\sigma)^2\left(\frac{1}{m^2}-\frac{1}{n^2}\right). \qquad (2)$$
$\varepsilon_F = ?$	(2) ni (1) ga qo'ysak,

$$\varepsilon_{\Phi} = hR\,(Z-\sigma)^2\left(\frac{1}{m^2}-\frac{1}{n^2}\right). \qquad (3)$$

Berilganlarni va kattaliklarning qiymatlarini (3) ga qo'yib quyidagini olamiz:

$$\varepsilon_F = 6{,}63\cdot10^{-34}\cdot3{,}29\cdot10^{15}(74-5{,}5)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)\text{J} =$$

$$= 24561{,}11\cdot10^{-19}\,\text{J} = 15350{,}69\text{eV} = 15{,}35\text{keV}.$$

Javob: $\varepsilon_F = 15{,}35\text{keV}$.

Mustaqil yechish uchun masalalar

1. Vodorod spektrining ko'zga ko'rinuvchi seriyasidagi (Balmer seriyasi) fotonning maksimal va minimal energiyalari aniqlansin. [3,41eV, 1,89 eV.]
2. Pashen seriyasidagi ikkinchi spektral chiziqning to'lqin uzunligi aniqlansin. [1,28 mkm.]
3. Layman, Balmer, Pashen seriyalarining chegaralariga mos keluvchi to'lqin uzunliklari aniqlansin. [91 nm; 364 nm; 820 nm.]
4. $4{,}86\cdot10^{-7}$m to'lqin uzunlikli foton chiqargan atomidagi elektronning kinetik energiyasi qanchaga o'zgaradi. [2,56 eV.]
5. Elektron yuqoriroq energetik sathdan pastki sathga o'tganida uning energiyasi 1eV ga kamayadi. Bu o'tishda chiqariladigan fotonning to'lqin uzunligi aniqlansin. [124 nm.]

6. Vodorod atomining uchinchi orbitasidagi elektronning tezligi aniqlansin. [0,731 Mm/s.]

7. Vodorod atomining uchinchi orbitasidagi elektronning aylanish chastotasi aniqlansin. [$2,42 \cdot 10^{14}$ Hz.]

8. Vodorod atomining ionlashuv energiyasi 13,6eV ekanligiga asoslanib, Balmer seriyasidagi eng katta to'lqin uzunlikli chiziqqa mos keluvchi fotonning energiyasi elektron-voltlarda aniqlansin. [1,89 eV.]

9. Vodorod atomining ikkinchi bor orbitasidagi elektronni yadroning tortishish doirasidan chiqarish uchun bajarilishi kerak bo'lgan ish aniqlansin. [$5,45 \cdot 10^{-19}$J.]

10. Vodorod atomining asosiy holatida bo'lgan elektron 17,7eV energiyali foton bilan urib chiqariladi. Elektronning atom tashqarisidagi tezligi ϑ aniqlansin. [1,2 Mm/s.]

11. Asosiy holatdagi vodorod atomi 12,12eV energiyali fotonni yutib, g'alayonlangan holatga o'tdi. Bu holatga mos keluvchi bosh kvant soni aniqlansin. [n=3.]

12. Yulduzlar spektrida, to'lqin uzunligi vodorod atomi spektri to'lqin uzunligidan 9 marta kichik bo'lgan vodorodsimon spektr topildi. Bu spektrni chiqaruvchi element aniqlansin. [Z=3, litiy.]

13. 1) elektron asosiy holatda bo'lsa; 2) elektron n=3 bosh kvant soniga mos keluvchi holatda bo'lsa uni bir marta ionlashgan geliy atomidan to'la uzib chiqarish uchun qanday energiya zarurligi aniqlansin. [154,4eV; 2) 6,04 eV.]

14. Agar rentgen trubkasi 150kV kuchlanish ostida ishlasa, rentgen nurlanishining eng kichik to'lqin uzunligi aniqlansin. [8,29 pm.]

15. 60kV kuchlanishda ishlovchi trubkadan olingan rentgen nurlari to'lqin uzunligining minimal qiymati 20,7 pm. Shularga asoslanib Plank doimiysining qiymati topilsin. [$6,62 \cdot 10^{-34}$ J · s.]

16. Rentgen trubkasining anodini bombardimon qiladigan elektronlarning tezligi 0,8c bo'lsa, yaxlit rentgen spektri qisqa to'lqinli chegarasining to'lqin uzunligi aniqlansin. [3,64 pm.]

17. Agar biror element K – seriyasi a – chizig'ining to'lqin uzunligi 72 pm bo'lsa, shu elementning D.I.Mendeleyev elementlar davriy sistemasidagi tartib nomeri aniqlansin. [42, molibden.]

18. Agar rentgen trubkasining anodi platinadan yasalgan bo'lsa, xarakteristik rentgen spektri K – seriyasining eng uzun to'lqin uzunlikli chizig'ining to'lqin uzunligi aniqlansin. Ekranlash doimiysi birga teng deb olinsin. [20,4 pm.]

19. Agar elektronning volfrom atomining M – qobig'idan L –qobig'iga o'tishi natijasida chiqarilgan fotonning to'lqin uzunligi 140pm ni tashkil qilsa, rentgen nurlanishining L – seriyasi uchun ekranlash doimiysi aniqlansin. [5,63.]

33-§. Kvant mexanikasi elementlari

Asosiy formulalar

De-Broyl to'lqin uzunligi va zarra impulsi orasidagi bog'lanish

$$\lambda = \frac{h}{p} = \frac{h}{m\vartheta},$$

bunda: m – zarra massasi, ϑ – tezligi.

Noaniqlik munosabatlari:

zarraning koordinatasi va impulsi uchun

$$\Delta x \cdot \Delta p_x \geq h, \ \Delta y \cdot \Delta p_y \geq h, \ \Delta Z \cdot \Delta p_Z \geq h,$$

bunda: Δx, Δy, ΔZ – koordinatalarning noaniqliklari; ΔP_x, ΔP_y, ΔP_Z – zarra impulsining mos koordinatalardagi proeksiyalarining noaniqliklari;

Energiya va vaqt uchun $\Delta E \cdot \Delta t \geq h$,

bunda: ΔE – berilgan kvant holati uchun energiyaning noaniqligi, Δt – sistemaning shu holatda bo'lish vaqti.

Shryodingerning umumiy tenglamasi:

$$-\frac{\hbar^2}{2m}\Delta\Psi + U(x,y,z,t)\Psi = i\hbar\frac{\delta\Psi}{\delta t}.$$

Bunda: $\Psi = \Psi(x,y,z,t)$ – zarraning holati funksiyasi; $\hbar = \frac{h}{2\pi}$;

$\Delta = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$ – Laplas operatori; $i = \sqrt{-1}$ – mavhum birlik;

$U = U(x,y,z,t)$ – zarraning harakatlanayotgan maydondagi potensial energiyasi.

Statsionar holat uchun Shryodinger tenglamasi:

$$\Delta\Psi + \frac{2m}{\hbar^2}(E - U)\Psi = 0,$$

bunda: $\Psi = \Psi(x,y,z)$ – to'lqin funksiyasining koordinatalarga bog'liq qismi, E – zarraning to'la energiyasi.

To‘lqin funksiyasini normallashtirish sharti $\int\limits_{-\infty}^{+\infty}|\Psi|^2 dV = 1$.

Zarrani x_1 da x_2 gacha oraliqda bo‘lish ehtimoli

$$W = \int\limits_{x_1}^{x_2}|\Psi(x)|^2 dx,$$

bunda: ψ – funksiya bilan tavsiflanuvchi holatda bo‘lgan zarrani xarakterlovchi L – fizik kattaliklarning o‘rtacha qiymati

$$< L >= \int\limits_{-\infty}^{+\infty} L|\Psi|^2 dV.$$

Erkin zarraning bir o‘lchamli harakatini tavsiflovchi to‘lqin funksiyasi

$$\Psi(x,t) = A \cdot e^{-\frac{i}{\hbar}(Et - P_x x)},$$

bunda: A –de Broyl to‘lqinining amplitudasi; $P_x = \hbar k$ – zarra impulsi, $E = \hbar\omega$ – energiyasi.

Bir o‘lchamli cheksiz chuqur potensial o‘raning n –energetik sathida bo‘lgan zarra energiyasining xususiy qiymatlari

$$E_n = n^2 \frac{\pi^2 h^2}{2ml^2}(n = 1,2,3,...),$$

bunda l – o‘raning kengligi.

Energiyaning yuqorida keltirilgan xusuiy qiymatlariga mos keluvchi xususiy to‘lqin funksiyalari,

$$\Psi_n(x) = \sqrt{\frac{2}{l}}\sin\frac{n\pi}{l}x(n = 1,2,3,...).$$

Chekli l –kenglikli to‘g‘ri burchakli potensial to‘siqning shaffoflik koeffitsienti

$$D = D_0 \exp\left[-\frac{2}{h}\sqrt{2m(U - E)}l\right],$$

bunda: D_0 – ko'paytuvchi, U – potensial to'siqning balandligi, E – zarra energiyasi.

Chiziqli garmonik ossillyator uchun Shryodinger tenglamasi

$$\frac{d^2\Psi}{dx^2}+\frac{2m}{h^2}\left(\mathrm{E}-\frac{m\omega_0^2x^2}{2}\right)\Psi=0,$$

bunda $\frac{m\omega_0^2x^2}{2}=U$ – ossilyatorning potensial energiyasi; ω_0 – ossilyator tebranishining xususiy chastotasi; m – zarra massasi.

Garmonik ossilyator energiyasining xususiy qiymatlari

$$E_n=\left(n+\frac{1}{2}\right)h\omega_0,\ (n=0,1,2,...)$$

Garmonik ossilyator nolinchi tebranishining energiyasi

$$E_0=\frac{1}{2}h\omega_0.$$

Masala yechishga misollar

1-misol. Harakatdagi massasi tinchlikdagi massasidan ikki marta katta bo'lgan elektronning de-Broyl to'lqin uzunligi aniqlansin.

Berilgan:	Yechish:
$m=2m_0$.	**Yechish:** Elektronning de-Broyl to'lqin uzunligi quyidagi ifoda yordamida aniqlanadi
$\lambda=?$	

$$\lambda=\frac{h}{p}=\frac{h}{m\vartheta}. \quad (1)$$

Masalaning shartiga ko'ra $m=2m_0$. Demak,

$$\lambda=\frac{h}{2m_0\vartheta}. \quad (2)$$

Elektronning tezligini uning relyativistik massasi ifodasidan aniqlaymiz

$$m = 2m_0 = \frac{m_0}{\sqrt{1 - \vartheta^2 / c^2}},$$

yoki $\quad 2\sqrt{1 - \frac{\vartheta^2}{c^2}} = 1$. (3)

(3) ifodadan ϑ ni topsak, $\quad \vartheta = \frac{\sqrt{3}}{2} c$. (4)

(4) ni (2) ga qo'yib topamiz: $\quad \lambda = \frac{h}{\sqrt{3} \cdot m_0 \cdot c}$. (5)

Standart kattaliklar $h = 6{,}63 \cdot 10^{-34}\ \mathrm{J \cdot s}$; $m_0 = 9{,}11 \cdot 10^{-31}\ \mathrm{kg}$; $c = 3 \cdot 10^8\ \mathrm{m/s}$ larni (5) ga qo'yamiz:

$$\lambda = \frac{6{,}63 \cdot 10^{-34}}{\sqrt{3} \cdot 9{,}11 \cdot 10^{-31} \cdot 3 \cdot 10^8}\ \mathrm{m} = 1{,}4\ \mathrm{pm}.$$

Javob: $\lambda = 1.4\ \mathrm{pm}$.

2-misol. Elektronning de-Broyl to'lqin uzunligi uning Kompton to'lqin uzunligiga teng bo'lishi uchun, elektron qanday kinetik energiyaga ega bo'lishi kerak?

Berilgan:

$\lambda = \lambda_K$.

$T = ?$

Yechish: Elektronning kinetik energiyasi uning to'la energiyasi va tinchlikdagi energiyalarining farqi sifatida aniqlanadi

$$T = E - E_0. \quad (1)$$

Bu yerda: $E_0 = m_0 c^2 = 0{,}51\mathrm{MeV}$ – elektronning tinchlikdagi energiyasi:

$$E = \sqrt{m_0^2 c^4 + p^2 c^2} = \sqrt{m_0^2 c^4 + \left(\frac{h}{\lambda}\right)^2 c^2}, \quad (2)$$

$\frac{h}{c}$ – relyativistik elektronning impulsi.

Masalaning shartiga ko'ra, $\lambda = \lambda_K = \frac{h}{m_0 c}$

ekanligi e'tiborga olsak, undan $\frac{h}{\lambda} = m_0 c$ (3)

ekanligini topamiz. (3) ni (2) ga qo'ysak va (1) ni e'tiborga olsak,

$$T = \sqrt{m_0^2 c^4 + (m_0 c)^2 c^2} - E_0 = \sqrt{2E_0^2} - E_0 = \sqrt{2}E_0 - E_0 = (\sqrt{2} - 1)E_0 = 0{,}41E_0 = 0{,}21\text{MeV}$$

Javob: $T = 0{,}21\text{MeV}$.

3-misol. Elektronning kinetik energiyasi o'zining tinchlikdagi energiyasiga teng. Bunda elektron koordinatasining eng kichik noaniqligi nimaga teng bo'ladi?

Berilgan:	**Yechish:** Koordinata va impuls uchun noaniqliklar munosabatini yozamiz
$T = E_0$.	$\Delta x \cdot \Delta p \geq \hbar$, (1)
$\Delta x = ?$	

bu yerda: $\hbar = \frac{h}{2\pi} = 1{,}05 \cdot 10^{-34}\, J \cdot s$ – Plank doimiysi.

Bundan:

$$\Delta x \geq \frac{\hbar}{\Delta p}. \quad (2)$$

Bu yerda: Δx – elektron koordinatasining, Δp – impulsining noaniqligi.

Impulsning noaniqligi Δp, impulsning o'zi p dan katta bo'lolmaydi.

$$\Delta p \leq p, \quad (3)$$

Agar elektron impulsi va kinetik energiyasi orasidagi

$$p = \sqrt{2m_0 T} \quad (4)$$

munosabatdan foydalansak

$$\Delta x \geq \frac{h}{\sqrt{2m_0 T}}. \qquad (5)$$

Kattaliklarning son qiymatlarini (5) ga qo'yib hisoblaymiz: $\left(m = 9{,}1\cdot10^{-3}\,\text{kg};\quad T = E_0 = m_0c^2 = 8{,}18\cdot10^{-14}\,\text{J}\right)$.

$$\Delta x = \frac{1{,}05\cdot10^{-34}}{\sqrt{2\cdot9{,}1\cdot10^{-31}\cdot8{,}18\cdot10^{-14}}}\,\text{m} = 0{,}27\cdot10^{-12}\,\text{m} = 0{,}27\,\text{pm}.$$

Javob: $\Delta x = 0{,}27\,pm$.

4-misol. Devorlari cheksiz baland, kengligi 200pm bo'lgan bir o'lchamli to'g'ri burchakli o'radagi elektron ($n=4$) g'alayonlangan holatda turibdi. Elektronning minimal energiyasi va o'raning birinchi choragida bo'lish ehtimoli aniqlansin.

Berilgan:
$l = 200pm = 2\cdot10^{-10}\,m$;
$x_1 = 0$;
$x_2 = l/4$.

$E_{min} = ?$
$W = ?$

Yechish: Devorlari cheksiz baland, bir o'lchamli to'g'ri burchakli potensial o'rada n –chi energetik sathda bo'lgan elektron energiyasining xususiy qiymatlari quyidagicha aniqlanadi:

$$E_n = n^2\frac{\pi^2\hbar^2}{2ml^2}, \quad (n = 1,2,3,...), \qquad (1)$$

bu yerda: m –elektronning massasi, $\hbar = \frac{h}{2\pi}$, h – Plank doimiysi. $n=1$ da elektronning energiyasi E_{min} qiymatni qabul qiladi:

$$E_{min} = \frac{\pi^2\hbar^2}{2ml^2} = \frac{h^2}{8ml^2}. \qquad (2)$$

Elektronni $x_1 < x < x_2$ intervalda bo'lish ehtimoli

$$W = \int_{x_1}^{x_2} |\Psi_n(x)|^2 dx \qquad (3)$$

kabi aniqlanadi. Bu yerda:

$$\Psi_n(x) = \sqrt{\frac{2}{l}} \sin\frac{n\pi}{l}x, \; (n = 1,2,3,...), \qquad (4)$$

elektronning n –holatga mos keluvchi normallashtirilgan xususiy funksiyasi. G'alayonlangan $n=4$ holatga quyidagi xususiy funksiya to'g'ri keladi:

$$\Psi_4(x) = \sqrt{\frac{2}{l}} \sin\frac{4\pi}{l}x. \qquad (5)$$

(5) ni (3) ga qo'yamiz: $W = \frac{2}{l}\int_0^{l/4} \sin^2\frac{4\pi}{l}x dx$. (6)

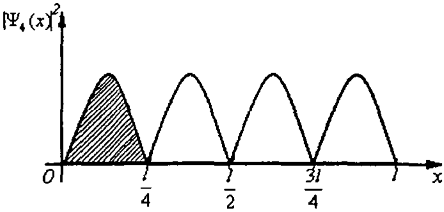
Kattaliklarning qiymatlarini (2) ga qo'ysak,

$$E_{min} = \frac{(6,63\cdot10^{-34})J}{8\cdot9,11\cdot10^{-31}\cdot(2\cdot10^{-2})^2} = 1,5\cdot10^{-18}\,J = 9,37eV.$$

Shuningdek, berilgan masalada $x_1 = 0$ va $x_2 = \frac{l}{4}$ ekanligidan (59-rasm) foydalansak va (6) da $\sin^2\frac{4\pi x}{l} = \frac{1}{2}\left(1-\cos\frac{8\pi x}{l}\right)$ almashtirish kiritsak,

$$W = \frac{1}{2}\left[\int_0^{l/4} dx - \int_0^{l/4}\cos\frac{8\pi x}{l}dx\right] = \frac{1}{l}\left[\frac{l}{4} - \frac{l}{8\pi}\sin\frac{8\pi}{l}x\Big|_0^{l/4}\right] =$$

$$\frac{l}{4} - \frac{l}{8\pi}(\sin 2\pi - \sin 0) = 0,25.$$



59-rasm

Javob: $E_{min} = 9{,}3\text{eV}$; $W = 0{,}25$.

5-misol. Ikkita bir xil 5 eV energiyali zarralar elektron va proton, x o'qining musbat yo'nalishi tomon harakatlanadi va balandligi 10 eV, kengligi 1pm bo'lgan to'g'ri burchakli potensial to'siqqa duch keladi. Zarralarning to'siqdan o'tish ehtimolliklarining nisbati topilsin.

Berilgan:
E=5eV=8 · 10^{-19}J;
U=10eV=16 · 10^{-19}J;
l=1pm=10^{-12}m.

$\frac{W_e}{W_p} = ?$

Yechish: Zarraning potensial to'siqdan o'tish ehtimoli shaffoflik koeffitsienti orqali aniqlanadi: $W = D$.
Demak,

$$W = D = D_0 \exp\left(-\frac{2}{\hbar}\sqrt{2m(U-E)}\cdot l\right), \qquad (1)$$

Bu yerda: D_0=1 (birga tenglashtirilgan ko'paytuvchi), t – zarra massasi, $\hbar = \frac{h}{2\pi}$ – Plank doimiysi.

Unda (1) yordamida so'ralgan nisbatni tuzamiz:

$$\frac{W_e}{W_p} = \frac{\exp\left(-\frac{2}{\hbar}\sqrt{2\cdot m_e(U-E)}l\right)}{\exp\left(-\frac{2}{\hbar}\sqrt{2\cdot m_p(U-E)}l\right)}, \qquad (2)$$

bu yerda: $m_e = 9{,}11\cdot 10^{-31}\,\text{kg}$, $m_p = 1{,}672\cdot 10^{-27}\,\text{kg}$.

Kattaliklarning qiymatlarini (2) ga qo'yib hisoblaymiz:

$$\frac{W_e}{W_p} = \exp\left[\frac{2}{\hbar}\sqrt{2\cdot m_p(U-E)}l - \frac{2}{\hbar}\sqrt{2\cdot m_e(U-E)}l\right] = \exp\left[\frac{2}{\hbar}\sqrt{2\cdot(U-E)}\cdot l\left(\sqrt{m_p}-\sqrt{m_e}\right)\right] =$$

$$\exp\left[\frac{4\cdot 3{,}14}{6{,}63\cdot 10^{-34}}\sqrt{2(16-8)\cdot 10^{-19}}\cdot 10^{-12}\left(\sqrt{1{,}672\cdot 10^{-27}}-\sqrt{9{,}11\cdot 10^{-31}}\right)\right] = 2{,}6$$

Javob: $\frac{W_e}{W_p} = 2{,}6$.

Mustaqil yechish uchun masalalar

1. 290 K haroratda o'rtacha kvadratik tezlik bilan harakatlanayotgan neytronning de-Broyl to'lqin uzunligi aniqlansin. [148 pm.]

2. Protonning de Broyl to'lqin uzunligi 1nm bo'lishi uchun, u qanday tezlashtiruvchi potensiallar farqidan o'tishi kerakligi aniqlansin. [0,821 MV.]

3. Zaryadlangan zarra 500V tezlashtiruvchi potensiallar farqidan o'tganda de Broyl to'lqin uzunligi 1,282 pm ga teng bo'ladi. Agar bu zarraning zaryadi elektronning zaryadiga teng bo'lsa, uning massasi nimaga teng? [$1,672 \cdot 10^{-27}$kg.]

4. Elektronning kinetik energiyasi 0,6MeV ga teng. Uning de Broyl to'lqin uzunligi aniqlansin. [1,26 pm.]

5. Elektronning qanday tezligida uning Kompton to'lqin uzunligi de Broyl to'lqin uzunligiga teng bo'ladi. [$2,12 \cdot 10^{8}$m/s.]

6. Elektron vodorod atomining birinchi bor orbitasida harakatlanmoqda. Tezlikni aniqlashdagi noaniqlik uning son qiymatining 10% ni tashkil qiladi deb hisoblab, elektron koordinatasining noaniqligi aniqlansin. [3,34 nm.]

7. Elektron, 0,3 nm diametrli atom ichida deb hisoblab, energiyasining noaniqligi elektron — voltlarda hisoblansin. $[h^2/[2m(\Delta x)^2] = 16{,}7eV.]$

8. Elektronning bir o'lchamli, to'g'ri burchakli cheksiz baland potensial oradagi normallashtirilgan xususiy funksiyasi $\Psi_n(x) = \sqrt{2/l} \cdot \sin\frac{n \cdot \pi}{l}x$ ko'rinishga ega, bu yerda l — o'raning kengligi. Elektron koordinatasining o'rtacha qiymati aniqlansin. [l/2.]

9. Zarra l kenglikli, bir o'lchamli, to'g'ri burchakli, cheksiz baland potensial o'rada asosiy holatda turibdi. Zarrani o'raning chapgi uchdan bir qismida bo'lish ehtimoli aniqlansin. [0,195.]

10. To'g'ri burchakli potensial to'siqning kengligi 0,1nm. Elektronning to'siqdan o'tish ehtimoli 0,99 bo'lishi uchun energiyalar farqi U — E necha elektron-volt bo'lishi aniqlansin. [10^{-4} eV.]

11. De Broyl to'lqin uzunligi 100pm bo'lgan elektron x o'qining musbat yo'nalishi bo'ylab harakatlanib, o'z yo'lida balandligi 100eV bo'lgan cheksiz keng to'g'ri burchakli to'siqqa duch keladi. To'siqdan o'tgandan keyin elektronning de-Broyl to'lqin uzunligi aniqlansin. [172 pm.]

12. Massasi 100 g va uzunligi 0,5 m bo'lgan matematik mayatnikni garmonik ossillyator sifatida qarab, mayatnikning, tebranishning nolinchi energiyasiga to'g'ri keluvchi klassik amplitudasi aniqlansin. [$1,54 \cdot 10^{-17}$m.]

34-§. Atom va molekulalar fizikasi elementlari

Asosiy formulalar

Vodorodsimon atomda elektronning yadro bilan o'zaro ta'sir potensial energiyasi: $U_{(r)} = -\frac{Ze^2}{4\pi\varepsilon_0 r}$,

bunda: r – elektron va yadro orasidagi masofa; Z – elementning tartib nomeri; ε_0 – elektrostatik doimiy.

Vodorodsimon atomdagi elektron energiyasining xususiy qiymatlari:

$$E_n = -\frac{1}{n^2}\frac{Zme^4}{8h^2\varepsilon_0^2}, \quad (n = 1,2,3,...).$$

Vodorod atomining ionlashuv energiyasi: $E_i = -E_1 = \frac{me^4}{8h^2\varepsilon_0^2}$.

Elektronning impuls momenti (orbital mexanik momenti):

$$L_e = h\sqrt{l(l+1)},$$

bunda: l – orbital kvant soni bo'lib, berilgan n uchun quyidagi qiymatlarni qabul qiladi:

$l=0, 1,...,(n-1)$ hammasi bo'lib n ta qiymat.

Impuls momentining, tashqi magnit maydonining Z yo'nalishidagi proeksiyasi: $L_{lZ} = \hbar m_e$,

bu yerda: m_e –magnit kvant soni bo'lib, berilgan l uchun quyidagi qiymatlarni qabul qiladi: $m_e = 0,\pm1,...,\pm l$ (hammasi bo'lib $(2l+1)$ta qiymat).

Orbital va magnit kvant sonlari uchun tanlash qoidasi:

$$\Delta l = \pm1 \quad \text{va} \quad \Delta m_e = 0,\pm1.$$

Vodorod atomidagi elektronning 1 S holatiga (asosiy holatiga) to'g'ri keluvchi normallashtirilgan to'lqin funksiyasi:

$$\Psi_{100}(r)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a},$$

bunda: $a=4\pi\varepsilon_0\hbar^2/(me^2)$ — birinchi bor radiusi bilan mos keluvchi kattalik.

Vodorod atomidagi 1s holatda bo'lgan elektronni r dan $r+dr$ gacha intervalda bo'lish ehtimoli:

$$dW=|\Psi_{100}|^2dV=|\Psi_{100}|^2\cdot 4\pi r^2dr.$$

Elektronning spini (impulsning xususiy mexanik momenti):

$$L_S=\hbar\sqrt{S(S+1)},$$

bunda: S — spin kvant soni $\left(S=\frac{1}{2}\right)$.

Spin momentining, tashqi magnit maydonining Z yo'nalishiga proeksiyasi:

$$L_{SZ}=\hbar m_s.$$

Bunda m_S —magnit spin kvant soni $\left(m_S=\pm\frac{1}{2}\right)$.

Pauli prinsipi: $Z(n,l,m_e,m_S)=0$ yoki 1,

bunda: $Z(n,l,m_e,m_S)$ — quyidagi to'rtta kvant soni bilan xarakterlanuvchi kvant holatida bo'lgan elektronlar soni; n —bosh, l —orbital, m_e —magnit, m_S — spin magnit kvant sonlari.

Berilgan n bosh kvant soni bilan aniqlanuvchi holatda bo'ladigan elektronlarning maksimal soni: $Z(n)=\sum_{l=0}^{n-1}2(2l+1)=2n^2$.

Masala yechishga misollar

1-misol. Vodorodning $n=1$ va $n=2$ lar uchun ionlashtiruvchi energiyalarining qiymatlari aniqlansin.

Berilgan:	**Yechish:** Vodorodning ionlashuv energiyasi ($n=1$):
$n=1$,	$$E_i=-E_1=\frac{me^4}{8h^2\varepsilon_0^2},\qquad(1)$$
$n=2$.	
$E_1=?$	n =2 uchun ionlashuv energiyasi:
$E_2=?$	

$$E_i=-E_2=\frac{me^4}{32h^2\varepsilon_0^2}.\qquad(2)$$

(1) va (2) larga kattaliklarning qiymatlarini qo‘yib olamiz:

$$E_i=-E_1=\frac{9{,}11\cdot10^{-31}\cdot(1{,}6\cdot10^{-19})^4}{8\cdot\left(6{,}63\cdot10^{-34}\right)^2\cdot\left(8{,}85\cdot10^{-12}\right)^2}\text{J}=$$

$$=\frac{9{,}11\cdot6{,}55}{8\cdot43{,}96\cdot78{,}32}\cdot\frac{10^{-31}\cdot10^{-76}}{10^{-68}\cdot10^{-24}}\text{J}=\frac{59{,}67}{275436}10^{-16}\text{J}=13{,}53\text{eV}.$$

$$E_i=-\text{E}_2=\frac{9{,}11\cdot10^{-31}\cdot(1{,}6\cdot10^{-19})^4}{32\cdot\left(6{,}63\cdot10^{-34}\right)^2\cdot\left(8{,}85\cdot10^{-12}\right)^2}\text{J}=$$

$$=\frac{9{,}11\cdot6{,}55}{32\cdot43{,}96\cdot78{,}32}\cdot\frac{10^{-31}\cdot10^{-76}}{10^{-68}\cdot10^{-24}}\text{J}=3{,}4\text{eV}.$$

Javob: $E_1=13{,}53\text{eV}$; $E_2=-3{,}4\text{eV}$.

2-misol. Bosh kvant soni n=4 uchun, orbital kvant soni l va magnit kvant soni m_e larning qabul qilishi mumkin bo‘lgan qiymatlari yozilsin.

Berilgan:	**Yechish:** Orbital kvant soni l berilgan n uchun quyidagi qiymatlarni qabul qiladi:
$n=4$.	$$l=0,1,...,\ (n-1).$$
$l=?$	
$m_e=?$	Demak, $l=0,1,2,3$.

Magnit kvant soni m_e, berilgan n uchun quyidagi qiymatlarni qabul qiladi: $m_e = 0,\pm1,\pm2,...,\pm l.$

Demak, $m_e = 0,\pm1,\pm2,\pm3.$

Javob: $l = 0,1,2,3;$ $m_e = 0,\pm1,\pm2,\pm3.$

3-misol. Vodorod atomida $1s$ – holatda (asosiy holatda) bo'lgan elektronning normallashgan to'lqin funksiyasi:

$$\Psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

ko'rinishga ega. Bunda: r – yadrodan elektrongacha bo'lgan masofa, a – birinchi bor radiusi. Elektronni atomda r=0,05a radiusli sfera ichidan topilish ehtimoli W aniqlansin.

Berilgan:	**Yechish:**
$\Psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$,	
$r = 0{,}05a$.	
$W = ?$	

Yechish: Vodorod atomidagi $1s$ –holatda bo'lgan elektronning ψ – funksiyasi sferosimmetrik (ya'ni faqat r ga bog'liq). Shuning uchun ham, bir xil ehtimollik zichligiga ega hajm elementini r – radiusli va dr qalinlikli hajm sifatida qarash mumkin. Unda $dV = 4\pi r^2 dr$.

Elektronni dV hajm elementida topilish ehtimoli:

$$dW = |\Psi_{100}(r)|^2 \cdot dV = \left|\frac{1}{\sqrt{\pi a^3}} e^{-r/a}\right|^2 \cdot 4\pi r^2 dr. \quad (1)$$

Uni $r_1 = 0$ dan $r_2 = 0{,}05a$ gacha chegarada integrallaymiz:

$$W = \frac{4}{a^3} \int_0^{0{,}05a} r^2 e^{-2r/a} dr. \quad (2)$$

Masalaning shartiga ko'ra r –kichik (r_2=0,05a; a=52,8pm), shuning uchun $e^{-r/a}$ hadni qatorga yoyish mumkin:

$$e^{-r/a} = 1 - \frac{2r}{a} + \frac{1}{21}\left(\frac{2r}{a}\right)^2 - \ldots \qquad (3)$$

Ikkinchi tartibli hadlarni e'tiborga olmay (3) ni (2) ga qo'ysak,

$$W = \frac{4}{a^3}\int_0^{0,05a} r^2\left(1 - \frac{2r}{a}\right)dr = \frac{4}{a^3}\left[\int_0^{0,05a} r^2 dr - \frac{2}{a}\int_0^{0,05a} r^3 dr\right] = \frac{4}{a^3}\left[\frac{r^3}{3}\Big|_0^{0,05a} - \frac{2}{a}\frac{r^4}{4}\Big|_0^{0,05}\right] = 1,54\cdot 10^{-4}.$$

Javob: $W = 1,54\cdot 10^{-4}$.

4-misol. Elektron atomda d – holatda turibdi: 1) elektronning impuls momenti; 2) impuls momentining tashqi magnit maydon yo'nalishidagi proeksiyasining maksimal qiymati aniqlansin.

Berilgan:	Yechish:
d holat.	**Yechish:** Elektronning d holati $l = 2$ orbital kvant soni bilan xarakterlanadi.
1) $L_e = ?$	Elektronning impuls momenti (mexanik
2) $(L_{eZ})_{max} = ?$	orbital momenti) esa quyidagicha aniqlanadi

$$L_e = \hbar\sqrt{l(l+1)}. \qquad (1)$$

Bu yerda $\hbar = 1,05\cdot 10^{-34}$ J $\cdot$ s – Plank doimiysi.

Impuls momentining tashqi magnit maydon Z yo'nalishidagi proeksiyasi:

$$L_{\ell Z} = \hbar\cdot m_e, \qquad (2)$$

bu yerda: $m_e = 0,\pm 1,\ldots,\pm l$. – magnit kvant soni. Mazkur masalada $m_e = 0,\pm 1,\pm 2$.

$$(L_{eZ})_{max} = h(m_e)_{max}. \qquad (3)$$

l va $(m_e)_{max}$ larning qiymatlarini (1) va (3) larga qo'yib quyidagini olamiz:

$$L_\ell = \hbar\sqrt{2(2+1)} = 2,45\hbar; \quad (L_{\ell Z})_{max} = 2\hbar.$$

Javob: $L_\ell = 2,45\hbar$; $(L_{\ell Z})_{max} = 2\hbar$.

Mustaqil yechish uchun masalalar

1. Vodorod atomidagi elektronning *1* s-holatini tavsiflovchi normallashgan to'lqin funksiyasi $\Psi_{100}(r)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$ ko'rinishga ega. Yadro maydonidagi elektron potensial energiyasining o'rtacha qiymati aniqlansin. [$-e^2/4\pi\varepsilon_0 a$.]

2. Vodorod atomidagi elektronning 1 s holatini tavsiflovchi normallashgan to'lqin funksiyasi $\Psi_{100}(r)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$ ko'rinishga ega. Elektronga ta'sir etuvchi Kulon kuchi modulining o'rtacha qiymati aniqlansin. [$e^2/(2pe_0a^2)$.]

3. Elektronning vodorod atomidagi 1 s holatini ifodalovchi normallashgan to'lqin funksiyasi $\Psi_{100}(r)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$ ko'riinishga ega. Elektron bo'lishi mumkin bo'lgan eng katta masofa aniqlansin. [$pe_0p^2/(me^2)$.]

4. Elektronning vodorod atomidagi 1 s-holatini ifodalovchi normallashgan to'lqin funksiyasi $\Psi_{100}(r)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$ ko'rinishga ega. Elektronning $r_1=0,5a$ va $r_2=1,5a$ radiusli $\Delta r=0,01a$ qalinlikli sferik qatlamda bo'lish ehtimolliklarining nisbati aniqlansin. [$W^1/W^2=0,825$.]

5. Atomda: 1) s-holatda; 2) P-holatda turgan elektron orbital harakatining impuls momenti L_e hisoblansin. [1)0; 2)$1,50\cdot10^{-34}$ J $\cdot$ s.]

6. Vodorod atomidagi elektron orbital harakatining impuls momenti $1,83\cdot10^{-34}\,J\cdot s$. Elektronning orbital harakati natijasida vujudga keladigan magnit momenti aniqlansin. [$1,61\cdot10^{-23}$J/Tl.]

7. Elektron impulsining spin momenti va shu momentning tashqi magnit maydonning Z yo'nalishidagi proeksiyasi aniqlansin. [$L_S=0,912\cdot10^{-34}$ J $\cdot$ s; $L_{SZ}=0,528\cdot10^{-34}$ J $\cdot$ s.]

8. *n=3* bosh kvant soniga to'g'ri keluvchi elektron qobig'i to'la. Shu qobiqdagi quyidagi kvant sonlari: 1) $m_S=-1/2$; 2) $m_e=0$; 3) $m_e=-1$, $m_S=1/2$ bir xil bo'lgan elektronlarning soni aniqlansin. [1) 9; 2) 6; 3) 2.]

35-§. Kvant statistikasi elementlari

Asosiy formulalar

E_i energiyali kvant holatida bo'ladigan bozonlarning o'rtacha soni $< n_i >$, Boze-Eynshteyn taqsimoti:

$$< n_i >= \frac{1}{e^{(E-E_F)/(kT)} - 1}$$

ga muvofiq aniqlanadi. Bu yerda: k – Bolsman doimiysi, T – termodinamik harorat, μ – kimyoviy potensial.

E_i energiyali kvant holatida bo'ladigan fermionlarning o'rtacha soni $< n_i >$ esa Fermi-Dirak taqsimoti:

$$< n_i >= \frac{1}{e^{(E-E_F)/(kT)} + 1}$$

bilan aniqlanadi.

Metalldagi erkin elektronlarning energiyalari bo'yicha Fermi-Dirak taqsimoti:

$$< n_i(E) >= \frac{1}{e^{(E-E_F)/(kT)} + 1},$$

bu yerda E_F – Fermi energiyasi.

T =0K da

$$<n(E)>= \begin{cases} 1,\text{agar } E>E_F \text{ bo'lsa,} \\ 0,\text{agar } E>E_F \text{ bo'lsa.} \end{cases}$$

T=0 K da metalldagi Fermi sathi, ya'ni elektron egallashi mumkin bo'lgan eng yuqori sath quyidagicha aniqlanadi

$$E_F = \frac{h^2}{2m}(3\pi^2 n)^{2/3}.$$

Bu yerda: h – Plank doimiysi, t – elektronning massasi, n – erkin elektronlar soni.

$T<< T_D$ da Debayning xarakteristik harorati

$$E_{max} = kT_D = h\omega_0$$

munosabatdan aniqlanadi. ω_0 — Kristall panjaradagi elastik tebranishlarning chegaraviy chastotasi.

(Ma'lumki $T \gg T_D$ da klassik fizika qonunlari o'rinli bo'ladi.)

Masala yechishga misollar

1-misol. Boze-Eynshteyn va Fermi-Dirak taqsimotlaridan foydalanib, Maksvell-Bolsman taqsimoti hosil qilinsin.

Yechish: Ma'lumki Maksvell-Bolsman taqsimoti yuqori haroratlar uchun o'rinlidir. Yuqori haroratlarda Beze-Eynshteyn va Fermi-Dirak taqsimotlaridagi $e^{\mu/kT} = A$ had yetarli darajada kichik bo'ladi. A<<1. bo'lgan holda esa $e^{(E_i-\mu)/(kT)} \gg 1$ shart bajarilib, taqsimotlarni quyidagicha yozish mumkin:

$$< n_i > = \frac{1}{e^{(E_i-\mu)/(kT)}} = e^{-E_i/(kT)} \cdot e^{\mu/kT} = Ae^{-E_i/(kT)}.$$

Demak,

$$< n_i > = Ae^{-E_i/(kT)}$$

klassik fizikadagi Maksvell-Bolsman taqsimotidir.

2-misol. 1) Fermi-Dirak; 2) Maksvell-Bolsman statistikalaridan foydalanib, $E - E_F \ll kT$ shartni qanoatlantiruvchi E energetik sath uchun elektronning taqsimot funksiyalari aniqlansin.

Berilgan:	**Yechish:** 1) E energetik sathdagi
$E - E_F \ll kT$.	elektronlar uchun Fermi-Dirak taqsimoti:
1) $(n)E_{FD} = ?$	$n(E) = \dfrac{1}{e^{(E-E_F)/(kT)} + 1}$ (1)
2) $(n)E_{MB} = ?$	ko'rinishga ega. Agar shartga ko'ra

$E - E_F \ll kT$, yoki $\dfrac{E - E_F}{kT} \ll 1$ ekanligidan,

bu nisbat nolga intilishini, ya'ni

$$\frac{E-E_F}{kT} \to 0. \qquad (2)$$

ligini hosil qilamiz. Ma'lumki, $e^0 = 1$. Unda (1) quyidagiga teng bo'ladi:

$$n(E)=\frac{1}{1+1}=\frac{1}{2}.$$

2) $E-$ energetik sathdagi elektronlar uchun Maksvell-Bolsman taqsimoti:

$$n(E)=e^{-(E-E_F)/(kT)}. \qquad (3)$$

ko'rinishga ega. (2) shart bajarilganda

$$n(E)=e^0=e^0=1$$

ni hosil qilamiz.

Javob: 1) $n(E)=\frac{1}{2}$; 2) $n(E)=1$.

3-misol. Fermi sathi $E_F = 6\text{eV}$ bo'lganda, 0 K haroratda metalldagi erkin elektronlar konsentratsiyasi aniqlansin.

Berilgan:

$E_F = 6\text{eV}$;

$T = 0\text{K}$.

$n = ?$

Yechish: $T = 0\text{K}$ da metalldagi Fermi sathi quyidagicha aniqlanadi

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}. \qquad (1)$$

bu yerda: $h = 6{,}63 \cdot 10^{-34}$ J · s – Plank doimiysi, $m = 9{,}1 \cdot 10^{-31}\text{kg}$ – elektronning massasi. (1) ifodadan elektronlar konsentratsiyasi n ni aniqlasak

$$n = \frac{(2m \cdot E_F)^{3/2}}{3\pi^2 \cdot h^3}. \qquad (2)$$

Kattaliklarning son qiymatlarini o'rniga qo'yib hisoblaymiz:

$$n = \frac{(2 \cdot 9{,}1 \cdot 10^{-31} \cdot 6 \cdot 1{,}6 \cdot 10^{-19})^{3/2}}{3 \cdot (3{,}14)^2 (6{,}63 \cdot 10^{-34})^3} \frac{1}{m^3} = 27{,}2 \cdot 10^{25}\, m^{-3}.$$

Javob: $n = 27{,}2 \cdot 10^{25}\, m^{-3}$.

4-misol. Krisstaldagi har bir alyuminiy atomiga uchtadan erkin elektron to‘g‘ri keladi deb hisoblab, T=0K da elektron ega bo‘lishi mumkin bo‘lgan eng katta energiya aniqlansin.

Berilgan:
T =0K;
$n = 3n_{Al}$;
$M = 27 \cdot 10^{-3}\, kg/mol.$

$E_{max} = ?$

Yechish: T=0K da elektronning maksimal energiyasi Fermi energiyasi, ya’ni elektron egallashi mumkin bo‘lgan eng yuqori sath energiyasi bilan aniqlanadi:

$$E_{max} = E_F = \frac{h^2}{2m}(3\pi^2 \cdot n)^{2/3}. \quad (1)$$

Elektronlarning konsentratsiyasini esa quyidagicha aniqlaymiz:

$$n = 3n_{Al} = 3\frac{\rho \cdot N_A}{m}. \quad (2)$$

Bu yerda: $\rho = 2{,}7 \cdot 10^{-3}\, kg/m^3$ – alyuminiyning zichligi,

$N_A = 6{,}02 \cdot 10^{23}\, mol^{-1}$ – Avogadro soni.

(2) ni (1)ga qo‘ysak,

$$E_{max} = \frac{h^2}{2m}\left(\frac{9\pi^2 \rho \cdot N_A}{M}\right)^{2/3}. \quad (3)$$

$$[E] = \frac{[h]^2}{[m]}\left(\frac{[\rho][N_A]}{[M]}\right)^{2/3} = \frac{J^2 s^2}{kg}\left(\frac{kg/m^3 \cdot mol^{-1}}{kg/mol}\right)^{2/3} = \frac{J^2 \cdot s^2}{kg \cdot m^2} = \frac{J^2 s^2}{kg \cdot m^2} = J$$

Kattaliklarning son qiymatlarini qo‘yib topamiz. $h = 6{,}63 \cdot 10^{-34}$ J · s -Plank doimiysi):

$$E_{max} = \frac{(6{,}63 \cdot 10^{-34})^2}{2 \cdot 9{,}1 \cdot 10^{-31}} \cdot \left(\frac{9 \cdot (3{,}14)^2 \cdot 2{,}7 \cdot 10^3 \cdot 6{,}02 \cdot 10^{23}}{27 \cdot 10^{-3}} \right)^{2/3} \text{J} =$$

$$= 0{,}65 \cdot 10^{-17} \text{J} = 34{,}21 \text{eV}$$

Javob: $E_{max} = 34{,}21 \text{eV}$.

5-misol. Debay xarakteristik temperaturasi 180 K bo‘lgan oltin kristalida vujudga keladigan fononning maksimal energiyasi aniqlansin.

Berilgan:

$T_D = 180\text{K}$;

$T << T_D$.

$E_{max} = ?$

$\lambda = ?$

Yechish: $T << T_D$ bo‘lganda fononning maksimal energiyasi:

$$E_{max} = k \cdot T_D, \qquad (1)$$

ifodadan aniqlanadi. Bu yerda $k = 1{,}38 \cdot 10^{-23}$ J/K – Bolsman doimiysı.

Shu bilan birga $E_{max} = h \cdot \nu = h\frac{c}{\lambda}$, (2)

ligidan $\lambda = \frac{hc}{E_{max}}$ (3)

$h = 6{,}63 \cdot 10^{-34}$ J · s – Plank doimiysi, $c = 3 \cdot 10^8$ m/s.

Kattaliklarning qiymatlarini (2) va (3) larga qo‘yib topamiz.

$$E_{max} = 1{,}38 \cdot 10^{-23} \cdot 180 \text{J} = 2{,}48 \cdot 10^{-21} \text{J} = 15{,}5 \text{MeV}.$$

$$\lambda = \frac{6{,}63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{2{,}48 \cdot 10^{-21}} \text{m} = 80{,}1 \cdot 10^{-6} \text{m} = 80{,}1 \text{mkm}.$$

Javob: $E_{max} = 15{,}5 MeV$; $\lambda = 80{,}1 mkm$.

Mustaqil yechish uchun masalalar

1. Qanday shartlar bajarilganda metaldagi elektronlar uchun Maksvell-Bolsman taqsimotidan foydalanish mumkin? Fermi-Dirak taqsimotidan foydalanib Maksvell-Bolsman taqsimotini hosil qiling.

2. Agar litiy va seziydagi Fermi sathlari mos ravishda $E_{F,1}$=4,72eV, $E_{F,2}$=1,53 eV larga tengligi ma'lum bo'lsa, 0 K da ulardagi erkin elektronlar konsentratsiyasining nisbati n_1/n_2 aniqlansin. [0,9.]

3. 0 K da natriyning bitta atomiga to'g'ri keluvchi erkin elektronlarning soni aniqlansin. Natriy uchun Fermi sathi E_F=3,12 eV. Natriyning zichligi 970 kg/m^3. [3 ta.]

4. Agar Fermi sathlari mos ravishda 11.7 eV va $E_{F,2}$=7,0 eV bo'lsa, 0 K da alyuminiyning bitta atomiga to'g'ri keluvchi erkin elektronlar soni misdagiga nisbatan necha marta kop? [3 marta.]

5. Agar Fermi sathi E_f=7eV bo'lsa, 0 K da metaldagi elektronlarning o'rtacha kinetik energiyasi aniqlansin. [4,2 eV.]

6. Agar Fermi sathi E_f=5 eV bo'lsa, 0 K da metaldagi elektronlarning maksimal tezligi aniqlansin. [1,32 Mm/s.]

7. Debay xarakteristik temperaturasi 320 K bo'lgan NaCl kristalida vujudga keladigan fononning maksimal energiyasi elektron-voltlarda aniqlansin. Shunday energiyali foton qanday to'lqin uzunligiga ega bo'lishi mumkin? [0,028 eV; 45 mkm.]

36-§. Qattiq jismlar fizikasi elementlari

Asosiy formulalar

Yarim o'tkazgichlarning xususiy solishtirma o'tkazuvchanligi (Zonalar nazariyasiga muvofiq):

$$\gamma = \gamma_0 e^{-\Delta E/(2kT)},$$

bu yerda: γ_0 – mazkur yarim o'tkazgichni xarakterlovchi doimiylik, ΔE – ma'n qilingan zonaning kengligi, k – Bolsman doimiysi, T– termodinamik temperatura.

Yarim o'tkazgichlar uchun Fermi sathi:

$$E_F = \Delta E/2.$$

Shuningdek, yarim o'tkazgichlarning xususiy solishtirma o'tkazuvchanligi quyidagicha ham aniqlanadi:

$$\gamma = e \cdot n(b_n + b_p),$$

bunda: e – elektronning zaryadi; n – zaryad tashuvchilarning konsentratsiyasi; b_n va b_p elektronlar va teshiklarning harakatchanligi.

Xoll effekti natijasida namunaning qirralarida vujudga keladigan kuchlanish:

$$U_H = R_H \cdot B_j l,$$

bunda: R_H – Xoll doimiysi, B – magnit maydon induksiyasi, l – namunaning uzunligi, j – tok zichligi.

Faqat bir xil ko'rinishdagi (n yoki p) zaryad tashuvchilarga ega bo'lgan yoqut, kremniy, germaniy kabi yarim o'tkazgichlar uchun Xoll doimiysi:

$$R_H = \frac{3\pi}{8} \cdot \frac{1}{e \cdot n},$$

n – zaryad tashuvchilarning konsentratsiyasi.

Masala yechishga misollar

1-misol. Germaniyning temperaturasi 273 K dan 290 K gacha orttirildi. Germaniyning ma'n qilish zonasining kengligi $\Delta E = 0{,}72 eV$ bo'lsa, uning solishtirma o'tkazuvchanligi necha marta ortadi?

Berilgan:
T_1=273 K;
T_2=290K;
ΔE=0,72 eV.

$\frac{\gamma_2}{\gamma_1} = ?$

Yechish: Germaniyning xususiy solishtirma o‘tkazuvchanligini

$$\gamma = \gamma_0 e^{-\Delta E/(2kT)}$$

ifoda yordamida aniqlaymiz.

Unda T_1 va T_2 temperaturalar uchun:

$$\gamma_1 = \gamma_0 e^{-\Delta E/(2kT_1)},$$

$$\gamma_2 = \gamma_0 e^{-\Delta E/(2kT_2)}.$$

So‘ralgan nisbatni aniqlasak,

$$\frac{\gamma_2}{\gamma_1} = \frac{\gamma_0 e^{-\Delta E/(2kT_1)}}{\gamma_0 e^{-\Delta E/(2kT_2)}} = \exp\left[\frac{\Delta E}{2k}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right].$$

Berilganlar yordamida topamiz:

$$\frac{\gamma_2}{\gamma_1} = \exp\left[\frac{0{,}72 \cdot 1{,}6 \cdot 10^{-19}}{2 \cdot 1{,}38 \cdot 10^{-23}}\left(\frac{1}{290} - \frac{1}{273}\right)\right] =$$

$$= \exp\left[\frac{1{,}15}{2{,}76}(3{,}44 - 3{,}66) \cdot 10\right] = \exp[-0{,}92].$$

$$\frac{\gamma_2}{\gamma_1} = e^{-0{,}92} = \frac{1}{e^{-0{,}92}} = 0{,}37.$$

Javob: $\frac{\gamma_2}{\gamma_1} = 0{,}37$.

2-misol. Kremniyning elektronlarining va teshiklarining harakatchanligi mos ravishda $b_n = 0{,}15\,\text{m}^2/(\text{V} \cdot \text{s})$ va $b_p = 5 \cdot 10^{-2}\,\text{m}^2/(\text{V} \cdot \text{s})$ ga teng. Agar kremniyning solishtirma qarshiligi $6{,}2 \cdot 10^2\,\Omega \cdot \text{m}$ bo‘lsa, u uchun Xoll doimiysini toping.

Berilgan:

$b_n = 0{,}15\,\mathrm{m^2/(V\cdot s)}$;

$b_p = 5\cdot 10^{-2}\,\mathrm{m^2/(V\cdot s)}$;

$\rho = 6{,}2\cdot 10^2\,\Omega\cdot\mathrm{m}$.

$R_n = ?$

Yechish: Kremniy uchun Xoll doimiysini

$$R_n = \frac{3\pi}{8}\frac{1}{en} \qquad (1)$$

ifodadan aniqlaymiz. Bu yerda n – zaryad tashuvchilarning konsentratsiyasi. Uni aniqlash uchun yarim o'tkazgichlarning xususiy solishtirma o'tkazuvchanligi ifodasini yozib olamiz:

$$\gamma = e\cdot n(b_n + b_p). \qquad (2)$$

(2) dan $e\cdot n$ ifodani topsak va $\gamma = \dfrac{1}{\rho}$ ligini e'tiborga olsak,

$$e\cdot n = \frac{\gamma}{b_n + b_p} = \frac{1}{\rho(b_n + b_p)}. \qquad (3)$$

(3) ni (1) ga qo'yib quyidagini hosil qilamiz:

$$R_H = \frac{3\pi}{8}\rho(b_n + b_p), \qquad (4)$$

$$[R_H] = [\rho][b] = \Omega\cdot\mathrm{m\,m^2/(V\cdot s)} = \frac{\mathrm{m^3}}{\mathrm{A\cdot s}} = \frac{\mathrm{m^3}}{\mathrm{C}}.$$

Berilganlar yordamida quyidagini hosil qilamiz:

$$R_H = \frac{3\cdot 3{,}14}{8}\cdot 6{,}2\cdot 10^2(0{,}15+0{,}05)\frac{\mathrm{m^3}}{\mathrm{C}} = \frac{3\cdot 3{,}14}{8}\cdot 124\frac{\mathrm{m^3}}{\mathrm{C}} = 146\frac{\mathrm{m^3}}{\mathrm{C}}.$$

Javob: $R_H = 146\dfrac{\mathrm{m^3}}{\mathrm{C}}$.

***3*-misol.** Magnit maydon induksiyasi 0,1 Tl bo'lgan bir jinsli magnit maydon induksiya chiziqlariga tik ravishda n –tipidagi germaniy plastinkasi joylashtirilgan. Plastinkaning bo'yi l=10 sm, eni b=6 mm. Plastinkaning uchlariga 250 V kuchlanish qo'yilganda 8,8 mV Xoll potensiallar ayirmasi vujudga keladi. 1) Xoll doimiysi R_H va, 2) zaryad tashuvchilarning konsentratsiyasi N_n aniqlansin. Germaniyning solishtirma o'tkazuvchanligi $\gamma = 80\ ^{sm}/_{m}$.

Berilgan:

$B = 0{,}1Tl$;

$l = 10sm = 0{,}1m$;

$b = 6mm = 6\cdot10^{-3}m$;

$U = 250V$;

$U_H = 8{,}8mV = 8{,}8\cdot10^{-3}V$;

$\gamma = 80\ sm/m$.

---

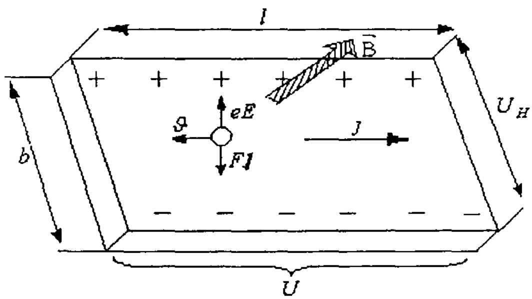
1) $R_n = ?$

2) $N_n = ?$

Yechish: 1) magnit maydonida joylashtirilgan yarim o'tgazgichga U potensiallar farqi qo'yilgan (60-rasm). Zaryad tashuvchi zarralar – elektronlar ko'ndalang yo'nalishda og'adi.

Lorens kuchi ta'sirida ro'y beradigan bunday og'ish natijasida plastinkaning yon sirtida zaryaldlarning "to'planishi" vujudga keladi. Buning natijasida hosil bo'ladigan Xoll potensiallar farqi U_H esa Lorens kuchini muvozanatlantiradi. Bu potensial esa quyidagi ifoda bilan aniqlanadi:

$$U_H = R_H Bjb. \quad (1)$$



60-rasm

Bundan Xoll doimiysini topsak,

$$R_H = \frac{U_H}{Bjb}. \quad (2)$$

Tok zichligi j ni Om qonunining differensial ko'rinishidan topamiz:

$$j = \gamma E, \qquad (3)$$

bunda E – yarim o'tkazgichdagi elektr maydon kuchlanganligi bo'lib, $E = \frac{U}{l}$ kabi aniqlanadi.

Unda
$$j = \gamma \frac{U}{l}, \qquad (4)$$

va uni (2) ga qo'ysak,

$$R_H = \frac{U_H \cdot l}{\gamma \cdot bUB}. \qquad (5)$$

(5) dan Xoll doimiysining birligi $\left(m^3/C\right)$ chiqishini tekshirib ko'ramiz:

$$[R_H] = \frac{[U_H]\cdot[l]}{[\gamma][b][U][B]} = \frac{1V \cdot 1m}{1\frac{sm}{m} \cdot 1m \cdot 1V \cdot 1Tl} = 1\frac{m}{Tl \cdot sm} = 1\frac{A \cdot m^2 \cdot V}{N \cdot A} = 1\frac{J \cdot m^2}{N \cdot C} = 1\,m^3/C.$$

Berilganlar yordamida topamiz:

$$R_H = \frac{8{,}8 \cdot 10^3 \cdot 0{,}1}{0{,}1 \cdot 250 \cdot 80 \cdot 6 \cdot 10^{-3}}\, m^3/C = 7{,}33 \cdot 10^{-5}\, m^3/C.$$

2) Germaniyning p-tip o'tkazuvchanligi uchun quyidagi munosabatni yozamiz:

$$R_H = \frac{3\pi}{8}\frac{1}{e \cdot n}. \qquad (6)$$

Bundan

$$n = \frac{3\pi}{8R_H \cdot e}. \qquad (7)$$

bu yerda: $e = 1{,}6 \cdot 10^{-19}\, C$ elementar zaryadi

$$n = \frac{3 \cdot 3{,}14}{8 \cdot 7{,}33 \cdot 10^{-5} \cdot 1{,}6 \cdot 10^{-19}} \frac{1}{m^3} = 10^{23} \frac{1}{m^3}.$$

Javob: 1) $R_H = 7{,}33 \cdot 10^{-5}\, m^3/C$; 2) $n = 10^{23} \frac{1}{m^3}$.

Mustaqil yechish uchun masalalar

1. 273K dan 301K gacha qizdirilgan kremniyning solishtirma o‘tkazuvchanligi 4,24 marta ortdi. Kremniy ma’n qilingan zonasining kengligi aniqlansin. [1,1 eV.]

2. Muayyan haroratda xususiy elektr o‘tkazuvchanlikli germaniyning solishtirma qarshiligi $0{,}48\Omega\cdot m$ ga teng. Agar elektronlarning va teshiklarning elektr o‘tkazuvchanligi mos ravishda $b_n = 0{,}36\,m^2/(V\cdot s)$ va $b_p = 0{,}16\,m^2/(V\cdot s)$ bo‘lsa, zaryad tashuvchilarning konsentratsiyasi aniqlansin. $[2{,}5\cdot 10^{19}m^{-3}.]$

3. Aralashmali kremniyning solishtirma o‘tkazuvchanligi 112 sm/m. Agar Xoll doimiysi $R_H = 3{,}66\cdot 10^{-4}\,C$ bo‘lsa, faqat teshikli o‘tkazuvchanlik mavjud deb hisoblab, teshiklarning harakatchanligi va ularning konsentratsiyasi aniqlansin. $\left[2{,}79\cdot 10^{-6}\,m^2/(V\cdot s);2\cdot 10^{22}\,m^{-3}.\right]$

4. Uzunligi 10 sm, kengligi 1sm bo‘lgan taxtacha ko‘rinishidagi yarim o‘tkazgich, 0,2 Tl induksiyali bir jinsli magnit maydoniga joylashtirilgan. Magnit induksiya vektori taxtacha tekisligiga tik yo‘nalgan va uchlariga (l yo‘nalish bo‘yicha) 300 V doimiy kuchlanish qo‘yilgan. Agar Xoll doimiysi $R_H = 0{,}1\,m^3/C$, yarim o‘tkazgichning solishtirma qarshiligi $0{,}5\Omega\cdot m$ bo‘lsa, taxtachaning yon tomonlaridagi Xoll potensiallar farqi aniqlansin. $[1{,}2_m V.]$

5. Kengligi 2sm bo‘lgan yupqa kremniy taxtacha induksiya chiziqlariga tik ravishda bir jinsli magnit maydoniga (B=0,5Tl) joylashtirilgan. Taxtacha bo‘ylab yo‘nalgan $j = 2mk\,A/mm^2$ tok zichligida Xoll potensiallar farqi 2,8V bo‘lsa, zaryad tashuvchilarning konsentratsiyasi aniqlansin. $[5{,}25\cdot 10^{10}m^{-3}.]$

VIII BOB. ATOM YADROSI VA ELEMENTAR ZARRALAR FIZIKASI

37-§. Atom yadrosining tuzilishi. Yadro bog'lanish energiyasi. Yadro reaksiyalari. Elementar zarralar fizikasi

Asosiy formulalar

Yadroning radiusi

$$R = R_0 A^{1/3}$$

munosabat bilan aniqlanadi. Bunda: $R_0 = 1{,}4 \cdot 10^{-15}$ m – proporsionallik koeffitsienti. A – massa soni (yadrodagi nuklonlar soni).

Yadroning massa deffekti:

$$\Delta m = [Z \cdot m_p + (A - Z)m_n] - m_{Ya} = Zm_p + Nm_n - m_{Ya}.$$

bu yerda: Z –yadrodagi protonlar, N –neytronlar soni, m_p m_n lar mos ravishda proton va neytronlarning massalari.

Yadro bog'lanish energiyasi:

$$E_{bog'} = [Zm_p + (A - Z)m_n - m_{Ya}]c^2 = [Z \cdot m_p + Nm_n - m_{Ya}]c^2.$$

Solishtirma bog'lanish energiyasi (bitta nuklonga to'g'ri keluvchi bog'lanish energiyasi):

$$E_{sol} = E_{bog'}/A.$$

Yadro reaksiyalari ramziy ravishda:
yoyiq

$${}_Z^A X + a \rightarrow {}_Z^A Y + b,$$

yoki qisqartirilgan

$${}_Z^A X(a,b){}_Z^A Y$$

ko'rinishlarda yozilishi mumkin.

Yadro reaksiyalari energiyalari:

$$Q = c^2[(m_1 + m_2) - (m_3 + m_4)],$$

bu yerda: m_1 va m_2 lar yadro nishon va yo'naltirilgan zarraning tinch holatdagi massalari, m_3 va m_4 lar esa reaksiya mahsullarining massalari.

Yadro reaksiyalarining energiyasi quyidagi ko'rinishda ham tasvirlanishi mumkin:

$$Q = (T_1 + T_2) - (T_3 + T_4)],$$

bu yerda T_1, T_2, T_3 va T_4 lar mos ravishda nishon yadro, yo'naltirilgan zarra, hosil bo'lgan zarra va mahsul yadroning kinetik energiyalari.

Uzluksiz zanjir reaksiyasining tezligi

$$\frac{dN}{dt} = \frac{N(k-1)}{T},$$

yoki boshqacha

$$N = N_0 e^{(k-1)t/T}.$$

Bu yerda: N_0 – boshlang'ich holatdagi neytronlar soni, $N - t$ vaqtdagi neytronlar soni, T – bir avlodning o'rtacha davri, k – neytronlarning ko'payish koeffitsienti.

Siljish qoidasi:

α -yemirilish uchun

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He;$$

β^- -yemirilish uchun

$$^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + ^{0}_{-1}e;$$

β^+ -yemirilish uchun

$$^{A}_{Z}X \rightarrow ^{A}_{Z-1}Y + ^{0}_{+1}e.$$

Masala yechishga misollar

1-misol. Neytral atom $^{12}_{6}$S massasining($m = 19{,}9272 \cdot 10^{-27}$ kg) qancha qismini elektron qobig'ining massasi tashkil qilishi aniqlansin.

Berilgan:

$m = 19{,}9272 \cdot 10^{-27}$ kg

$\frac{m'_E}{m} = ?$

Yechish: So'ralgan nisbat quyidagi munosabatdan aniqlanadi:

$$\frac{m'_E}{m},$$

Bu yerda $m_E - {}^{12}_{E}$ S atomidagi elektronlarning massalari, ya'ni $m'_E = 6m_e$. Demak,

$$\frac{m'_E}{m} = \frac{6 \cdot 9{,}11 \cdot 10^{-31}\,\text{kg}}{19{,}9272 \cdot 10^{-27}\,\text{kg}} = 2{,}74 \cdot 10^{-4}.$$

Javob: $2{,}74 \cdot 10^{-4}$.

2-misol. Xlor, nisbiy atom massalari $A_{r_1} = 34{,}469$ va $A_{r_2} = 36{,}966$ bo'lgan ikkita izotopning aralashmasidan iborat. Agar ularning massaviy ulushlari mos ravishda $\omega_1 = 0{,}754$ va $\omega_2 = 0{,}246$ bo'lsa, xlorning nisbiy atom massasi A_2 hisoblansin.

Berilgan:

$A_{r_1} = 34{,}469$;

$A_{r_2} = 36{,}966$;

$\omega_1 = 0{,}754$;

$\omega_2 = 0{,}246$.

$A_2 = ?$

Yechish: Izotopdan tashkil topgan elementning nisbiy atom massasi quyidagi ifoda yordamida aniqlanadi:

$$A_2 = \frac{A_{r_1} \cdot A_{r_2}}{\omega_1 A_{r_2} + \omega_2 A_{r_1}}.$$

Ushbu ifoda va berilganlar yordamida xlorning nisbiy atom massasini topamiz:

$$A_2 = \frac{34{,}469 \cdot 36{,}968}{0{,}754 \cdot 36{,}968 + 0{,}246 \cdot 34{,}469} = \frac{1274{,}250}{27{,}874 + 8{,}480} = \frac{1274{,}250}{36{,}353} = 35{,}052$$

Javob: $A_2 = 35{,}052$.

3-misol. ${}^{48}_{20}\text{Ca}$ yadrosi uchun massa defekti, bog'lanish energiyasi va solishtirma bog'lanish energiyasi hisoblansin.

Berilgan:	**Yechish:** Atom yadrosining massa defekti quyidagi ifoda bilan aniqlanadi:
${}^{48}_{20}Ca$.	$\Delta m = Z \cdot m_p + N \cdot m_n - m_я$.
$\Delta m = ?$	${}^{48}_{20}Ca$ yadrosi uchun Z=20, N=20, A=40.
$E_{bog'} = ?$	Proton va neytronlar massasi esa mos ravishda
$E_{sol} = ?$	

$m_p = 1{,}67 \cdot 10^{-27}$ kg, $m_n = 1{,}68 \cdot 10^{-27}$ kg, $m_я = 66{,}53 \cdot 10^{-27}$ kg.

Yadroning bog'lanish energiyasi: $E_{bog'} = \Delta mc^2$.

Solishtirma bog'lanish energiyasi: $E_{sol} = \dfrac{E_{bog'}}{A}$.

Kattaliklarning qiymatlarini yuqoridagi ifodalarga qo'yamiz:

$$\Delta m = \left[20 \cdot 1{,}67 \cdot 10^{-27} + 20 \cdot 1{,}68 \cdot 10^{-27} - 66{,}53 \cdot 10^{-27}\right]\text{kg} = 0{,}47 \cdot 10^{-27}\,\text{kg};$$

$$E_{bog'} = 0{,}47 \cdot 10^{-27}\,\text{kg} \cdot \left(3 \cdot 10^8 \,\frac{\text{m}}{\text{s}}\right)^2 = 4{,}2 \cdot 10^{-11}\,\text{J} = 262{,}3 \cdot 10^6\,\text{eV} = 262{,}3\,\text{MeV};$$

$$E_{sol} = \frac{262{,}3}{40}\,\text{MeV} = 6{,}56\,\text{MeV}.$$

Javob: $\Delta m = 0{,}47 \cdot 10^{-27}$ kg; $E_{bog'} = 262{,}3$ MeV; $E_{sol} = 6{,}56$ MeV.

4-misol. ${}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0 n$ yadro reaksiyasining energiyasi hisoblansin.

Berilgan:	**Yechish:** $A + B \rightarrow C + D$ Yadro reaksiyasining energiyasi reaksiyagacha va undan keyingi to'la energiyalarning farqidek, ya'ni quyidagi ifoda yordamida aniqlanadi:
${}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0 n$.	
$Q = ?$	

$$Q = (E_A + E_B) - (E_S + E_D). \qquad (1)$$

Agar yadrolarning energiyasi $E=mc^2$ ifoda yordamida aniqlanishini e'tiborga olsak, (1) ni quyidagidek qayta yozishimiz mumkin:

$$Q = c^2[(m_A + m_B) - (m_S + m_D)]. \qquad (2)$$

Bu yerda: $c = 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$ – yorug'likning bo'shliqdagi tezligi.

Mazkur masalada:

$$m_A = m_{\text{H}_1^2} = 3{,}0141\text{a.m.b.}; \quad m_B = m_{\text{H}_1^3} = 3{,}0160\text{a.m.b.}$$

$$m_S = m_{\text{He}_2^4} = 4{,}002\text{a.m.b.}; \quad m_D = m_{n_1^0} = 1{,}0087\text{a.m.b.}$$

$$c^2 = 931{,}4\,\text{MeV}/\text{a.m.b.}$$

Kattaliklarning qiymatlarini (2) ga qo'yib hisoblaymiz:

$$Q = 931{,}4[(2{,}0141 + 3{,}0160) - (4{,}0026 + 1{,}0087)]\,\text{MeV} = 17{,}58\text{eV}.$$

Javob: $Q = 17{,}58$ MeV miqdordagi energiya ajraladi.

5-misol. $^2\text{H}(d,\gamma)^4\text{He}$ reaksiyasining energiyasi hisoblansin.

Berilgan:

$^2\text{H}(d,\gamma)^4\text{He}.$

$Q = ?$

Yechish:

$$_1^2\text{H} + _1^2\text{H} \rightarrow _2^4\text{He} + \gamma$$

reaksiyasi uchun energiyaning saqlanish qonunini yozamiz:

$$c^2 \cdot m_{_1^2\text{H}} + c^2 \cdot m_{_1^2\text{H}} = c^2 \cdot m_{_2^4\text{He}} + T + \varepsilon_F, \qquad (1)$$

bu yerda: ε_Φ – fotonning energiyasi, $T - _2^4$ He ning kinetik energiyasi.

Yadro reaksiyasining energiyasi

$$Q = T + \varepsilon_F = [m_{_1^2\text{H}} + m_{_1^2\text{H}} - m_{_2^4\text{He}}] \cdot c^2 = [2m_{_1^2\text{H}} - 2m_{_2^4\text{He}}]c^2. \quad (2)$$

ga teng bo'ladi. Bu yerda: $m_{_1^2\text{H}} = 2{,}0141\text{a.m.b.}$ – og'ir vodorodning (deytireyning) massasi; $m_{_2^4\text{He}} = 4{,}0026\text{a.m.b.}$ – geliy yadrosining massasi; c – yorug'lik tezligi, $c^2 = 931{,}4\frac{\text{MeV}}{\text{a.m.b.}}$.

Kattaliklarning qiymatlarini (2) ga qo‘yib hisoblaymiz:

$Q = (2 \cdot 2{,}0141 - 4{,}0026) \cdot 931{,}4\text{MeV} = 23{,}8\text{MeV}$.

Javob: $Q = 23{,}8$ MeV energiya ajraladi.

6-misol. $^{238}_{92}$U uchta α va ikkita β^--yemirilishdan keyin qanday elementga aylanadi?

Berilgan:

$^{238}_{92}$U;

3 ta α;

2 ta β^-.

$^{A}_{Z}Y = ?$

Yechish: Reaksiyaning ramziy ko‘rinishi quyidagicha bo‘ladi:

$$^{238}_{92}\text{U} \rightarrow ^{A}_{Z}\text{Y} + 3^{4}_{2}\text{He} + 2 \cdot ^{0}_{-1}e.$$

Massa soni A va zaryad soni Z larning saqlanish qonunlariga muvofiq

$$238 = A + 3 \cdot 4 + 2 \cdot 0; \qquad (1)$$

$$92 = Z + 3 \cdot 2 + 2 \cdot (-1). \qquad (2)$$

(1) va (2) lardan A va Z larni topamiz:

$$A = 238 - 12 = 226;$$

$$Z = 92 - 6 + 2 = 88.$$

Demak, hosil bo‘lgan element uchun $A = 226$, $Z = 88$.

Bu $^{226}_{88}$Ra elementidir.

Javob: $^{226}_{88}$Ra.

7-misol. $^{2}_{1}\text{H} + ^{2}_{1}\text{H} \rightarrow ^{3}_{2}\text{He} + ^{1}_{0}n$ yadro reaksiyasida $\Delta E = 3{,}27\text{MeV}$ miqdoridagi energiya ajraladi. Agar $m_{^2_1\text{H}} = 3{,}34461 \cdot 10^{-27}$ kg bo‘lsa, $^{3}_{2}$He atomining massasi aniqlansin.

Berilgan:

$\Delta E = 3{,}27\text{MeV}$;

$m_{^2_1\text{H}} = 3{,}34461 \cdot 10^{-27}$ kg.

Yechish: Reaksiya uchun energiyaning saqlanish qonuni quyidagi ko‘rinishga ega bo‘ladi:

$m_{^3_2\text{He}} = ?$	$2m_{^2_1\text{H}} \cdot c^2 = m_{^3_2\text{He}} \cdot c^2 + m_n c^2 + \Delta E$. (1)

Bu ifodadan so'ralgan massani topamiz:

$$m_{^3_2\text{He}} = \left(2m_{^2_1\text{H}} - m_n\right) - \frac{\Delta E}{c^2}. \qquad (2)$$

Massalarni a.m.b. da ifodalash hisob-kitoblarni osonlashtiradi:

$$m_{^2_1\text{H}} = 2{,}01410\text{a.m.b.}; \quad m_n = 1{,}00867\text{a.m.b.}$$

$$c^2 = 931{,}4\frac{\text{MeV}}{\text{a.m.b.}},$$

Kattaliklarning qiymatlarini (2) ga qo'ysak,

$$m_{^3_2\text{He}} = \left[(2 \cdot 2{,}01410 - 1{,}00867) - \frac{3{,}27}{931{,}4}\right]\text{a.m.b.} = (3{,}01953 - 0{,}00351)\text{a.m.b.} =$$

$$= 3{,}01603\text{a.m.b.} = 5{,}00841 \cdot 10^{-27}\,\text{kg}.$$

Javob: $m_{^3_2\text{He}} = 3{,}01603\text{a.m.b.} = 5{,}00841 \cdot 10^{-27}\,\text{kg}.$

8-misol. Agar har bir uran $^{235}_{92}\text{U}$ yadrosi emirilganda 200 MeV energiya ajralsa, 1 g uran emirilganda qancha miqdordagi energiya ajraladi?

Berilgan:	**Yechish:** So'ralgan energiya
$E_0 = 200\text{MeV}$;	$E = N \cdot E_0$, (1)
$m = 1\text{г} = 10^{-3}\,\text{kg}$.	ifoda yordamida aniqlanadi. Bu erda N
$E = ?$	berilgan massadagi yadrolar soni. Uni
	$N = \frac{m}{M_\text{U}} N_A$. (2)

ifoda yordamida aniqlashimiz mumkin. Bu erda: M_U – urarning molyar massasi. $M = 235 \cdot 10^{-3}\,\text{kg}/\text{mol}$, $N_A = 6.02 \cdot 10^{23}\,\text{mol}^{-1}$ – Avogadro soni.

(2) ni (1) ga qo'yamiz: $$E=\frac{m\cdot N_A}{M_U}\cdot E_0. \quad (3)$$

$$[E]=\frac{[m]\cdot[N_A]}{[M]}\cdot[E]=\frac{1\text{kg}\cdot 1\text{mol}^{-1}}{1\frac{\text{kg}}{\text{mol}}}1\cdot\text{J}=1\text{J}.$$

Kattaliklarning qiymatlarini (3) ga qo'yib olamiz

$$E=\frac{10^{-3}\cdot 6{,}02\cdot 10^{23}}{235\cdot 10^{-3}}\cdot 200\text{MeV}=5{,}12\cdot 10^{23}\text{MeV}.$$

Javob: $E=5{,}12\cdot 10^{23}\text{MeV}$.

Mustaqil yechish uchun masalalar

1. Xrom $^{52}_{24}Cr$ neytral atomining massasi aniqlansin. $[8{,}64\cdot 10^{-26}\text{kg}]$.

2. Kislorodning uchta izotoni: 1) $^{16}_{8}O$; 2) $^{17}_{8}O$; 3) $^{18}_{8}O$ larning tarkibiga kiruvchi protonlar va neytronlar soni aniqlansin.

3. Bor elementi nisbiy atom massalari A_{r1}=10,013 va A_{r2}=11,009 bo'lgan ikkita izotopning aralashmasidan iborat. Borning nisbiy atom massasi A_r=10,811 bo'lsa, izotoplarning massaviy ulushlari topilsin.[0,186; 0,814].

4. Agar neytral litiy atomining massasi 7,01601 a.m.b. ga teng bo'lsa, litiy yadrosining massasi aniqlansin. [7,01436 a.m.b.]

5. Kobalt atomi hajmining qancha qismini uning yadrosining hajmi tashkil qilishi baholansin. Kobaltning zichligi $4{,}5\cdot 10^3$ kg/m^3. [$3\cdot 10^{-14}$.]

6. Alfa zarraning massa defekta, bog'lanish energiyasi va solishtirma bog'lanish energiyasi aniqlansin. [$0{,}06\cdot 10^{-27}$ kg; 33,7 MeV; 8,42 MeV.]

7. $^3H(P,\gamma)^4He$ yadro reaksiyasining energiyasi aniqlansin. [19,4MeV.]

8. Uchta proton va to'rtta neytrondan iborat yadroning boglanish energiyasi 39,3 MeV ga teng. Shunday yadroga ega bo'lgan neytral atomning massasi aniqlansin. [$1{,}165\cdot 10^{-26}$ kg.]

9. $^{23}_{12}Mg\rightarrow^{23}_{11}Na+^{0}_{1}e+^{0}_{0}\gamma$ reaksiyasi natijasida ajraladigan energiya aniqlansin. Magniy va natriy neytral atomlarining massalari mos ravishda $3{,}8184\cdot 10^{-26}$ kg va $3{,}8177\cdot 10^{-26}$ kg ga teng. [2,9 MeV.]

10. ${}^{14}_{7}N+{}^{4}_{2}He\rightarrow{}^{1}_{1}H+{}^{17}_{8}O$ reaksiyasi natijasida energiya ajralishi yoki yutilishi aniqlansin.

$m_{{}^{14}_{7}H}=2{,}3253\cdot10^{-26}\,kg,\ m_{{}^{4}_{2}He}=6{,}6467\cdot10^{-27}\,kg,$

$m_{{}^{1}_{1}H}=1{,}6737\cdot10^{-27}\,kg,\ m_{{}^{17}_{8}O}=2{,}8229\cdot10^{-27}\,kg.$

11. ${}^{2}_{1}H+{}^{2}_{1}H\rightarrow{}^{3}_{2}He+{}^{1}_{0}n$ yadro reaksiyasi natijasida 3,27 MeV energiya ajraladi. Agar $m_{{}^{2}_{1}H}=3{,}34461\cdot10^{-27}\,kg$ bo'lsa, ${}^{3}_{2}He$ atomining massasi aniqlansin. [$5{,}00841\cdot10^{-27}$ kg.]

12. Tinch holatdagi ${}^{220}_{86}Rn$ radon yadrosi $16\cdot10^{6}$ m/s tezlik bilan a - zarra chiqardi. Qanday yadro hosil bo'lishi aniqlansin. Tepki natijasida u qanday tezlik olgan? [${}^{216}_{84}Po$, $296\,km/s$]

13. ${}^{210}_{81}Tl$ talliy yadrosi ${}^{206}_{82}Pb$ – qo'rg'oshin yadrosiga aylanganda nechta α va β – zarralar ajraladi (Bitta α – zarra, uchta β – zarralar)

14. Uran ${}^{235}_{92}U$ yadrosi issiq neytron yutganida ikkita bo'linish parchasi va ikkita neytron hosil bo'ladi. Agar bu bo'laklarning biri stronsiy ${}^{95}_{38}Sr$ yadrosi bo'lsa, ikkinchi bo'lakning tartib nomeri Z va massa soni A aniqlansin. [54; 139.]

15. Issiq neytronlar bilan ishlovchi yadro reaktorida bir avlod neytronlarning o'rtacha yashash vaqti T=90 ms ga teng. Neytronlarning ko'payish koeffitsienti $k\approx1{,}002$ deb hisoblab, reaktor davri, ya'ni reaktordagi issiqlik neytronlari oqimi e marta ortish vaqti aniqlansin. [45 s.]

16. Nima uchun termoyadro reaksiyasi ro'y berishi uchun juda yuqori harorat zarur?

38-§. Radioaktivlik. Dozimetriya elementlari

Asosiy formulalar

Radioaktiv yemirilishning asosiy qonuni:

$$N = N_0 e^{-\lambda t},$$

bunda: N — vaqtning t momentida yemirilmagan yadrolar soni, N_0—$t=0$ momentda yemirilmagan yadrolar soni, λ — radioaktiv yemirilish doimiysi.

t — vaqtda yemirilgan yadrolar soni:

$$\Delta N = N_0 - N = N_0\left(1 - e^{-\lambda t}\right).$$

Yarim yemirilish davri $T_{1/2}$ — yemirilish doimiysi bilan quyidagicha bog'langan:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.$$

Radioaktiv yadroning o'rtacha yashash vaqti τ — yemirilish doimiysiga teskari kattalikdir:

$$\tau = \frac{1}{\lambda}.$$

Radioaktiv izotopdagi atomlar soni:

$$N = \frac{m}{M} N_A.$$

Radioaktiv manbadagi nuklidning faolligi:

$$A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}.$$

A_0 — nuklidning boshlangich momentdagi ($t=0$) faolligi.

Moddadan o'tganda radioaktiv nurlanish intensivligining susayishi

$$I = I_0 e^{-\mu x},$$

bunda: I_0 — moddaga tushayotgan intensivlik, x — moddaning qalinligi, μ — susayishning chiziqli koeffitsienti.

Yarim susaytirish qatlami, ya'ni nurlanish intensivligini ikki marta kamaytiradigan qatlam:

$$x_{1/2} = \frac{\ln 2}{\mu} = \frac{0{,}693}{\mu}.$$

Nurlanish dozasi (yutilgan nurlanish dozasi):

$$\mathcal{D} = \frac{\Delta W}{\Delta m},$$

bunda: ΔW – moddaga tushayotgan ionlashtiruvchi nurlanish energiyasi, Δm – modda massasi. $[\mathcal{D}] = 1\text{Grey} = 1\,\frac{J}{kg}$.

Nurlanish dozasining quvvati:

$$\mathcal{D} = \frac{\Delta \mathcal{D}}{\Delta t}.$$

$$[\mathcal{D}] = 1\,\frac{Gr}{s}.$$

Ekspozitsion nurlanish dozasi:

$$x = \frac{\Sigma Q}{m}.$$

Bunda: $\Sigma Q - m$ massali havoda hosil bo'lgan bir xil ishorali barcha ionlar elektr zaryadlarining yig'indisi $[x] = 1\frac{\text{C}}{\text{kg}}$.

Ekspozitsion nurlanish dozasining quvvati:

$$\dot{x} = \frac{\Delta x}{\Delta t}.$$

$$[\dot{x}] = 1\,\frac{\text{C}}{\text{kg} \cdot \text{s}} = 1\,\frac{\text{A}}{\text{kg}}.$$

Himoya qatlamidan o'tayotgan nurlanish ekspozitsion dozasining o'zgarishi:

$$x = x_0 e^{-\mu x}.$$

bunda x – himoya qatlamining qalinligi.

Masala yechishga misollar

1-misol. Kobalt $^{60}_{27}Co$ radioaktiv izotopi yadrosining o'rtacha yashash vaqti topilsin.

Berilgan:

$^{60}_{27}Co$;

$T_{1/2} = 5,3\,\text{yil}$.

$\tau = ?$

Yechish: Yadroning o'rtacha yashash vaqti

$$\tau = \frac{1}{\lambda} \qquad (1)$$

ifodasidan aniqlanadi. Yemirilish doimiysi λ esa yarim emirilish davri $T_{1/2}$ bilan quyidagicha bog'langan

$$T_{1/2} = \frac{\ln 2}{\lambda}. \qquad (2)$$

Bundan λ ni topsak va natijani (1) ga qo'ysak, quyidagi hosil bo'ladi:

$$\lambda = \frac{T_{1/2}}{\ln 2}.$$

Kattaliklarning son qiymatlarini qo'yamiz:

$$\tau = \frac{5,3\,\text{yil}}{0,693} = 7,65\,\text{yil}.$$

Javob: $\tau = 7,65$ yil.

2-misol. Fosforning massasi m=1g bo'lgan radioaktiv izotopida: 1) $t_1 = 1\,\text{min}$; 2) $t_2 = 5$ sutka, – davomida parchalanadigan yadrolar soni aniqlansin.

Berilgan:

$m = 1\text{mg} = 10^{-6}\,\text{kg}$;

$T_{1/2} = 14,3\,\text{sutka} = 1235520\,\text{s}$;

1) $t_1 = 1\,\text{min} = 60\,\text{s}$;

Yechish: t vaqt davomida parchalanadigan yadrolar soni

$$\Delta N = N_0\left(1 - e^{-\lambda t}\right). \qquad (1)$$

ifoda yordamida aniqlanadi.

2) $t_2 = 5\text{sutka} = 432000\text{s}$.	Bu yerda $N_0 - t = 0$ da parchalanmagan yadrolar soni.
$\Delta N = ?$	

$$N_0 = \frac{m}{\mu} \cdot N_A. \qquad (2)$$

Bu yerda: $\mu = 32 \cdot 10^{-3} \dfrac{\text{kg}}{\text{mol}}$; $N_A = 6{,}02 \cdot 10^{23}\,\text{mol}^{-1}$ – Avogadro soni.

Radioaktiv parchalanish doimiysi λ ni

$$T_{1/2} = \frac{0{,}693}{\lambda}$$

ifodadan aniqlaymiz:

$$\lambda = \frac{0{,}693}{T_{1/2}}. \qquad (3)$$

(2) va (3) ni (1) ga qo'yib quyidagini olamiz:

$$\Delta N = \frac{mN_A}{M}\left[1 - \exp\left(-\frac{0{,}693}{T_{1/2}} \cdot t\right)\right]. \qquad (4)$$

Kattaliklarning son qiymatlarini qo'yib hisoblaymiz:

1) $$\Delta N = \frac{10^{-6} \cdot 6{,}02 \cdot 10^{23}}{32 \cdot 10^{-3}}\left[1 - \exp\left(-\frac{0{,}693 \cdot 60}{1235520}\right)\right] = 6{,}3 \cdot 10^{14};$$

2) $$\Delta N = \frac{10^{-6} \cdot 6{,}02 \cdot 10^{23}}{32 \cdot 10^{-3}}\left[1 - \exp\left(-\frac{0{,}693 \cdot 432000}{1235520}\right)\right] = 4{,}04 \cdot 10^{18}.$$

Javob: 1) $6{,}3 \cdot 10^{14}$; 2) $4{,}04 \cdot 10^{18}$.

3-misol. Izotopning aktivligi 10 sutka davomida 20% kamaysa, uning yarim yemirilish davri $T_{1/2}$ aniqlansin.

Berilgan:
$t = 10\text{sutka} = 864000\,\text{s}$;
$\Delta A = 20\%$.

$T_{1/2} = ?$

Yechish: Yarim yemirilish davri

$$T_{1/2} = \frac{\ln 2}{\lambda}, \qquad (1)$$

ifodadan aniqlanadi. Bu yerda λ — radioaktiv yemirilish doimiysi. Uni

$$A = A_0 e^{-\lambda t} \qquad (2)$$

ifodadan aniqlab olamiz: $$\lambda = -\frac{\ln\frac{A}{A_0}}{t} = \frac{\ln\frac{A_0}{A}}{t}. \qquad (3)$$

(3) ni (1) ga qo'ysak, $$T_{1/2} = \frac{t\cdot\ln 2}{\ln\frac{A_0}{A}}. \qquad (4)$$

$\frac{A_0}{A}$ nisbatni quyidagi mulohazalar asosida topamiz.

$$\frac{A}{A_0} = \frac{A_0 - 4A}{A_0} = 1 - \frac{\Delta A}{A_0} = 1 - \frac{20\%}{100\%} = 1 - \frac{1}{5} = \frac{4}{5}.$$

Demak, $$\frac{A_0}{A} = \frac{5}{4}. \qquad (5)$$

(5) dan foydalanib, t ning son qiymati yordamida aniqlaymiz:

$$T_{1/2} = \frac{864000\cdot 0{,}693}{\ln\frac{5}{4}} = 2683260\,\text{s} = 31{,}1\,\text{sutka}$$

Javob: $T_{1/2} = 31{,}1\text{sutka}$.

4-misol. $m = 0{,}4$ mg radioaktiv magniy ^{27}Mg ning boshlang'ich faolligi va 2 soatdan keyingi faolligi aniqlansin. Magniyning barcha atomlari radioaktiv deb olinsin.

Berilgan:

$m = 0{,}4\,\text{mg} = 4 \cdot 10^{-7}\,\text{kg};$

$t_1 = 0;$

$t_2 = 2\,\text{soat} = 7200\,\text{s}.$

$A_0 = ?$

$A = ?$

Yechish: Magniyning boshlang'ich faolligi ($t = 0$) quyidagicha aniqlanadi:

$$A_0 = \lambda N_0. \qquad (1)$$

Radioaktiv yemirilish doimiysi λ quyidagicha aniqlanadi:

$$\lambda = \frac{\ln 2}{T_{1/2}}. \qquad (2)$$

Shuningdek, boshlang'ich momentda ($t = 0$)izotopdagi atomlar soni

$$N_0 = \frac{m}{M} N_A. \qquad (3)$$

Unda (1) quyidagi ko'rinishni oladi: $A_0 = \dfrac{m \cdot N_A}{M \cdot T_{1/2}} \cdot \ln 2.$ (4)

Bu yerda: $M = 27 \cdot 10^{-3}\,\dfrac{\text{kg}}{\text{mol}}$ – magniyning atomar massasi, $N_A = 6{,}02 \cdot 10^{23}\,\text{mol}^{-1}$.

Izotopning faolligi vaqt o'tishi bilan $A = A_0 e^{-\lambda t}$ (5) qonunga muvofiq o'zgaradi.

Agar λ ning o'rniga (2) dagi ifodasini qo'ysak,

$$A = A_0 \exp\left(-\frac{\ln 2}{T_{1/2}} \cdot t\right). \qquad (6)$$

Agar magniy $^{27}_{12}Mg$ uchun $T_{1/2} = 10\,\text{min} = 600\,\text{s}$ ekanligini e'tiborga olsak va kattaliklarning qiymatlarini (4) va (6) larga qo'ysak,

$$A_0 = \frac{4 \cdot 10^{-7} \cdot 6{,}02 \cdot 10^{23} \cdot 0{,}693}{27 \cdot 10^{-3} \cdot 600} \text{Bk} = 1{,}03 \cdot 10^{13} \text{Bk}.$$

$$A = 1{,}03 \cdot 10^{13} \cdot \exp\left(-\frac{0{,}693 \cdot 7200}{600}\right) \text{Bk} = 1{,}03 \cdot 10^{13} e^{-8{,}116} \text{Bk} = .$$

$= 1{,}03 \cdot 10^{13} \cdot 1{,}24 \cdot 10^{2} \text{Bk} = 1{,}25 \cdot 10^{15} \text{Bk}$ olamiz.

Javob: $A_0 = 1{,}03 \cdot 10^{13} \text{Bk}$; $A = 1{,}25 \cdot 10^{15} \text{Bk}$.

5-misol. Agar susayishning chiziqli koeffitsienti 0,047 sm^{-1} bo'lsa, suv uchun γ – nurlar parallel dastasining yarim susaytirish qatlamining qalinligi $x_{1/2}$ aniqlansin.

Berilgan:	**Yechish:** Muhitdan o'tishda γ - nurlarning susayishi
$\mu = 0{,}047 \text{sm}^{-1} = 4{,}7 \text{m}^{-1}$.	$I = I_0 e^{-\mu x}$, (1)
$x_{1/2} = ?$	

ifoda yordamida aniqlanishi ma'lum.

γ -nurlar yarim susaytirish qalinligi $x_{1/2}$ dan o'tganda uning intensivligi I dastlabki intensivlikning yarmiga teng bo'ladi, ya'ni

$$I = \frac{I_0}{2}.$$

Shunday qilib $x = x_{1/2}$ da $I = \frac{I_0}{2}$. Unda (1) quyidagi ko'rinishni oladi $\frac{I_0}{2} = I_0 e^{-\mu x}$, yoki $\frac{1}{2} = e^{-\mu x_{1/2}}$.

Bundan $x_{1/2}$ ni topsak $x_{1/2} = \frac{\ln 2}{\mu}$.

μ ning qiymatidan foydalansak, $x_{1/2} = \frac{0{,}693}{4{,}7} \text{m} = 0{,}147 \text{m}$.

Javob: $x_{1/2} = 0{,}147 \text{m}$.

6-misol. Kosmik nurlar ekvatorda, dengiz sathida 10 s va 1sm³ hajmda 24ta ionlar juftini hosil qiladi. Bir yil davomida odam oladigan ekspozitsion doza aniqlansin.

Berilgan:

$t_1 = 10s;$

$V = 1sm^3 = 10^{-6} m^3;$

$N = 24;$

$t_2 = 1 yil = 365 \cdot 24 \cdot 3600 s.$

$x = ?$

Yechish: Odam oladigan ekspozitsion dozani

$$x = \dot{x} \cdot t_2 \qquad (1)$$

ifoda yordamida aniqlash mumkin. Bu yerda $\dot{x}$ — ekspozitsion nurlanish dozasining quvvati bo'lib u quyidagicha aniqlanishi mumkin:

$$\dot{x} = \frac{\Sigma Q}{m \cdot t_1}, \qquad (2)$$

Bu yerda: ΣQ — bir xil ishorali barcha ionlar elektr zaryadlarining yig'indisi bo'lib, bizning holimizda y e · N ga teng bo'ladi. Shunday qilib

$$x = \frac{e \cdot N}{m \cdot t_1}. \qquad (3)$$

Agar (3) ni (1) ga qo'ysak va $m = \rho \cdot V$ ekanligini nazarda tutsak

$$\left(\rho = 1{,}29 \, {}^{kg}\!/_{m^3}\right) \qquad x = \frac{e \cdot N \cdot t_2}{\rho \cdot V \cdot t_1}. \qquad (4)$$

ni hosil qilamiz.

Berilganlarni (4) ga qo'yib quyidagini topamiz:

$$x = \frac{1{,}6 \cdot 10^{-19} \cdot 24C}{1{,}29 \cdot 10^{-6} \cdot 10kg} = 9{,}41 \cdot 10^{-6} \frac{C}{kg} = 9{,}41 \frac{mkC}{kg}$$

Javob: $x = 9{,}41 \frac{mkC}{kg}$.

8-misol. x=258mkC/kg ekspozitsion dozada normal sharoitdagi barcha havo molekulalarining qancha qismi rentgen nurlari tomonidan ionlashtiriladi?

Berilgan:	**Yechish:** Ekspozitsion dozani
$x = 258\dfrac{\text{mkC}}{\text{kg}}$.	$$x = \frac{\Sigma Q}{m} \qquad (1)$$
$\omega = ?$	ifoda yordamida aniqlaymiz.

Bu yerda: $\Sigma Q = eN_{ion}$ – hosil bo‘lgan bir xil ismli zaryadlarning yig‘inidisi.Massani esa barcha ionlar soni N ning havo molekulasining massasi $M_x$ga ko‘paytmasi sifatida aniqlash mumkin. Ya’ni, $m = T \cdot M_x$.

O‘z navbatida havo molekulasining massasini 1 m³ havo massasini, ya’ni havo zichligi ρ ni, 1 m³ havoda mavjud bo‘lgan molekulalarning soni $n_0$ga, ya’ni molekulalar konsentratsiyasiga nisbati sifatida aniqlash mumkin:

$$M_k = \frac{\rho}{n_0}.$$

Shunday qilib, $$x = \frac{e \cdot N_{ion}}{M_k \cdot N} = \frac{e}{M_k}\omega = \frac{e \cdot \omega \cdot n_0}{\rho}. \qquad (2)$$

Oxirgi ifodadan $\omega = \dfrac{N_{ion}}{N}$ ni aniqlasak, $$\omega = \frac{x \cdot \rho}{e \cdot n_0}. \qquad (3)$$

Bu yerda: $\rho = 1{,}29\dfrac{\text{kg}}{\text{m}^3}$ – havoning zichligi, $e = 1{,}6 \cdot 10^{-19}\,\text{C}$ – elementar zaryad, $n_0 = 2{,}65 \cdot 10^{25}\,\text{m}^{-3}$ – normal sharoitda havo molekulalari konsentratsiyasi.

Kattaliklar yordamida quyidagini topamiz:

$$\omega = \frac{258 \cdot 10^{-6} \cdot 1{,}29}{1{,}6 \cdot 10^{-19}} = 7{,}73 \cdot 10^{-11}.$$

Javob: $\omega = 7{,}73 \cdot 10^{-11}$.

Mustaqil yechish uchun masalalar

1. Radiyning $^{219}_{88}Ra$ va $^{226}_{88}Ra$ izotopning yemirilish doimiysi λ aniqlansin. [700 s^{-1}; 13,6 ps^{-1}.]

2. Bir yil davomida radioaktiv izotopning boshlang'ich miqdori uch marta kamaydi. Ikki yil davomida necha marta kamayadi? [9 marta.]

3. Radioaktiv nuklidning yarim yemirilish davri 1 soat. Bu nuklidning o'rtacha yashash vaqti aniqlansin. [1,44 yil.]

4. Ikkita yarim yemirilish davriga teng vaqtda radioaktiv izotop yadrolarining qancha qismi parchalanishi aniqlansin. [0,75.]

5. $^{210}_{82}Pb$ izotopining radioaktiv yemirilish doimiysi $10^{-9}s^{-1}$ ga teng. Bu izotop boshlang'ich yadrolarining 2/5 qismi qancha vaqtda parchalanishi aniqlansin. [16,2 yil.]

6. Bir sutka davomida izotopning faolligi 118 GBk. dan 7,4 GBk gacha kamaydi. Bu nuklidning yarim yemirilish davri aniqlansin. $\left[\dfrac{69{,}3\cdot 864}{\ln 16}\,\text{yil.}\right]$

7. $^{60}_{27}Co$ izotopining solishtirma faolligi aniqlansin. [40,7 mBk/kg.]

8. $^{238}_{92}U$ izotopining yarim yemirilish davri $4{,}5\cdot 10^9$ yilligi ma'lum bo'lsa, uning solishtirma faolligi aniqlansin. [12,3 mBk/kg.]

9. $^{131}_{53}I$ yod izotopining barcha izotoplari radioaktiv deb hisoblab, uning 1g uchun: 1) boshlang'ich faollik; 2) uch sutkadan keyingi faollik aniqlansin. Izotopning yarim yemirilish davri 3 sutka. [1) 4,61 mBk; 2) 3,55 tBk.]

10. Faolligi 148 GBk bo'lgan nuqtaviy izotrop radioaktiv manbaidan 5 sm masofada $\gamma-$ nurlanishning intensivligi aniqlansin. Har bir yemirilishda o'rtacha 1,8 tadan 0,51 MeV energiyali foton chiqariladi deb hisoblansin. [0,6 W/m^2.]

11. Fotonlarining energiyasi 0,6 MeV bo'lgan $\gamma-$ nurlanishning ingichka dastasini yarim susaytiruvchi beton qatlamining qalinligi aniqlansin. [3,85 sm.]

12 Havo normal sharoitda $\gamma-$ nurlanish bilan nurlantirilmoqda. 258 mkC/kg ekspozitsion nurlanish dozasida 5g havo tomonidan yutiladigan energiya aniqlansin. [8,77 mkJ.]

ILOVA

Fizik kattaliklarning jadvali

1. Falakiyotning ba'zi kattaliklari

Yerning radiusi	$6{,}37 \cdot 10^6$ m
Yerning massasi	$5{,}98 \cdot 10^{24}$ kg
Quyoshning radiusi	$6{,}95 \cdot 10^8$ m
Quyoshning massasi	$1{,}98 \cdot 10^{30}$ kg
Oyning radiusi	$1{,}74 \cdot 10^6$ m
Oyning massasi	$7{,}33 \cdot 10^{22}$ kg
Yerning markazidan quyoshning markazigacha bo'lgan masofa	$1{,}49 \cdot 10^{11}$ m
Yerning markazidan Oyning markazigacha bo'lgan masofa	$3{,}84 \cdot 10^8$ m
Oyning Yer atrofidan aylanish davri	27,3 kecha-kunduz $= 2{,}36 \cdot 10^6$ s

2. Qattiq jismlarning va suyuqliklarning zichliklari (kg/m^3)

Qattiq jismlar

Aluminiy	2700
Temir (cho'yan, po'lat)	7870
Oltin	19300
Osh tuzi	2200
Mis	8930
Nikel	8800
Qo'rg'oshin	11300

Suv(sof 4°C da)	1000
Glitserin	1260
Kerosin	800
Moy	900
Kanakunjut moyi	960
Simob	13600
Uglerod sulfid	1260
Spirt	800

3. Normal sharoitlarda gazlarning zichliklari

Azot	1,25
Vodorod	0,09
Havo	1,29
Geliy	0,18
Kislorod	1,43

4. Qattiq jismlarning elastiklik doimiysi (yaxlitlangan qiymatlar)

Modda	Yung moduli E (GPa)	Siljish moduli (GPa)
Aluminiy	69	24
Temir (po'lat)	200	76
Mis	98	44
Kumush	74	27

5. Normal sharoitda gazlarning ba'zi xarakteristikalari

Modda	Effektiv diametr d (nm)	Dinamik qovushqoqlik η $(mkPa \cdot s)$	Issiqlik O'tkazuvchanlik $\lambda\left(mW/(m \cdot K)\right)$
Azot	0,38	16,6	24,3
Vodorod	0,28	8,66	168
Havo	–	17,2	24,1
Kislorod	0,36	19,8	24,4
Suv bug'lari	–	8,32	15,8

6. Kritik parametrlar va Van-der-Vaals tuzatmalari

Gaz	Kritik harorat T_{kr}(K)	Kritik bosim P_{kr}(MPa)	Van-der-Vaals tuzatmasi	
			$a\left(N \cdot m^4/mol^2\right)$	$b \cdot 10^{-5}\ m^3/mol^2$
Azot	126	3,39	0,135	3,86
Suv bug'i	647	22,1	0,545	3,04
Kislorod	155	5,08	0,136	3,17
Karbonot angidrid	304	7,38	0,361	4,28

7. 20°C da suyuqliklarning dinamik qovushqoqligi $\eta(mPa \cdot s)$

Suv	1,00
Glitserin	1480
Kanakunjut moyi	987
Simob	1,58

8. 20°C da suyuqliklarning sirt tarangligi σ (mN/m)

Suv	73
Glitserin	62
Sovunli suv	40
Simob	$5{,}0 \cdot 10^2$
Spirt	22

9. Dielektrik singdiruvchanlik ε

Suv	81
Moy (transformator moyi)	2,2
Parafin	2,0
Sluda	7,0
Shisha	7,0
Chinni	5,0
Ebonit	3,0

10. O'tkazgichlarning solishtirma qarshiligi ρ va harorat koeffitsiyenti α

Modda	20° C da ρ, $n\Omega\ m$	α, $^{0}C^{-1}$
Temir	98	$6,2 \cdot 10^{-3}$
Miss	17	$4,2 \cdot 10^{-3}$
Aluminiy	26	$3,6 \cdot 10^{-3}$
Grafit	$3,9 \cdot 10^{3}$	$-0,8 \cdot 10^{3}$

11. Sindirish ko'rsatkichi, *n*

Olmos	2,42
Suv	1,33
Dolchin moyi	1,60
Uglerod sulfid	1,63
Shisha	1,50

Izoh: Shishaning sindirish ko'rsatkichi, shishaning naviga va undan o'tayotgan nurlanishning to'lqin uzunligiga bog'liq bo'ladi. Shuning uchun ham bu yerda keltirilgan qiymatni shartli ravishda qabul qilish va undan sindirish ko'rsatkichi masala shartida berilmagan holdagina foydalanish kerak.

12. Elektronlarning metalldan chiqish ishi

Metall	*A*, eM	$A \cdot 10^{-19}, J$
Kaliy	2,2	3,5
Litiy	2,3	3,7
Natriy	2,5	4,0
Platina	6,3	10,1
Kumush	4,7	7,5
Rux	4,0	6,4

13. Neytral atomlarning massalari

Element	Tartib nomeri	Izotopi	Massa a.m.b.
Neytron	0	n	1,00867
Vodorod	1	^{1}H ^{2}H ^{3}H	1,00783 2,01410 3,01605
Geliy	2	^{3}He ^{4}He	3,01603 4,00260
Litiy	3	^{6}Li ^{7}Li	6,01513 7,01601
Berilliy	4	^{7}Be ^{9}Be ^{10}Be	7,01693 9,01219 10,01354
Bor	5	^{9}B ^{10}B ^{11}B	9,01333 10,01294 11,00931
Uglerod	6	^{10}S ^{12}S ^{13}S ^{14}S	10,00168 12,00000 13,00335 14,00324
Azot	7	^{13}N ^{14}N ^{15}N	13,00574 14,00307 15,00011
Kislorod	8	^{16}O ^{17}O ^{18}O	15,99491 16,99913 17,99916
Ftor	9	^{19}F	18,99840
Natriy	11	^{22}Na ^{23}Na	21,99444 22,98977
Magniy	12	^{23}Mg	22,99414
Aluminiy	13	^{30}Al	29,99817
Kremniy	14	^{31}Si	30,97535
Fosfor	15	^{31}P	30,97376
Kaliy	19	^{41}K	40,96184
Kalsiy	20	^{41}Ca	43,95549
Qurg'oshin	82	^{206}Pb	205,97446
Poloniy	84	^{210}Po	209,98297

14. Ba'zi elementar va yengil yadrolarning massalari va tinchlikdagi energiyalari

Zarralar	Massa			
	m_0, kg	m_0, a.m.b.	E_0, J	E_0, MeV
Elektron	$9{,}11\ 10^{-31}$ kg	0,00055	$8{,}16\cdot10^{-14}$	0,511
Neytral	$2{,}41\cdot10^{-38}$ kg	0,14526	–	135
Proton	$1{,}67\cdot10^{-27}$ kg	1,00728	$1{,}50\cdot10^{-10}$	938
Neytron	$1{,}68\ 10^{-27}$ kg	1,00867	$1{,}51\ 10^{-10}$	939
Deytron	$3{,}35\ 10^{-27}$ kg	2,01355	$3{,}00\ 10^{-10}$	1876
α – zarra	$6{,}64\cdot10^{\ 27}$ kg	4,00149	$5{,}96\cdot10^{-10}$	3733

15. Radioaktiv izotoplarning yarim emirilish davrlari

Izotop	Izotopning belgisi	Parchalanish turi	Yarim yemirilish davri
Aktiniy	$^{225}_{89}Ac$	α	10 kecha-kunduz
Yod	$^{131}_{53}I$	β, γ	8 kecha-kunduz
Iridiy	$^{192}_{77}I$	β, γ	75 kecha-kunduz
Kobalt	$^{60}_{27}Co$	β, γ	5,3 y il
Magniy	$^{27}_{12}Mg$	β	10 min
Radiy	$^{219}_{88}Ra$	α	10^{-3}S
Radiy	$^{226}_{88}R$	α, β	$1{,}62\cdot10^{3}$ *yil*
Radon	$^{222}_{86}Rn$	α	3,8 kecha-kunduz
Stronsiy	$^{90}_{38}Sr$	β	28 yil
Toriy	$^{229}_{90}Th$	α, γ	$7\cdot10^{3}$ *yil*
Uran	$^{238}_{92}U$	α, γ	$4{,}5\cdot10^{9}$ *yil*
Fosfor	$^{32}_{15}P$	β	14,3 kecha-kunduz
Natriy	$^{22}_{11}Na$	γ	2,6 yil

16. Asosiy fizik kattaliklar (uchta ahamiyatli raqamgacha yaxlitlangan)

Erkin tushishning normal tezlanishi	$g = 9{,}81\, m/s^2$
Tortishish doimiysi	$G = 6{,}67 \cdot 10^{-11}\, m^3/kg \cdot s^2$
Avogadro doimiysi	$N_A = 6{,}02 \cdot 10^{23}\, mol^{-1}$
Molyar gaz doimiysi	$R = 8{,}31\, J/(K \cdot mol)$
Standart hajm	$V_m = 22{,}4 \cdot 10^{-3}\, m^3/mol$
Bolsman doimiysi	$k = 1{,}38\ 10^{23}\, K$
Faradey doimiysi	$F = 9{,}65 \cdot 10^4\, C/mol$
Elementar zaryad	$e = 1{,}60 \cdot 10^{-19}\, C$
Elektronning massasi	$m_e = 9{,}11 \cdot 10^{-31}\, kg$
Elektronning solishtirma zaryadi	$e/m = 1{,}76 \cdot 10^{11}\, C/kg$
Yorug'likning bo'shliqdagi tezligi	$C = 3{,}00 \cdot 10^8\, m/s$
Stefan-Bolsman doimiysi	$\sigma = 5{,}67 \cdot 10^{-8}\, W/(m^2 \cdot K^4)$
Vin siljish qonunining doimiysi	$S = 2{,}90 \cdot 10^{-3}\, m.k.$
Plank doimiysi	$h = 6{,}63 \cdot 10^{-34}\, J\ s.$ $\hbar = h/2\pi = 1{,}05 \cdot 10^{-34}\, J\ s.$
Ridberg doimiysi	$R' = 1{,}10 \cdot 10^7\, m^{-1} = 3{,}29 \cdot 10^{15}\, s^{-1}$
Birinchi bor orbitasining radiusi	$a = 5{,}29 \cdot 10^{-11}\, m$
Elektronning Kompton to'lqin uzunligi	$\lambda_c = 2{,}43 \cdot 10^{-12}\, m$
Bor magnetoni	$\mu_B = 9{,}27 \cdot 10^{-24}\, J/Tl$
Vodorod atomining ionlashuv energiyasi	$E_i = 2{,}16 \cdot 10^{-18}\, J$
Atom massa birligi	$1\, a.m.b. = 1{,}66 \cdot 10^{-27}\, kg$
Yadro magnetoni	$\mu_N = 5{,}05 \cdot 10^{-27}\, J/Tl$

Muqaddima

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