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IQTISODCHILAR UCHUN MATEMATIKA

Masalalar to‘plami

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Tuzuvchilar S.Turdaxunova, G.Xujaniyazova. Iqtisodchilar uchun matematika. Masalalar to‘plami I qism. TMI, 2017 y. 356 bet.

Ushbu masalalar to‘plami institutning barcha bakalavriat ta’lim yo‘nalishlari talabalari uchun mo‘ljallangan bo‘lib, unga “Iqtisodchilar uchun matematika” fanidan matritsalar, aniqlovchilar, chiziqli tenglamalar sistemasi va ularni yechish usullari, vektorlar sistemasi, chiziqli fazo, kvadratik shakllar, tekislikdagi va fazodagi analitik geometriya elementlari, sonli ketma-ketliklar, bir va ko‘p o‘zgaruvchili funksiya, funksiya limiti va uzluksizligi haqida tushunchalar keltirilgan. Har bir mavzuga oid asosiy tushunchalar, formulalar, iqtisodga bog‘liqligi, namunaviy masalalar yechimlari, qo‘shimcha masalalar keltirilgan.

Masalalar to‘plami Toshkent moliya instituti O‘quv-uslubiy Kengashida muhokama qilingan va nashrga tavsiya etilgan (“__”_______ 2017 yil, qaror №__).

Taqrizchilar:

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I BOB
MATRITSA VA DETERMINANTLAR
1.1. Matritsalar. Texnologik matritsa

Matritsalar ustida quyidagi amallarni bajarish mumkin:

1. *Matritsani songa ko'paytirish* uchun uning barcha elementlari shu songa ko'paytiriladi. $k \neq 0$ son hamda

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ matritsa berilgan bo'lsa,

$Ak = kA = \begin{pmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{pmatrix}$ tenglik o'rinli bo'ladi.

2. O'lchamlari bir hil bo'lgan A va B *matritsalarni qo'shish* uchun mos elementlari qo'shiladi:

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ va $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$ bo'lsa,

$A + B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{pmatrix}$ matritsa hosil bo'ladi.

Bu keltirilgan ikki amal matritsalar ustida *chiziqli amallar* deb yuritiladi.

3. *Matritsalarni ko'paytirish*.

Agar A matritsaning ustunlari soni B matritsaning satrlar soniga teng bo'lsa A ni B ga ko'paytirish mumkin, $n \times m$ o'lchovli $A = (a_{ij})$ matritsani $m \times p$ o'lchovli $B = (b_{jk})$ matritsaga quyidagi formula bo'yicha ko'paytiriladi.

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$$

Hosil bo'lgan $C = (c_{ik})$ matritsa $n \times p$ o'lchamlidir.

4. Matritsani *darajaga oshirish* $A^n = \underbrace{A \cdot A \cdot ... \cdot A}_{nta}$ kabi amalga oshiriladi.

1.1.1. Quyidagi matritsalarning $2A+3B$ chiziqli kombinatsiyasini toping, bu yerda

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix},\ B=\begin{pmatrix} -2 & 3 & 0 \\ 2 & 1 & 1 \end{pmatrix}.$$

Yechish. $2A+3B=2\cdot\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix}+3\cdot\begin{pmatrix} -2 & 3 & 0 \\ 2 & 1 & 1 \end{pmatrix}=\begin{pmatrix} 2 & 4 & 6 \\ 0 & 2 & -2 \end{pmatrix}+$

$$+\begin{pmatrix} -6 & 9 & 0 \\ 6 & 3 & 3 \end{pmatrix}=\begin{pmatrix} 2-6 & 4+9 & 6+0 \\ 0+6 & 2+3 & -2+3 \end{pmatrix}=\begin{pmatrix} -4 & 13 & 6 \\ 6 & 5 & 1 \end{pmatrix}.$$

Berilgan matritsalarning chiziqli kombinatsiyasini toping:

1.1.2. $A-\lambda E,\ A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}.$

1.1.3. $4A-5B,\ A=\begin{pmatrix} 2 & -1 & 0 \\ 3 & 4 & -2 \\ -3 & 1 & 5 \end{pmatrix},\ B=\begin{pmatrix} 3 & 1 & 2 \\ -2 & 1 & 3 \\ 0 & 2 & -4 \end{pmatrix}.$

1.1.4. $A=\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \end{pmatrix}$ va $B=\begin{pmatrix} 3 & 4 & 5 \\ 6 & 0 & -2 \\ 7 & 1 & 8 \end{pmatrix}$ matritsalar berilgan. AB va BA matritsalar ko‘paytmasi (agar mumkin bo‘lsa)ni toping.

Yechish. $AB=\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 3 & 4 & 5 \\ 6 & 0 & -2 \\ 7 & 1 & 8 \end{pmatrix}=$

$$=\begin{pmatrix} 1\cdot 3+2\cdot 6+3\cdot 7 & 1\cdot 4+2\cdot 0+3\cdot 1 & 1\cdot 5+2\cdot(-2)+3\cdot 8 \\ 1\cdot 3+0\cdot 6+(-1)\cdot 7 & 1\cdot 4+0\cdot 0+(-1)\cdot 1 & 1\cdot 5+0\cdot(-2)+(-1)\cdot 8 \end{pmatrix}=$$

$$=\begin{pmatrix} 36 & 7 & 25 \\ -4 & 3 & -3 \end{pmatrix}.$$

BA ko‘paytma mavjud emas, B matritsaning ustunlari soni A matritsaning satrlari soniga mos kelmaydi.

AB va BA matritsalar ko‘paytmasi (agar mumkin bo‘lsa)ni toping:

1.1.5. $A=\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$.

1.1.6. $A=\begin{pmatrix} 4 & 0 & -2 & 3 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 3 \\ 1 \\ -1 \\ 5 \\ 2 \end{pmatrix}$.

1.1.7. $\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}^9$ matritsani toping.

Yechish. $A=\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ belgilash kiritib A matritsaning kvadratini hisoblaymiz: $A^2=\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. $A^2=E$ - birlik matritsa. Shuning uchun $A^9=\left(A^2\right)^4\cdot A=E^4A=A$ va $\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}^9=\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

Matritsalarni hisoblang:

1.1.8. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^5$.

1.1.9. $\begin{pmatrix} 3 & 2 \\ -4 & -2 \end{pmatrix}^5$.

1.1.10. $\begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}^3$.

1.1.11. Agar $f(x)=-2x^2+5x+9$, $A=\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$ bo‘lsa, $f(A)$ matritsali ko‘phadning qiymatini toping.

Yechish. $A^2 = A\cdot A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 1\cdot 1 + 2\cdot 3 & 1\cdot 2 + 2\cdot 0 \\ 3\cdot 1 + 0\cdot 3 & 3\cdot 2 + 0\cdot 0 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 3 & 6 \end{pmatrix},$

$$f(A) = -2A^2 + 5A + 9E = -2\cdot \begin{pmatrix} 7 & 2 \\ 3 & 6 \end{pmatrix} + 5\cdot \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} + 9\cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -14 & -4 \\ -6 & -12 \end{pmatrix} + \begin{pmatrix} 5 & 10 \\ 15 & 0 \end{pmatrix} + \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ 9 & -3 \end{pmatrix}.$$

$f(A)$ matritsali ko‘phadning qiymatini toping:

1.1.12. $f(x) = 3x^3 + x^2 + 2,\ A = \begin{pmatrix} 1 & 5 \\ 0 & -3 \end{pmatrix}.$

1.1.13. $f(x) = 2x^3 - 3x^2 + 5,\ A = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}.$

1.1.14. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ matritsani transponirlang.

Yechish. $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}.$

Quyidagi matritsalarni transponirlang:

1.1.15. $A = \begin{pmatrix} 3 & 0 \\ 2 & -5 \end{pmatrix}.$

1.1.16. $A = \begin{pmatrix} 1 & 0 \\ -3 & 2 \\ 5 & -1 \end{pmatrix}.$

1.1.17. Satrlari ustida elementar almashtirishlar yordamida A matritsani pog‘onasimon ko‘rinishga keltiring:

$$A = \begin{pmatrix} 0 & -1 & -1 & -3 \\ 1 & 2 & 4 & 7 \\ 5 & 0 & 10 & 5 \end{pmatrix}$$

Yechish. $A=\begin{pmatrix}0 & -1 & -1 & -3\\ 1 & 2 & 4 & 7\\ 5 & 0 & 10 & 5\end{pmatrix} I\leftrightarrow II \sim \begin{pmatrix}1 & 2 & 4 & 7\\ 0 & -1 & -1 & -3\\ 5 & 0 & 10 & 5\end{pmatrix} III-5\cdot I\sim$

$$\sim\begin{pmatrix}1 & 2 & 4 & 7\\ 0 & -1 & -1 & -3\\ 0 & -10 & -10 & -30\end{pmatrix} III-10\cdot II\sim$$

$$\sim B=\begin{pmatrix}1 & 2 & 4 & 7\\ 0 & -1 & -1 & -3\\ 0 & 0 & 0 & 0\end{pmatrix} - pog'onasimon\ matritsa.$$

Matritsalarni pog‘onasimon ko‘rinishga keltiring:

1.1.18. $\begin{pmatrix}2 & 3 & -2\\ 3 & 1 & 1\\ 1 & 5 & -5\end{pmatrix}$.

1.1.19. $\begin{pmatrix}1 & -2 & 1 & 11\\ 3 & -1 & 2 & 5\\ 2 & 1 & -3 & -18\\ 5 & 0 & -1 & -13\end{pmatrix}$.

1.1.20. Korxona 3 xil mahsulot ishlab chiqarish uchun 2 xil xomashyodan foydalanadi. Xomashyo xarajatlari $A=\begin{pmatrix}2 & 3\\ 5 & 2\\ 1 & 4\end{pmatrix}$ matritsa bilan berilgan. Mahsulot ishlab chiqarish rejasi $C=\begin{pmatrix}100 & 80 & 130\end{pmatrix}$ - satr-matritsa ko‘rinishida berilgan. Har bir xomashyo turining bir birligi bahosi (pul.birl.) $B=\begin{pmatrix}30\\ 50\end{pmatrix}$ - ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan xomashyo miqdorini va xomashyoning umumiy bahosini aniqlang.

Yechish. 1-usul. Har bir xomashyo sarfi

$$S=C\cdot A=\begin{pmatrix}100 & 80 & 130\end{pmatrix}\cdot\begin{pmatrix}2 & 3\\ 5 & 2\\ 1 & 4\end{pmatrix}=\begin{pmatrix}730 & 980\end{pmatrix}$$

bo‘lsa, xomashyoning umumiy bahosi

$$Q = S \cdot B = (C \cdot A) \cdot B = \begin{pmatrix} 730 & 980 \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 50 \end{pmatrix} = \begin{pmatrix} 70900 \end{pmatrix}$$

bo‘ladi.

2-usul. Avval har bir mahsulot turiga sarflanuvchi xomashyo miqdori

$$R = A \cdot B = \begin{pmatrix} 2 & 3 \\ 5 & 2 \\ 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 50 \end{pmatrix} = \begin{pmatrix} 210 \\ 250 \\ 230 \end{pmatrix}$$

So‘ngra, xom ashyoning umumiy bahosini aniqlaymiz

$$Q = C \cdot R = \begin{pmatrix} 100 & 80 & 130 \end{pmatrix} \cdot \begin{pmatrix} 210 \\ 250 \\ 230 \end{pmatrix} = \begin{pmatrix} 70900 \end{pmatrix}$$

Quyidagi iqtisodiy mazmundagi masalani yeching:

1.1.21. Kopxona 3 xil mahsulot ishlab chiqarish uchun 2 xil xomashyodan foydalanadi. Xomashyo harajatlari $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 3 & 4 \end{pmatrix}$ matritsa bilan berilgan. Maxsulot ishlab chiqarish rejasi $C = \begin{pmatrix} 100 & 200 & 150 \end{pmatrix}$ – satr-matritsa ko‘rinishida berilgan. Har bir xomashyo turining bir birligi bahosi (pul.birl.) $B = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$ – ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan xomashyo miqdorini va xomashyoning umumiy bahosini aniqlang.

Qo‘shimcha masalalar

Berilgan matritsalarning chiziqli kombinatsiyasini toping:

1.1.22. $3A+4B,\ A=\begin{pmatrix} 7 & -2 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 3 & 2 & 0 \end{pmatrix},\ B=\begin{pmatrix} 2 & -1 & -3 & 1 \\ 7 & -1 & 0 & 4 \\ 8 & -2 & 1 & 5 \end{pmatrix}.$

1.1.23. $2A-B,\ A=\begin{pmatrix} 1 & 5 \\ 2 & -4 \end{pmatrix},\ B=\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}.$

1.1.24. $3A-2B,\ A=\begin{pmatrix} 1 & -1 & -3 \\ 2 & 1 & 5 \end{pmatrix},B=\begin{pmatrix} 0 & 3 & 2 \\ -1 & 4 & 1 \end{pmatrix}.$

1.1.25. $2A+5B,\ A=\begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix},\ B=\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}.$

AB va BA matritsalar ko‘paytmasi (agar mumkin bo‘lsa)ni toping:

1.1.26. $A=\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix},\ B=\begin{pmatrix} 2 & 6 \\ -1 & -3 \end{pmatrix}.$

1.1.27. $A=\begin{pmatrix} 1 & 2 & 3 \end{pmatrix},\ B=\begin{pmatrix} 4 & -3 \\ 1 & 2 \\ 0 & 2 \end{pmatrix}.$

1.1.28. $A=\begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},\ B=\begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}.$

1.1.29. $A=\begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 4 \\ -4 & 5 & 1 \end{pmatrix},\ B=\begin{pmatrix} 3 & 4 & 1 \\ 0 & 2 & 5 \\ 1 & -1 & 4 \end{pmatrix}.$

1.1.30. $A=\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 5 \end{pmatrix},\ B=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$

1.1.31. $A=\begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix},\ B=\begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 0 \end{pmatrix}.$

1.1.32. $A=\begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 2 \\ 4 & -1 & 5 \end{pmatrix},\ B=\begin{pmatrix} -3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{pmatrix}.$

1.1.33. $A=\begin{pmatrix} 1 & -3 & 0 \\ 2 & 5 & 1 \end{pmatrix},\ B=\begin{pmatrix} 0 & -1 & 3 \\ 3 & 5 & 2 \\ 4 & -2 & 1 \end{pmatrix}.$

1.1.34. $A=\begin{pmatrix} 1 & -3 & 2 \\ 3 & -4 & 1 \\ 2 & -5 & 3 \end{pmatrix},\ B=\begin{pmatrix} 2 & 5 & 6 \\ 1 & 2 & 5 \\ 1 & 3 & 2 \end{pmatrix}.$

1.1.35. $A=\begin{pmatrix} 2 & -1 & 3 & -4 \\ 3 & -2 & 4 & -3 \\ 5 & -3 & -2 & 1 \\ 3 & -3 & -1 & 2 \end{pmatrix},\ B=\begin{pmatrix} 7 & 8 & 6 & 9 \\ 5 & 7 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 2 & 1 & 1 & 2 \end{pmatrix}.$

1.1.36. $A=\begin{pmatrix} 5 & 7 & -3 & -4 \\ 7 & 6 & -4 & -5 \\ 6 & 4 & -3 & -2 \\ 8 & 5 & -6 & -1 \end{pmatrix},\ B=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{pmatrix}.$

$f(A)$ matritsali ko‘phadning qiymatini toping:

1.1.37. $f(x)=3x^2-5x+2,\ A=\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & -1 \\ -2 & 1 & 4 \end{pmatrix}.$

1.1.38. $f(x)=x^3-6x^2+9x+4,\ A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 1 & 4 \end{pmatrix}.$

1.1.39. $f(x)=x^2+x+1,\ A=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$

Matritsalarni pog‘onasimon ko‘rinishga keltiring:

1.1.40. $\begin{pmatrix} 2 & 3 & -2 & 3 \\ 3 & 1 & 1 & 2 \\ 1 & 5 & -5 & 4 \end{pmatrix}$.

1.1.41. $\begin{pmatrix} 1 & -3 & 1 & 13 \\ 3 & 1 & -7 & 9 \\ -1 & 2 & 0 & -10 \\ 2 & 1 & -5 & 5 \end{pmatrix}$.

Quyidagi iqtisodiy mazmundagi masalalarni yeching:

1.1.42. Kopxona 3 xil mahsulot ishlab chiqarish uchun 2 xil xomashyodan foydalanadi. Xomashyo harajatlari $A = \begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 5 & 4 \end{pmatrix}$ matritsa bilan berilgan . Maxsulot ishlab chiqarish rejasi $C = \begin{pmatrix} 150 & 120 & 100 \end{pmatrix}$ – satr-matritsa ko‘rinishida berilgan. Har bir xomashyo turining bir birligi bahosi (pul.birl.) $B = \begin{pmatrix} 40 \\ 60 \end{pmatrix}$ – ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan xomashyo miqdorini va xomashyoning umumiy bahosini aniqlang.

1.1.43. Kopxona 4 xil mahsulot ishlab chiqarish uchun 2 xil xomashyodan foydalanadi. Xomashyo harajatlari $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 4 \\ 3 & 2 \end{pmatrix}$ matritsa bilan berilgan. Mahsulot ishlab chiqarish rejasi $C = \begin{pmatrix} 120 & 80 & 150 & 130 \end{pmatrix}$ – satr-matritsa ko‘rinishida berilgan. Har bir xomashyo turining bir birligi bahosi (pul.birl.) $B = \begin{pmatrix} 80 \\ 60 \end{pmatrix}$ – ustun-matritsa ko‘rinishida berilgan. Rejani bajarish uchun sarflanadigan xomashyo miqdorini va xomashyoning umumiy bahosini aniqlang.

1.2. Determinantlar nazariyasi

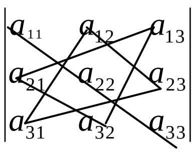
$a_{11}, a_{12}, a_{21}, a_{22}$ haqiqiy sonlar berilgan bo‘lsin ikkinchi tartibli determinant (yoki aniqlovchi) deb, $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ kabi belgilanuvchi va

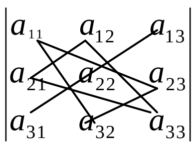
$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$ tenglik bilan aniqlanuvchi songa aytiladi.

Berilgan $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}$ haqiqiy sonlardan tuzilgan $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$ yig‘indiga teng va

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ kabi berilgan songa uchinchi tartibli determinant deb ataladi. Uchinchi tartibli determinantlarni uchburchaklar usulida, Sarryus usulida hamda biror satr (ustun) elementlari bo‘yicha yoyib hisoblash mumkin.

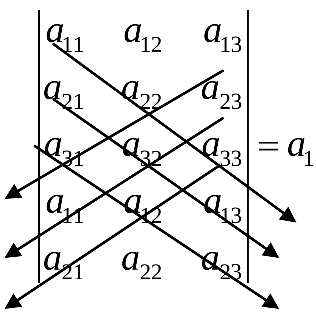
1. Uchburchaklar usuli:

(+) 

(-) 

$$a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

2. Sarryus usuli:



$$= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{33}a_{12}a_{21}$$

3. Birinchi ustun elementlari bo‘yicha yoyib hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

n - tartibli determinantning a_{ij} elementi minori M_{ij} deb i -satr va j -ustunni o’chirishdan hosil bo’lgan $(n-1)$ -tartibli determinantga aytiladi.

n - tartibli determinantning a_{ij} elementi algebraik to‘ldiruvchisi $A_{ij} = (-1)^{i+j} M_{ij}$ formula bo‘yicha hisoblanadi.

Determinantning ixtiyoriy satr yoki ustun elementlarining o‘z algebraik to‘ldiruvchilariga ko‘paytmalarining yig‘indisi uning kattaligiga teng degan xossaga ko‘ra, har qanday determinantni ixtiyoriy satr (ustun) bo‘yicha yoyib yozish mumkin.

Determinantning asosiy xossalari yordamida yuqori tartibli determinantlar quyi tartibli determinantga keltiriladi.

1.2.1. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ ikkinchi tartibli determinantni hisoblang:

Yechish. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1\cdot 4 - 2\cdot 3 = -2.$

Ikkinchi tartibli determinantni hisoblang:

1.2.2. $\begin{vmatrix} -7 & 6 \\ 5 & -4 \end{vmatrix}.$

1.2.3. $\begin{vmatrix} 10 & -5 \\ 9 & -8 \end{vmatrix}.$

Tenglamani yeching:

1.2.4. $\begin{vmatrix} 2x+1 & 3 \\ x+5 & 2 \end{vmatrix} = 0.$

1.2.5. $\begin{vmatrix} x+3 & x-1 \\ 7-x & x-1 \end{vmatrix} = 0.$

1.2.6. Uchinchi tartibli determinantni hisoblang: $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$.

Yechish. Determinantni birinchi satr elementlari bo‘yicha yoyib hisoblaymiz:

$$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = 3\cdot\begin{vmatrix} 5 & 3 \\ 4 & 2 \end{vmatrix} - 2\cdot\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} + 1\cdot\begin{vmatrix} 2 & 5 \\ 3 & 4 \end{vmatrix} =$$

$$= 3\cdot(5\cdot 2 - 3\cdot 4) - 2\cdot(2\cdot 2 - 3\cdot 3) + 1\cdot(2\cdot 4 - 5\cdot 3) =$$

$$= 3\cdot(-2) - 2\cdot(-5) + 1\cdot(-7) = -3.$$

Uchinchi tartibli determinantlarni ixtiyoriy satr (ustun) elementlari bo‘yicha yoyib hisoblang:

1.2.7. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$.

1.2.8. $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$.

1.2.9. Uchinchi tartibli determinantni uchburchak qoidasidan foydalanib hisoblang: $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 5 & -6 \\ 7 & 8 & 9 \end{vmatrix}$.

Yechish. $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 5 & -6 \\ 7 & 8 & 9 \end{vmatrix} = 1\cdot 5\cdot 9 + 2\cdot(-6)\cdot 7 + (-4)\cdot 3\cdot 8 -$

$$-3\cdot 5\cdot 7 - (-4)\cdot 2\cdot 9 - 1\cdot(-6)\cdot 7 = 45 - 84 - 96 - 105 + 72 + 42 = -126.$$

Uchburchak qoidasidan foydalanib determinantlarni hisoblang:

1.2.10. $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{vmatrix}.$

1.2.11. $\begin{vmatrix} 0 & x & 0 \\ y & 0 & 0 \\ 0 & 0 & z \end{vmatrix}.$

1.2.12. $\begin{vmatrix} 0 & 1 & 0 \\ 2 & 3 & 4 \\ 0 & 5 & 0 \end{vmatrix}.$

1.2.13. Determinantning xossalaridan foydalanib tenglikni isbotlang:

$$\begin{vmatrix} a_1 & b_1 & c_1 + a_1x + b_1y \\ a_2 & b_2 & c_2 + a_2x + b_2y \\ a_3 & b_3 & c_3 + a_3x + b_3y \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.$$

Chap determinantning **2.1.** uchunchi ustunini uchta ustun yig‘indisi ko‘rinishida ifodalash mumkin, bu determinantni uchta determinant yig‘indisi ko‘rinishida ifodalaymiz: $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & a_1x \\ a_2 & b_2 & a_2x \\ a_3 & b_3 & a_3x \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & b_1y \\ a_2 & b_2 & b_2y \\ a_3 & b_3 & b_3y \end{vmatrix}.$

Ikkinchi determinantning uchunchi ustuni birinchi ustuniga proporsional, uchunchi determinantning uchunchi ustuni ikkinchi ustuniga proporsional. Shuning uchun ikkinchi va uchinchi determinantlar nolga teng.

Tenglikni isbotlang:

1.2.14. $\begin{vmatrix} a_1 + b_1x & a_1 - b_1x & c_1 \\ a_2 + b_2x & a_2 - b_2x & c_2 \\ a_3 + b_3x & a_3 - b_3x & c_3 \end{vmatrix} = -2x \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.$

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

1.2.15. To‘rtinchi tartibli determinantni hisoblang:

$$\Delta = \begin{vmatrix} a & 0 & 3 & 5 \\ 0 & 0 & b & 2 \\ 1 & c & 2 & 3 \\ 0 & 0 & 0 & d \end{vmatrix}.$$

Yechish. Determinantni to‘rtinchi satr elementlari bo‘yicha yoyib hisoblaymiz:

$$\Delta = (+d) \cdot \begin{vmatrix} a & 0 & 3 \\ 0 & 0 & b \\ 1 & c & 2 \end{vmatrix} = \begin{bmatrix} \det er \min antni \\ 2 - satr\, bo'yicha\; yoyamiz \end{bmatrix} =$$

$$= d \cdot (-b) \cdot \begin{vmatrix} a & 0 \\ 1 & c \end{vmatrix} = -d \cdot b \cdot a \cdot c.$$

Satr yoki ustun elementlari bo‘yicha yoyish orqali determinantlarni hisoblang:

1.2.16. $\begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \end{vmatrix}.$

1.2.17. $\begin{vmatrix} 1 & 2 & 0 & -3 \\ 3 & 1 & 0 & 4 \\ 1 & 5 & -1 & 7 \\ -2 & 1 & 0 & 1 \end{vmatrix}.$

Qo‘shimcha masalalar

Ikkinchi tartibli determinantni hisoblang:

1.2.18. $\begin{vmatrix} \sqrt{a}+\sqrt{b} & \sqrt{a}-\sqrt{b} \\ \sqrt{a}-\sqrt{b} & \sqrt{a}+\sqrt{b} \end{vmatrix}.$

1.2.20. $\begin{vmatrix} \frac{x+y}{x} & \frac{2x}{x-y} \\ \frac{y-x}{x^2-y^2} & \frac{y-x}{x^2-y^2} \end{vmatrix}.$

1.2.19. $\begin{vmatrix} \sin 1^0 & \sin 89^0 \\ -\cos 1^0 & \cos 89^0 \end{vmatrix}.$

1.2.21. $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha \\ \sin^2\beta & \cos^2\beta \end{vmatrix}.$

1.2.22. $\begin{vmatrix} \sqrt{5}-a^{\frac{1}{2}} & a^{\frac{1}{2}} \\ -a^{\frac{1}{2}} & \sqrt{5}+a^{\frac{1}{2}} \end{vmatrix}.$

1.2.23. $\begin{vmatrix} \sin 60^0 & \cos 45^0 \\ \sin 45^0 & tg30^0 \end{vmatrix}.$

1.2.24. $\begin{vmatrix} tga & -1 \\ 4 & ctga \end{vmatrix}.$

1.2.25. $\begin{vmatrix} 1,(3) & 2,25 \\ 23/3 & 6 \end{vmatrix}.$

1.2.26. $\begin{vmatrix} \dfrac{a-1}{2\sqrt{a}} & \dfrac{a+\sqrt{a}}{\sqrt{a}-1} \\ \dfrac{a\sqrt{a}-\sqrt{a}}{2a} & \dfrac{a-\sqrt{a}}{\sqrt{a}+1} \end{vmatrix}.$

Tenglamani yeching:

1.2.27. $\begin{vmatrix} 2x-1 & x+1 \\ x+2 & x-1 \end{vmatrix} = -6.$

1.2.28. $\begin{vmatrix} x-2 & y+3 \\ -y-3 & x-2 \end{vmatrix} = 0.$

1.2.29. $\begin{vmatrix} \sin 2x & \sin x \\ \cos x & \cos 2x \end{vmatrix} = 0.$

Uchinchi tartibli determinantlarni ixtiyoriy satr (ustun) elementlari bo‘yicha yoyib hisoblang:

1.2.30. $\begin{vmatrix} 3 & 2 & -1 \\ -2 & 2 & 3 \\ 4 & 2 & -3 \end{vmatrix}.$

1.2.31. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}.$

1.2.32. $\begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 9 \\ 16 & 25 & 81 \end{vmatrix}.$

Uchinchi tartibli determinantlarni qulay usulda hisoblang:

1.2.33. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}.$

1.2.34. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}.$

1.2.35. $\begin{vmatrix} 1 & 2 & 3 \\ 8 & 1 & 4 \\ 2 & 1 & 1 \end{vmatrix}.$

1.2.36. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}.$

1.2.37. $\begin{vmatrix} 3 & -1 & -2 \\ 1 & 2 & 5 \\ -4 & 1 & 6 \end{vmatrix}$.

1.2.38. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}$.

1.2.39. $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$.

1.2.40. $\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix}$.

1.2.41. $\begin{vmatrix} \cos\alpha & \sin\alpha\cos\beta & \sin\alpha\sin\beta \\ -\sin\alpha & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ 0 & -\sin\beta & \cos\beta \end{vmatrix}$.

1.2.42. $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$.

1.2.43. $\begin{vmatrix} m+a & m-a & a \\ n+a & 2n-a & a \\ a & -a & a \end{vmatrix}$.

1.2.44. $\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix}$.

1.2.45. $\begin{vmatrix} \sin 3\alpha & \cos 3\alpha & 1 \\ \sin 2\alpha & \cos 2\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \end{vmatrix}$.

1.2.46. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

1.2.47. $\begin{vmatrix} a & x & x \\ x & b & x \\ x & x & c \end{vmatrix}$.

Tenglama va tengsizliklarni yeching:

1.2.48. $\begin{vmatrix} 2 & 0 & 3 \\ -1 & 7 & x-3 \\ 5 & -3 & 6 \end{vmatrix} = 0.$

1.2.50. $\begin{vmatrix} -1 & 0 & 2x+3 \\ 3-x & 1 & 1 \\ 2x+1 & -1 & 2 \end{vmatrix} = 0.$

1.2.49. $\begin{vmatrix} -1 & 3 & -2 \\ 2-3x & 0 & 5 \\ 3 & 2 & 1 \end{vmatrix} \geq 0.$

1.2.51. $\begin{vmatrix} 6 & 3 & x-1 \\ 2x & 1 & 0 \\ 4 & x+2 & 2 \end{vmatrix} = 0.$

Tengliklarni isbotlang:

1.2.52. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (b-a)(c-a)(c-b).$

1.2.53. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-a)(c-a)(c-b).$

1.2.54. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}.$

1.2.55. $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}.$

1.2.56. $\begin{vmatrix} (a+b)^2 & c^2 & c^2 \\ a^2 & (b+c)^2 & a^2 \\ b^2 & b^2 & (c+a)^2 \end{vmatrix} = 2bc(a+b+c)^3.$

Satr yoki ustun elementlari bo‘yicha yoyish orqali determinantlarni hisoblang:

1.2.57. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -9 & -9 & -9 & -9 \\ 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 0 \end{vmatrix}.$

1.2.58. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ -2 & -4 & -6 & 1 \end{vmatrix}.$

1.2.59. $\begin{vmatrix} 1 & 2 & -3 & 1 \\ 3 & 0 & 1 & -1 \\ 2 & 0 & 4 & 1 \\ 5 & 1 & 2 & 1 \end{vmatrix}$.

1.2.60. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$.

1.2.61. $\begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}$.

1.2.62. $\begin{vmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \end{vmatrix}$.

1.2.63. $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$.

1.2.64. $\begin{vmatrix} 3 & -1 & 2 & -1 & 1 \\ 5 & 1 & -2 & 1 & 2 \\ 9 & -1 & 1 & 3 & 4 \\ 3 & 0 & 6 & -1 & 3 \\ 5 & 2 & 3 & -2 & 1 \end{vmatrix}$.

1.2.65. $\begin{vmatrix} 0 & 6 & 3 & 5 & 1 \\ -3 & 2 & 4 & 1 & 0 \\ 5 & 1 & 4 & 3 & 2 \\ -3 & 8 & 7 & 6 & 1 \\ 1 & 0 & 3 & 4 & 0 \end{vmatrix}$.

1.3. Matritsa rangi. Teskari matrisa

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \qquad (1)$$

A matritsaning rangi deb noldan farqli minorlarning eng yuqori tartibiga aytiladi va $rang\left(A\right);r(A)$ kabi ifodalanadi. $r(A)\leq \min(m,n)$, bunda m – matritsa satrlari soni, n – matritsaning ustunlari soni.

Matritsa rangi ikki usulda topiladi:

1. Matritsa rangi ta’rifga asoslangan “minorlar ajratish” usuli;

2. Matritsa ustun va satrlarida nollar yig‘ib hisoblashga asoslangan “Gauss algoritmi”.

A matritsa uchun teskari matritsa 2 usulda topiladi:

1. Klassik usuli;
2. Jordan usuli.

Klassik usulda teskari matritsa $A^{(-1)}=1/|A|\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$ (2)

formula bo‘yicha hisoblanadi. Bu yerda $\left|A\right|$ berilgan matritsa determinanti.

$\left(A|E\right)\sim\left(E|A^{-1}\right)$ - Jordan usuli algoritmi.

Berilgan matritsani birlik matritsa hisobida kengaytirib, elementar almashtirishlar bajaramiz, bu usulni to chap tomonda A matritsa o‘rnida birlik matritsa hosil bo‘lguncha davom ettiramiz, o‘ng tomonda hosil bo‘lgan matritsa berilgan matritsaga nisbatan teskari matritsa bo‘ladi.

1.3.1. Matritlmsa rangini ta'rifga asosan hisoblang: $A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$.

A matritsa 3×5 o'lchamli, demak uning rangi 3 dan yuqori bo'lmaydi. Uchinchi tartibli minorlarni hisoblaymiz:

$$M_1=\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix}=-4-10-12+12+4+10=0;$$

$$M_2=\begin{vmatrix} 2 & -1 & -2 \\ 4 & -2 & 1 \\ 2 & -1 & 8 \end{vmatrix}=-32-2+8-8+32+2=0;$$

$$M_3=\begin{vmatrix} 2 & -1 & 4 \\ 4 & -2 & 7 \\ 2 & -1 & 2 \end{vmatrix}=-8-14-16+16+8+14=0;$$

$$M_4=\begin{vmatrix} -1 & 3 & -2 \\ -2 & 5 & 1 \\ -1 & 1 & 8 \end{vmatrix}=-40-3+4-10+48+1=0;$$

$$M_5=\begin{vmatrix} 3 & -2 & 4 \\ 5 & 1 & 7 \\ 1 & 8 & 2 \end{vmatrix}=6-14+160-4+20-168=0;...$$

Barcha uchinchi tartibli minorlar nolga teng. Ikkinchi tartibli minorlarni hisoblaymiz: $M_1^1=\begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix}=-5+6=1$ $\quad M_1^1\neq0,\quad r(A)=2.$

Bu usulda noldan farqli minor topilgunga qadar hisoblashlar davom etadi. Shuning uchun 3 va undan kattaroq tartibli matritsa rangini hisoblash birmuncha qiyinchiliklarga olib keladi.

1.3.2. Matritsa rangini elementar almashtirishlar yordamida nollar yig'ib hisoblang: $A=\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix}$

Yechish:

$$A=\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix} \sim \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 1 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Bu matritsaning rangi $\begin{pmatrix} 25 & 31 & 17 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ matritsa rangiga teng.

$$\begin{vmatrix} 25 & 31 & 17 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 25 \neq 0 \qquad\qquad r\begin{pmatrix} 25 & 31 & 17 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = 3$$

Demak, berilgan matritsaning rangi ham 3 ga teng. $r(A)=3.$

1.3.3. Berilgan kvadrat matritsaning rangini toping. Xosmas matritsaning teskarisini toping:

$a)\, A=\begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix};$ $\qquad b)\, B=\begin{pmatrix} 2 & 5 \\ -4 & 2 \end{pmatrix};$

1.3.4. $A=\begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix}$ matritsa uchun teskari A^{-1} matritsani klassik usulda toping.

Yechish. $A_{ij}\left(i=1,2,3;\ j=1,2,3\right)$ A matritsa elementlarining algebraik to‘ldiruvchilari. $|A|=\begin{vmatrix}2 & 3 & 2\\ 5 & 1 & 4\\ 1 & -2 & -1\end{vmatrix}=-2+12-20-2+15+16=43-24=19\neq 0$

Demak A xosmas matritsa, va A^{-1} teskari matritsa mavjud. Algebraik to‘ldiruvchilarni hisoblaymiz:

$A_{11}=\begin{vmatrix}1 & 4\\ -2 & -1\end{vmatrix}=-1+8=7;$ $A_{21}=-\begin{vmatrix}3 & 2\\ -2 & -1\end{vmatrix}=-\left(-3+4\right)=-1;$

$A_{31}=\begin{vmatrix}3 & 2\\ 1 & 4\end{vmatrix}=12-2=10;$ $A_{12}=-\begin{vmatrix}5 & 4\\ 1 & -1\end{vmatrix}=-\left(-5-4\right)=9;$

$A_{22}=\begin{vmatrix}2 & 2\\ 1 & -1\end{vmatrix}=-2-2=-4;$ $A_{32}=-\begin{vmatrix}2 & 2\\ 5 & 4\end{vmatrix}=-\left(8-10\right)=2;$

$A_{13}=\begin{vmatrix}5 & 1\\ 1 & -2\end{vmatrix}=-10-1=-11;$ $A_{23}=-\begin{vmatrix}2 & 3\\ 1 & -2\end{vmatrix}=-\left(-4-3\right)=7;$

$A_{33}=\begin{vmatrix}2 & 3\\ 5 & 1\end{vmatrix}=2-15=-13;$ topilganlarni (2) formulaga qo‘yamiz va teskari $A^{-1}=1/19\begin{pmatrix}7 & -1 & 10\\ 9 & -4 & 2\\ -11 & 7 & -13\end{pmatrix}$ matritsani olamiz.

Teskari matritsaning to‘griligini tekshirish uchun quyidagi tenglikni tekshiramiz:

$$AA^{-1}=A^{-1}A=E \qquad (3)$$

$$\begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix} \cdot 1/19 \begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix} =$$

$$= 1/19 \cdot \begin{pmatrix} 14+27-22 & -2-12+14 & 20+6-26 \\ 35+9-44 & -5-4+28 & 50+2-52 \\ 7-18+11 & -1+8-7 & 10-4+13 \end{pmatrix} =$$

$$= 1/19 \cdot \begin{pmatrix} 19 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 19 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$$

Demak, A^{-1} to‘g‘ri topilgan.

1.3.5. $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 4 & 3 & -2 \end{pmatrix}$ matritsa uchun A^{-1} matritsani Gauss-Jordan usulida toping.

Yechish: $|A| = -16 \neq 0$ teskari matritsa mavjud. Berilgan matritsani birlik matritsa hisobida kengaytirib, elementar almashtirishlar bajaramiz, bu usulni to chap tomonda A matritsa o‘rnida birlik matritsa hosil bo‘lguncha davom ettiramiz, o‘ng tomonda hosil bo‘lgan matritsa berilgan matritsaga nisbatan teskari matritsa bo‘ladi.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ -1 & -1 & -3 & 0 & 1 & 0 \\ 4 & 3 & -2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & -5 & -6 & -4 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & -16 & 1 & 5 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 5 & -1 & -2 & 0 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -11/16 & -7/16 & 5/16 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right)$$

$A^{-1} = -1/16\begin{pmatrix} 11 & 7 & -5 \\ -14 & -6 & 2 \\ 1 & 5 & 1 \end{pmatrix}$ teskari matritsa to‘g‘ri topilganini (3) formulaga qo‘yib tekshiramiz:

$$AA^{-1} = -1/16\begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 4 & 3 & -2 \end{pmatrix}\cdot\begin{pmatrix} 11 & 7 & -5 \\ -14 & -6 & 2 \\ 1 & 5 & 1 \end{pmatrix} =$$

$$= -1/16\begin{pmatrix} 11-28+1 & 7-12+5 & -5+4+1 \\ -11+14-3 & -7+6-15 & 5-2-3 \\ 44-42-2 & 28-18-10 & -20+6-2 \end{pmatrix} =$$

$$= -1/16\begin{pmatrix} -16 & 0 & 0 \\ 0 & -16 & 0 \\ 0 & 0 & -16 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ demak, teskari matritsa to‘g’ri topilgan.

1.3.6. Berilgan kvadrat matritsalar uchun teskari matritsani ikki usulda toping:

$a)\begin{pmatrix} -1 & 1 \\ 4 & 2 \end{pmatrix}$; $\qquad b)\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$.

1.3.7. Berilgan kvadrat matritsalar uchun teskari matritsani qulay usulda toping:

$a)\begin{pmatrix} 1 & 5 & 7 \\ 3 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$; $\qquad b)\begin{pmatrix} 2 & -1 & 7 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{pmatrix}$;

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1.3.8. Berilgan kvadrat matritsalarning rangini toping. Xosmas matritsaning teskarisini toping:

$a)\begin{pmatrix} -1 & 0 & 8 \\ 5 & 9 & 0 \\ 0 & 4 & 3 \end{pmatrix}$; $\qquad b)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}$; $\qquad c)\begin{pmatrix} 1 & 0 & 5 \\ 4 & -2 & -1 \\ 2 & 1 & 3 \end{pmatrix}$

$d)\begin{pmatrix} 2 & 3 & 4 & 0 \\ 1 & 5 & 7 & 0 \\ 3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};\qquad e)\begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \end{pmatrix}$

1.3.9. Quyidagi matritsalar rangini minorlar ajratish usuli bilan hisoblang:

$a)\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix};\quad b)\begin{pmatrix} 1 & 2 & 3 & 6 \\ 2 & 3 & 1 & 6 \\ 3 & 1 & 2 & 6 \end{pmatrix};\quad c)\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix};\quad d)\begin{pmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \end{pmatrix}$

$e)\begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix};\quad j)\begin{pmatrix} 1 & 2 & 1 & 3 \\ 4 & -1 & -5 & -6 \\ 1 & -3 & -4 & -7 \\ 2 & 1 & -1 & 0 \end{pmatrix};\quad k)\begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$

Misollarda matritsalar rangini elementar almashtirish usuli bilan hisoblang:

1.3.10. $\begin{pmatrix} 1 & 2 & 1 & 3 & 4 \\ 3 & 4 & 2 & 6 & 8 \\ 1 & 2 & 1 & 8 & 4 \end{pmatrix};$

1.3.11. $\begin{pmatrix} 1 & 7 & 5 & 8 & 9 & 2 \\ 3 & 21 & 15 & 24 & 27 & 6 \\ 2 & 14 & 10 & 16 & 18 & 4 \end{pmatrix};$

1.3.12. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix};$

1.3.13. $\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \end{pmatrix};$

1.3.14. $\begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix};$

1.3.15. $\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix};$

1.3.16. $\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{pmatrix};$

1.3.17. $\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 6 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$;

1.3.18. $\begin{pmatrix} 4 & 5 & 2 & 1 & -3 \\ 0 & 2 & 1 & 1 & 2 \\ 4 & 7 & 3 & 3 & -1 \\ 8 & 12 & 5 & 3 & -4 \end{pmatrix}$;

1.3.19. $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$;

1.3.20. $\begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix}$.

1.3.21. Berilgan kvadrat matritsalar uchun teskari matritsani ikki usulda toping:

$$a)\begin{pmatrix} tg\alpha & 1 \\ 2 & ctg\alpha \end{pmatrix}; \qquad b)\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 2 \end{pmatrix};$$

$$c)\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}; \qquad d)\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix};$$

$$e)\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}.$$

1.3.22. Berilgan kvadrat matritsalar uchun teskari matritsani qulay usulda toping:

$$a)\begin{pmatrix} 1 & 0 & -2 \\ 3 & 1 & 0 \\ -1 & 2 & 4 \end{pmatrix}; \qquad b)\begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix};$$

$$c)\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}; \qquad d)\begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ a & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & a & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & a & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & a & 1 \end{pmatrix};$$

$$e)\begin{pmatrix} 1 & a & a^2 & a^3 & \dots & a^n \\ 0 & 1 & a & a^2 & \dots & a^{n-1} \\ 0 & 0 & 1 & a & \dots & a^{n-2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}.$$